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THE

PHYSICAL REVIEW

*A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS*

CONDUCTED WITH THE CO-OPERATION
OF THE
AMERICAN PHYSICAL SOCIETY
BY
EDWARD L. NICHOLS, ERNEST MERRITT,
AND FREDERICK BEDELL

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Volume XXIV. *January, 1907.* *Number 1.*

THE

PHYSICAL REVIEW.

ON THE COEFFICENT OF EXPANSION OF FUSED QUARTZ.

BY HOWARD D. MINCHIN.

THE increasing use of fused quartz in physical and chemical work and especially in optical problems renders the accurate knowledge of its physical properties very desirable. Its extremely small coefficient of expansion adapts it to investigations in which sudden changes of temperature are unavoidable and even greatly to be desired, while its transparency and high fusing point render it peculiarly suitable for the optical treatment of all problems involving wide ranges of temperature. The study of the thermal expansion of fused quartz has formed the subject of numerous recent investigations among which may be mentioned those of Le Chatelier,[1] Callendar,[2] and Holborn and Henning.[3]

Le Chatelier used a differential method, having two prisms of the same length, one of fused quartz and the other of porcelain. The latter was used as the standard, its coefficient of expansion having been determined by direct measurement. The two prisms were placed in a crucible of plumbago set in a bed of carbon which was heated to incandescence, and the expansions were measured by a cathetometer reading to 0.01 mm. The temperature was determined by means of an air thermometer. The mean coefficient of expansion of the fused quartz was found to be 70×10^{-8} between the temperatures of 0° and 1000° C. A contraction was reported as taking place at about 800° C.

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[1] Comp. Rendus, 130, p. 1703, 1900.

[2] Chem. News, 83, p. 151, 1901.

[3] Ann. der Physik, 10, p. 446, 1903.

Callendar used a rod of fused quartz about 40 cm. long, clamped at one end, and measured the expansion by means of a micrometer microscope reading to 0.001 mm., focussed upon the free end. The rod was heated by placing it in a platinum tube carrying an electric current. The expansion of the platinum tube was determined and the temperature to which the quartz was heated was computed from the coefficient of expansion of platinum. The mean coefficient of expansion of the fused quartz between room temperature and 1000° C. was found to be 59×10^{-8}. The expansion was found to be regular up to 1000° but from 1000° to 1400° C. it increased rapidly, and beyond 1400° a contraction took place.

Holborn and Henning used a rod of the material about 52 cm. long and observed the change in length by means of telescopes focussed upon marks cut near the ends of the rod. The rod was heated electrically[1] by placing it in a porcelain tube 80 cm. long mounted horizontally. Temperature measurements were made by means of a thermoelement, and the temperature of the rod was determined at nine different points throughout its length. Observations were made at room temperature, at 250°, 500°, 750°, and 1000° C. As a mean of fourteen determinations 54×10^{-8} is given as the coefficient of expansion.

The marked discrepancy in the results thus far obtained seemed to justify a redetermination of this important constant, with more delicate apparatus for measuring the expansion and especial devices for maintaining the temperature of the expanding solid both uniform and constant.

METHOD.

The method used in this investigation for measuring the coefficient of expansion is briefly as follows: A ring of the fused quartz about 1 cm. long is placed in an electrically heated oven and the expansion is determined by the displacement of interference fringes. The higher temperatures are determined by means of a thermoelement and the lower temperatures by means of mercury thermometers. The method of determining the expansion by the displacement of interference fringes is a modification of the method first employed by Fizeau.

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[1] Ann. der Physik, 2, p. 505, 1900.

The fused quartz system (Fig. 1) is arranged for the measurement of the expansion in the manner first devised by Pulfrich[1] and used by Reimerdes[2] and later by Randall.[3] The bed-plate *A* is of fused quartz 38.4 mm. in diameter and 10 mm. thick, and with its two faces ground approximately parallel. Its upper surface is plane polished, while the lower surface is left rough and ground in such a way as to leave three equidistant feet so that when the plate is mounted upon a level surface its upper surface is horizontal. The piece of fused quartz *Q*, whose coefficient of expansion was to be studied, is in the form of a ring approximately 9.9 mm. long, 31.8 mm. in external diameter, and 24.4 mm. internal diameter. This ring was made from a circular plate whose two surfaces have been ground and polished accurately plane parallel. From the center of this plate was cut a cylinder 24.4 mm. in diameter. Each end has been ground away so as to leave three equidistant feet as shown in the figure. The cover-plate *B*, also of fused quartz, is 7.5 mm. thick and 36.0 mm. in diameter. Its surfaces are plane polished and inclined to each other at an angle of about 20 minutes in order to avoid the disturbing effect due to the light reflected at the upper surface. A small plate *C*, not shown in the figure, made from the same material as the other pieces is approximately 19.8 mm. in diameter and about 0.2 mm. thick. Its upper surface is plane polished and its lower surface is rough and ground away so as to leave three feet similar to the arrangement of the bed-plate.

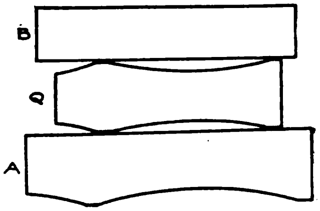

Fig. 1.

In arranging the parts preparatory to making a series of observations the plate *C* is placed upon the bed-plate *A* and the ring *Q* is then placed upon *A* symmetrically about *C*. The cover-plate *B* is next placed upon the ring *Q*. One of the feet of *C* was ground off sufficiently to make the angle included between C and the lower surface of B of the proper size to give interference fringes of the desired width. For the purpose of measurement the cover-plate B has at the center of its lower surface a small fine circle and around this a number of concentric circles. The inner circle serves as a fixed point in the field of view from which measurements are made. Upon the upper surface of B is a fine mark cut parallel to the thin edge of the plate. This mark is to enable the operator to place the parts in the same relative position, each time the system is set up.

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[1] Zeitschr. für Krystall., 31, p. 372, 1899.
[2] Inaug. Diss., Jena, 1896.
[3] Phys. Rev., 20, p. 10, 1905.

The fused quartz parts were made from material produced in the smelting works of the firm of Carl Zeiss, Jena, and the ring and optical surfaces were also made by Zeiss.

When the apparatus is clean and properly adjusted and illuminated by monochromatic light the eye will see the surface of the plate C crossed by a system of parallel interference fringes. When the illumination is produced by light of different colors a system of rainbow colored fringes can be observed. Each dark band of the fringe system represents a definite thickness of the air layer between B and C. Let λ be the wave-length of the light used, then the thickness of the air layer under any dark band differs from the thickness under the next dark band on either side by $\frac{1}{2}\lambda$. When the fused quartz system is heated the ring Q expands and the plates B and C are separated and the interference fringes move toward the thin edge of the included air layer. The number of dark bands passing under the small reference circle on B represents the change in the thickness of the air layer immediately under this circle and thus measures the expansion of the ring Q in terms of the wave-length of the light employed. The total expansion for any temperature interval $(t_2 - t_1)$ is $f \times \lambda/2$, where f is the number of dark bands passing under the reference circle for a temperature change from t_1 to t_2, and λ is the wave-length of light used. We then have the coefficient of expansion a, given by the equation

$$a = \frac{f\frac{\lambda}{2}}{L(t_2 - t_1)},$$

where L is the effective length of the fused quartz expanding, that is, the length of the ring *Q*, minus the length of the small plate *C*.

The spectral lines used for measurements were the mercury green ($\lambda = 546\ \mu\mu$) and the yellow lines of mercury ($\lambda = 578.8\ \mu\mu$ and $\lambda = 576.8\ \mu\mu$). The hydrogen red ($\lambda = 656.2\ \mu\mu$) was also used for some measurements but the intensity of illumination was not satisfactory and its use was abandoned.

APPARATUS.

The interferometer employed was an Abbé-Fizeau instrument as modified and improved by Pulfrich, and constructed by Carl Zeiss, Jena. The same instrument was used by Randall,[1] who has given a full description with detailed directions for its manipulation in connection with his investigation upon crystalline quartz. As a source of light the ordinary H-form of Geissler tube containing hydrogen and vapor of mercury was used.

THE SUPPORTING SYSTEM.

To avoid the presence of metal in the furnace the support for the quartz system was constructed as follows: Two rods *FF* (Fig. 2), of Berlin porcelain, about 50 cm. long, are clamped to a metal block *E*, by means of metal straps and held in position by screws. These rods carry at their lower ends a porcelain plate *M*. The block *E*, is made tapering in order to admit the interferometer tube between *FF*, and also as an aid in holding *M* in place. The rods *FF*, are notched at their lower ends to fit into slots cut in *M* as shown in the figure. This holds *M* sufficiently rigid to furnish a stable support for the optical system.

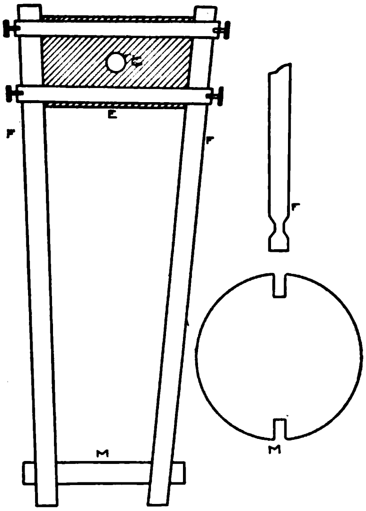

Fig. 2.

To mount the support a threaded hole *G* (Fig. 2), is made in the center of *E*, into which is screwed a thirteen millimeter steel rod.

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[1] Loc. cit.

This rod is held by a right angled clamp to a larger rod attached rigidly to the pier. This method of mounting gives to the support two adjustments for levelling. It may be revolved on the rod attached to E in a plane at right angles to the axis of the interferometer tube, and also around the second rod in the plane of the axis of the interferometer tube. This gives sufficient freedom of movement to permit of accurate adjustment of the interference system.

THE ELECTRIC FURNACE.

The furnace consists of an unglazed porcelain battery jar 14.6 cm. inside length and 6.9 cm. inside diameter. A thin sheet of chemically pure asbestos paper was wound around this cylinder and then upon this two coils of rolled platinum wire were wound with the strands about 2 mm. apart. Fourteen feet of number 20 platinum wire was used. This was rolled so that its width was about twice its original diameter when its length was about 28 feet. The coils were wound separately so that either coil may be used alone, both coils may be used in parallel, or the two coils may be used in series. The plan of winding and the connections are shown in cross-section in Fig. 3. The resistance of each coil at room temperature is about 2.5 ohms.

The coils were covered with about a half inch coating of ganister made into a mortar. This material is a mixture of six parts of white sand and one part of clay ground together very fine. This was then covered by several layers of asbestos paper with small strips of asbestos between so as to keep the layers separated from each other for the purpose of lessening the radiation. The asbestos winding was carried about two inches above the porcelain cylinder. The whole was then packed in mineral wool, in a sheet-iron cylinder and mounted upon a table capable of being raised or lowered by

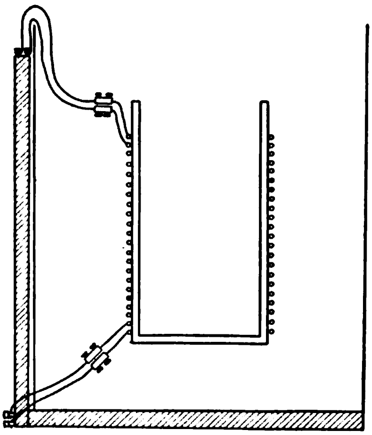

Fig. 3.

means of a screw actuated by a crank. This table was sufficiently rigid to prevent any lateral displacement of the oven, or jarring of the optical parts.

The cover was a porcelain plate similar to *M*, and split through the middle so that it could be put in place when the supporting system was adjusted in the furnace. Openings were made in it for the thermometers and thermoelements and also to admit the light. The arrangement of the cover is shown in Fig. 4, where *A* is the central opening about one inch in diameter to admit the light, *bb* are openings for the thermometers or thermoelements, and *cc* are the openings for the supporting rods *FF*.

In order to prevent radiation through *A* and yet not cut off the light a series of air chambers was built up. The opening *A* was covered with a thin sheet of mica, then a collar or ring of glass, cut from a glass tube about one inch in diameter, was placed on the mica and over the opening, this ring was covered by a second sheet of mica, another glass collar added and another covering of mica. In this way four air chambers were built up, which served as an excellent check to radiation and did not noticeably diminish the illumination of the fringe system. A cross-section of the arrangement of the air chambers is shown in Fig. 5, where *C* is the cover, *M* the mica sheets and *r* the glass collars.

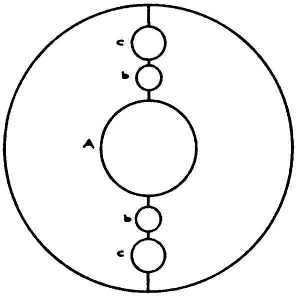

Fig. 4.

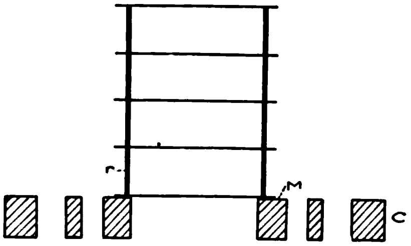

Fig. 5.

The effectiveness of the heat insulation was shown by the fact that the sheet-iron cylinder in which the furnace was packed showed but little rise in temperature when the furnace was heated up to 1000° C. Two thermometers were hung above the furnace, one close to the supports *FF* and within a few centimeters of the interior of the furnace, near the air chambers above *A*, the other just outside of the asbestos packing, and these showed temperatures but a few degrees above that of the room in each case. The small mass of the furnace renders it possible to heat up very quickly and at the same time allows the temperature of the interior to follow very closely every change in the heating current. The temperature can be held constant, to the full limit of the accuracy of the temperature measurements, for hours at a time. A current of 5 amperes is sufficient to heat up to 1000° C. in about two hours. In order to test the uniformity of the temperature within the heated space a series of temperature determinations was made in the different regions of the furnace. Two thermoelements were used and so adjusted that the temperatures of two separate regions of the furnace were determined at the same time. The temperature was found to be constant to within 0°.5 throughout the whole interior, no variation being detected until the one element was brought within a centimeter of the top of the furnace and this variation disappeared when the cover and insulating material had been properly adjusted.

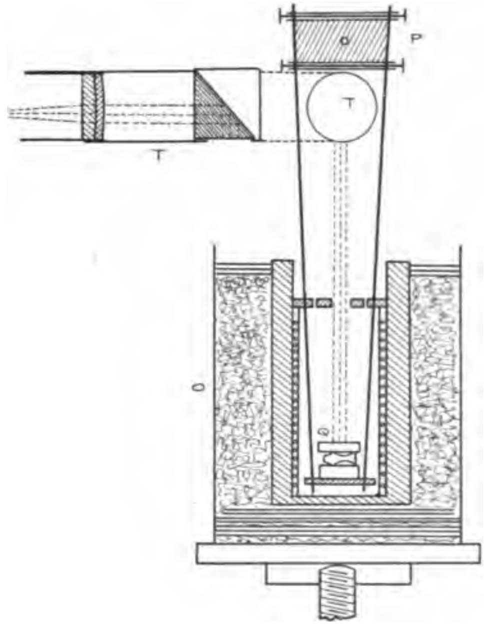

Fig. 6.

The apparatus was set up in a basement room of the Physical Laboratory of the University of Michigan. A brick pier served as a support for the interferometer and the supporting system. The electric oven and other devices were mounted upon the asphalt floor. In Fig. 6 is shown a vertical section of the end of the interferometer tube *T*, the electric oven *O*, the supporting system *P* and the quartz system *Q* when adjusted for work.

During the first part of the investigation the direct current, at 220 volts pressure, from the university lighting plant was used. Later it became necessary to use current from the storage battery for heating. This battery gave but 70 volts, yet owing to the low resistance of the coils of the furnace sufficient current was available for heating up to 1000° C. or more, while the extreme steadiness of the current proved to be a great advantage when it was necessary to keep the temperature constant at any point above the temperature of the room.

Table I. gives an idea of the performance of the furnace and its condition as regards uniformity and efficiency for the various temperature intervals worked over.

Table I.

Temperature Interval Worked Over.	Mean Current Used to Heat Up.	Time Required to Heat Up.		Current Required to Hold Temperature Constant.	Time During Which the Temperature was Held Constant.	
	Amperes.	Hours.	Min.	Amperes.	Hours.	Min.
Room temperature to 300°	3.0	1	05	2.4 to 2.5	2	20
" " " 400	3.0	1	10	3.2	2	30
" " " 500	3.5	1	20	3.3 to 3.4	2	50
" " " 600	5.0	1	16	3.6 to 3.8	2	00
" " " 700	5.0	1	30	3.9	2	45
" " " 800	5.0	1	50	4.6 to 4.8	2	20
" " " 900	5.0	2	00	4.5 to 4.6	2	00

In calibrating the thermoelements mentioned above, the furnace was heated up to 1350° C. and it was later heated up to 1200° C. on a trial test for the time required to heat up. For this test 5.5 amperes were used and the time required to reach 1200° C. was two hours and ten minutes.

THERMOMETERS.

Four thermometers were used. For room temperatures a Peters and Rost thermometer reading to 60° C. and calibrated in the physical laboratory was used. For temperatures up to 200° C. two standard thermometers, made by Haak of Jena, were used, the one graduated from — 20° C. to 105° C., the other from 90° C. to 210° C. These were calibrated at the Reichsanstalt. The fourth thermometer is graduated in degrees, having a short scale at 0° C. and after an enlargement of the bore the graduation extends from 195° C. to 550° C. This thermometer was also calibrated at the Reichsanstalt. For temperatures between 300° C. and 1000° C. a platinum platinum-rhodium thermoelement was used. This element was certified by the Reichsanstalt. As a check on the latter a platinum platinum-iridium thermoelement, made by myself and calibrated in the physical laboratory, was used. As a further check both these thermoelements were compared with a platinum platinum-rhodium element, certified by the Reichsanstalt, and belonging to Professor E. D. Campbell, director of the chemical laboratory.

In measuring the temperature by means of the thermoelement the method used was the compensation method described by Lindeck and Rothe.[1] The apparatus was made by Siemens and Halske according to specifications of the Reichsanstalt.

THE SPHEROMETER.

The spherometer used in measuring the fused quartz ring and plates was one made by the Geneva Society and reads to 0.001 mm. The dimensions of the different parts of the fused quartz used in the computations were obtained by taking the mean of several series of readings. The length of Q is the mean of thirty readings, and the length of C is the mean of twenty-four readings.

ADJUSTMENT OF THE APPARATUS.

The adjustment of the interferometer and the Geissler tube was made according to the method described by Randall.[2] The support

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[1] Zeitschr. für Instr., 20, p. 285, 1900.

[2] Loc. cit.

P (Figs. 2 and 6) was mounted independently upon rods clamped to the pier and so arranged that it could be readily adjusted for the proper level. The thermoelement *K* (Fig. 7), was fastened to one of the supporting rods *F*, so that it passed through one of the openings of the cover of the furnace. Its junction *J*, was placed within about two millimeters of the ring *Q*, and between *A* and *B* as indicated in the figure.

In the adjustment of the system great care was necessary to avoid the presence of dust or lint between the surfaces at points where contact was desired, therefore each piece was thoroughly cleaned before being set in place. Plate *A* was placed centrally upon *M* of the support, *C* was placed in the center of *A*, the ring *Q* was then placed symmetrically around *C*, and the plate *B* placed on *Q* with the circles down. The support was then levelled and the optical system adjusted until interference fringes were obtained. The different pieces were marked to aid in placing them in the same relative position at each mounting of the system. When the fringes were erect and sufficiently bright readings were made upon two adjacent dark bands to determine the width of the fringes. To insure against the presence of dust or foreign particles between the surfaces in contact, the fused quartz system was taken down and remounted several times, the width of the fringes being determined each time. When several successive determinations gave the same band width it was assumed that the apparatus was in proper adjustment for work. The reference circle was then adjusted so that it was in the center of the field in order that any rectification of the fringes should still leave them symmetrically placed in the field.

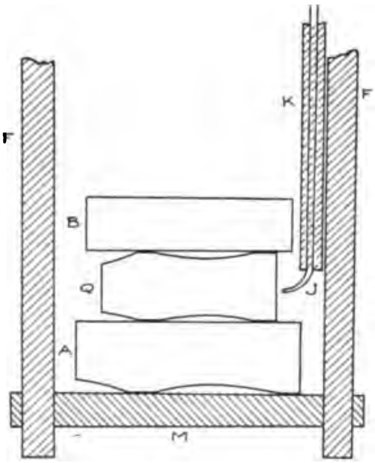

Fig. 7.

The furnace was then raised until the fused quartz system was near the bottom and symmetrically placed in the interior, the cover

C, was then put in place, the air chambers built over the opening *A*, and the space around these chambers packed with asbestos up to the asbestos winding on the furnace. For the lower temperatures a thermometer was introduced in the second opening *b*, and for the higher temperatures the second thermoelement was used in a manner similar to the one already described. The apparatus was allowed to stand over night in order to allow it to acquire a stationary temperature. The position of the fringes was read and the temperature and barometric pressure determined. The current was then turned on and the system heated up to the desired temperature. The current was then adjusted so as to maintain this temperature constant and micrometer readings were made upon some one of the dark bands in the field of view. The temperature was kept constant and at intervals of about 20 minutes the position of the dark band selected was again determined. After the temperature had been held constant for about an hour the position of the band did not change. This indicated that the system had acquired a stationary temperature. The final readings on the bands were taken after the temperature had been held constant for a period of not less than two hours. In all micrometer readings made upon the fringes a mean of ten or more readings was taken. During the heating of the quartz the number of dark bands passing the reference circle were counted. When the readings at the desired temperature had been made the current was shut off and the system was allowed to cool to room temperature and another set of readings made. Thus for each temperature interval worked over, two separate determinations were made, one from the lower temperature to the higher and a second from the higher to the lower temperature. In this way separate determinations were made over the temperature intervals between room temperature and each of the temperatures 200°, 300°, 400°, 500°, 600°, 700°, 800°, 900° and 950° C. After each of the series of readings 1, 2, 5 and 6 given in Table II., the quartz was removed from the furnace, taken down, remounted and the furnace again adjusted for the next series of readings. The quartz pieces were all carefully measured after the heating was completed and their lengths were found to be the same as at the beginning.

Two methods were used for determining the number of fringes displaced, the one by direct observation, the other by computation.

By means of the micrometer attached to the eye-piece of the interferometer the exact position of the bands was determined and then the number of whole bands passing the reference circle was counted while the system was being heated up to the desired temperature. To this whole number was added the fraction of a band by which the last dark band had passed the reference circle, and from this was subtracted the fraction of a band by which the first dark band was past when the counting began. The total fringe displacement was then given by the formula

$$f = M + \partial_{t_2} - \partial_{t_1},$$

where M represents the number of whole bands observed and ∂_{t_2} and ∂_{t_1} the fractions of a band at temperatures t_2 and t_1. These quantities were determined from the micrometer readings. The heating and the expansion were so regular that once having determined the time of passage of a dark band it was an easy matter to check up the result.

The method by computation is that first described by Abbé.[1]

As a change in the temperature and pressure results in a change in the index of refraction of the air layer between the reflecting surfaces, a correction was made upon the observed number of fringes displaced. The correction was computed by the following formula given by Pulfrich:[2]

$$K = d(t_2 - t_1)\frac{b_1}{760}\cdot\frac{1}{1 + at_1}\cdot\frac{1}{1 + at_2}\left[2\frac{(N-1)a}{\lambda}\right] + d(b_2 - b_1)\frac{1}{1 + at_2}\left[2\frac{N-1}{\lambda}\cdot\frac{1}{760}\right]$$

in which K represents the correction, d the thickness of the air layer $= Q - C$ (Fig. 1), b_1 the barometric pressure observed at the lower temperature t_1, b_2 the barometric pressure at the higher temperature t_2, a the coefficient of expansion of air, N the index of refraction of air under standard conditions and λ the wave length of light used.

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[1] Wied. Ann., 38, p. 473, 1889.

[2] Zeitschr. für Instr., 13, p. 455, 1893. While the relation here assumed to exist between the index of refraction of a gas and its temperature and pressure, is known to hold for temperatures up to 100° C., no data, so far as is known to the writer, exist for the behavior of a gas at such temperatures as are reached in this work. An investigation of the validity of this correction for temperatures between 100° and 900° C., is being made by Dr. Randall.

THE DEGREE OF ACCURACY.

The formula for the coefficient of expansion

$$a = \frac{f\frac{\lambda}{2}}{L(t_2 - t_1)}.$$

shows that a is dependent upon the variables f, L, t_2, and t_1. In the expression $f = M + \partial_{t_2} - \partial_{t_1}$ the ∂'s could be determined with an error affecting f by ± 0.02 of a fringe and the resulting error in the value of a for the larger temperature intervals is $\pm 0.1 \times 10^{-8}$ and for the smaller temperature intervals it is $\pm 0.5 \times 10^{-8}$. The length of L could be determined to within ± 0.001 mm. and the error in a due to this error is 0.01×10^{-8}. The temperature measurements made with the thermometers could be determined with an error of ± 0.1° C. With the thermoelement the temperature could be determined with an error of ± 0°.5 at the lower temperatures and ± 2° at the higher temperatures. The error in a resulting from the measurement of the temperature intervals is $\pm 0.1 \times 10^{-8}$ and $\pm 1.1 \times 10^{-8}$ respectively. The total effect of these errors upon the final result is given by the equation

$$E = \sqrt{E_f^2 + E_L^2 + E_t^2}$$

where E_f, E_L, and E_t represent the error in the measurement of the number of bands, the length of the ring, and the temperature interval, respectively. For the shorter temperature intervals at the higher temperatures, this amounts to 1.6×10^{-8} and for the longer intervals it is 0.5×10^{-8}.

The series of readings in Table II. were made for the temperature intervals indicated.

After the above series of readings were made the apparatus was dismantled and the quartz system, the furnace, and the thermoele-

TABLE II.

Date.	Series.	Temperature Interval Worked Over.	
		(1)	(2)
1905, Sept. 2	1	Room temperature to 200°	Back to room temperature.
" 7		" " " 300	" " " "
" 8		" " " 400	" " " "
" 9		" " " 500	" " " "
" 10		" " " 600	" " " "
" 11		" " " 700	" " " "
" 12		" " " 800	" " " "
" 13		" " " 900	" " " "
" 13		" " " 950	" " " "
" 14	2	" " " 600	" " " "
" 14	3	" " " 800	" " " "
" 15	4	" " " 300	" " " "
" 15		300° " " 400	
" 15		400 " " 500	
" 15		500 " " 600	
" 15		600 " " 700	Back to room temperature.
" 16	5	Room temperature to 800°	
" 16		800° " " 900	
" 16		900 " " 950	

ments were taken to the University of Rochester where the apparatus was again set up. A new Abbé-Fizeau interferometer belonging to the University of Rochester was used, and for measuring the temperature an Otto Wollf potentiometer was used in connection with the thermoelements in place of the compensation device of Siemens and Halske. The series of readings given in Table III. were then made.

TABLE III.

Date.	Series.	Temperature Interval Worked Over.	
1905.		(1)	(2
Nov. 8	6	Room temperature to 300°	Back to room temperature.
" 10		" " " 500	" " "
" 11		" " " 700	" " "
" 22		" " " 900	" " "
Dec. 5	7	" " " 200	
" 18	8	200° to 400	
" 19		400 to 600	
1906.			
Jan. 16		600 to 700	
" 18		700 to 800	

EXAMPLE OF A COMPLETE DETERMINATION.

September 7, 1905.

Length of fused quartz ring, 9.9009 mm.

Length of fused quartz cylinder, 1.9496 mm.

Effective length of fused quartz expanding, 7.9513 mm.

Thickness of air layer, 7.9513 mm.

Computation for one set of readings:

$t_1 = 18.4$ C. , $b_1 = 742.0$ mm.

$t_2 = 300°$ C. , $b_2 = 742.6$ mm.

$\delta_{t_1} = 0.2736$, $\delta_{t_2} = 0.3981$ mm.

M observed $= 3$, $\lambda = 0.000546$ mm.

$f = 3 + 0.398 - 0.274 = 3.124.$

$K = 0.385.$

$f_0 = 3.124 + 0.385 = 3.509.$

$f_0 =$ number of bands when correction is applied.

$$a = \frac{f\lambda/2}{L(t_2 - t_1)} = 43 \times 10^{-8}.$$

RESULTS.

Table IV. contains the data obtained in the physical laboratory of the University of Michigan. Table V. contains the data obtained in the physical laboratory of the University of Rochester.

TABLE IV.

Series.	Temperature Interval Worked Over.			
	Lower to Higher.	a	Higher to Lower.	a
1	Room temperature to 200°	43.2×10^{-8}	Back to room temperature.	43.3×10^{-8}
	" " " 300	43.0 "	" " " "	43.2 "
	" " " 400	44.0 "	" " " "	44.5 "
	" " " 500	46.0 "	" " " "	45.7 "
	" " " 600	44.0 "	" " " "	44.8 "
	" " " 700	44.0 "	" " " "	44.1 "
	" " " 800	46.0 "	" " " "	45.8 "
	" " " 900	43.0 "	" " " "	44.0 "
	" " " 950	44.0 "	" " " "	44.2 "
2	" " " 600	46.0 "	" " " "	45.7 "
3	" " " 800	46.0 "	" " " "	45.8 "
4	" " " 300	44.0 "		
	300° " " 400	44.0 "		
	400 " " 500	46.0 "		
	500 " " 600	45.0 "		
	600 " " 700	45.0 "		
5	Room temperature " 800	46.0 "		
	800° " " 900	44.0 "		
	900 " " 950	45.0 "		

TABLE V.

Series.	**Temperature Interval Worked Over.**			
	Lower to Higher.	α	**Higher to Lower.**	α
6	Room temperature to 200°	44.0×10^{-8}	Back to room temperature.	44.5×10^{-8}
	" " " 500	46.0 "	" " " "	45.4 "
	" " " 700	44.8 "	" " " "	45.2 "
	" " " 900	45.0 "	" " " "	45.1 "
	" " " 200	43.0 "		
7	200° " " 400	45.0 "		
8	400 " " 600	44.5 "		
	600 " " 700	45.0 "		
	700 " " 800	46.0 "		

CONDITIONS FOR OBSERVATIONS.

At about 400° C. the interior of the furnace began to appear bright and at about 500° C. it gave a reddish tint. This light, however, did not interfere with the readings upon the interference fringes. Between 700° C. and 800° C. a band of reddish light emitted by the fused quartz appeared in the telescope of the interferometer in the neighborhood of the hydrogen red of the Geissler tube. At about 825° C. this light became so bright that it annulled the hydrogen red in the field, and above 950° C. the intensity of the light from the fused quartz was so great that further readings were impossible. The colors used in the interference phenomenon were the yellow and the green lines of mercury, and up to 950° C. the brilliancy and sharpness of the fringes due to these lines were as marked as at the lower temperatures. Even at this high temperature there was no wandering or quivering of the fringes, and readings could be taken as readily as at the lower temperatures. This was due to the freedom from air currents and to the perfect insulation of the furnace. In so far as the writer is aware interference fringes have not been previously used for measurements at temperatures above 500° C.

In the construction of the interferometer the light is analyzed by a direct vision prism train in the eye-piece, and it is possible that some modification of this set of prisms might render it possible to extend the temperature limit at which the fringes could be read to 1100° C. or 1200° C. The use of some element in the Geissler tube which would give brilliant lines in the blue or violet would certainly extend this range.

Comparison of Results with Those of Other Observers.

The results obtained in the investigations referred to in the introduction are given below together with that obtained by the writer.

	α
Le Chatelier	50×10^{-8},
Callendar	59×10^{-8},
Holborn and Henning	54×10^{-8},
Minchin	45×10^{-8}.

It is seen that the result obtained by the writer is considerably below the other results. Le Chatelier in his work used the differential method, using porcelain as his standard. He found certain irregularities in the expansion of the fused quartz, but Callendar thinks it probable that this was due to the irregularities inherent in the expansion of porcelain. Shenstone[1] says the fused quartz used by Le Chatelier was probably impure, and Le Chatelier himself expresses doubt as to its purity.

Callendar placed a rod of the fused quartz in a platinum tube and clamped both at one end and read the expansion by means of a micrometer microscope reading to 0.001 mm. His method of determining temperatures was, to say the least, indirect and liable to error.

The value $\alpha = 54 \times 10^{-8}$, given by Messrs. Holborn and Henning as the result of their measurements, was a mean value obtained from several different ranges of temperature. The following table contains, in the first two columns the data given by Holborn and Henning,[2] to which has been added in the third column the corresponding value of α computed by the writer.

The effective length of the fused quartz expanding was 486.9 mm.

As will be noted from the table, the individual values of α vary considerably, and while several of these values are comparable to the result obtained in this investigation, still the differences between individual values of the series are sufficiently large to warrant some consideration of the method employed.

The method used by Messrs. Holborn and Henning, while offering the seeming advantage of a great length of the expanding substance, was weak in that it failed to secure uniform temperature throughout the length of the rod, the temperature of the ends being ten per cent. lower than that of the middle.[1] It was also found necessary to correct for the expansion of the supports of the reading microscopes and it seems possible that the method employed might involve other errors due to expansion, which were not entirely eliminated by the foregoing correction. In discussing the rather wide variation between their individual results, they suggest that this may possibly be due to the irregular form of the fused quartz rod under examination.[2] By means of their apparatus Messrs. Holborn and Henning were able to measure elongations to 0.001 mm., which in a rod of 50 cm. length would amount to 1 part in 500,000.

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[1] Nature, 64, p. 65, 1901.

[2] Ann. der Physik, 10, p. 447, 1903.

Table VI.

Temperature Interval Worked Over.	Expansion for Interval in μ.	α	Temperature Interval Worked Over.	Expansion for Interval in μ.	α
17° to 250°	54.0	47.6×10^{-8}	500° to 750°	68.0	55.9×10^{-8}
	57.5	50.7 "		62.0	50.9 "
				60.0	49.3 "
	54.0	47.6 "		62.5	51.3 "
	52.0	45.8 "	16° to 750°	192.5	53.9 "
250° to 500°	79.5	65.3 "	750° to 1000°	66.0	54.2 "
	74.0	60.8 "		61.0	50.1 "
	77.0	63.3 "			

The method and the apparatus employed in the present investigation seem to justify confidence in the results obtained for the following important reasons.

(*a*) *Uniformity of Temperature Throughout the Expanding Body.* — As has been pointed out, the temperature of the interior of the electric furnace did not vary by as much as 0°.5 throughout a range of about 14 cm., and the length of the expanding quartz, being approximately 1 cm., the error from inequalities of temperature in its different parts may be considered as eliminated.

(*b*) *Constancy of Temperature During Measurements.* —The temperature of the furnace and its contents was held constant during the measurements or at least the variations of temperature were smaller than could be detected by means of the apparatus employed, *i. e.*, the compensation apparatus, furnished by Siemens and Halske. The use of the current from the storage battery contributed greatly to this result.

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[1] Ann. der Physik, 2, p. 507, 1900.

[2] Ibid., 10, p. 447, 1903.

(*c*) *Uniformity and Homogeneity of the Quartz Specimen Examined.* — The quartz ring employed, received from the smelting works of the "Zeiss Fabrik" may safely be assumed to be an excellent sample of pure fused quartz, while its small length and the symmetry of its parts render errors from any want of symmetry, or lack of homogeneity extremely unlikely.

(*d*) *Close Agreement Between Temperature Measured and Actual Temperature of the Quartz Ring.* — The junction of the thermoelement was placed within two millimeters of the surface of the quartz ring and since the variation of temperature in the interior of the furnace was practically *nil*, and since the temperature of the junction was constant both before and during the measurements, it may be assumed that the indicated temperature was very accurately that of the expanding quartz.

(*e*) *Variety of Temperature Ranges and of Conditions of Measurement.* — No pains were spared to make the variety of temperature intervals over which the expansion could be measured as great as possible, in order to detect any variation in the rate of expansion of the fused quartz within that limit, if such variation existed. Also the repetition of the measurements, in another laboratory, and under new conditions, and with new instruments is thought to give added value to the result obtained.

(*f*) *Delicacy of Measurements of Elongation.* — With the system as finally arranged for measurements, the fringe width was equivalent to 100 divisions of the divided head of the micrometer, for this rendered it possible to measure displacements to one two-hundredth of a wave-length. Hence elongations of double this amount, or 0.01 of a wave-length of the light employed, could be measured with ease. This gives, in the case of the yellow light from mercury vapor, the possibility of measuring elongations of the order of 0.000578 × 0.01 = 0.000005 mm., and this when used in connection with a length of 7.9 mm. of the expanding substance, amounts to one part in 1,500,000.

SUMMARY OF RESULTS.

I. In this investigation it has been shown that interference methods are applicable to the measurement of minute lengths up to 950° C. when the fringes are produced between surfaces of fused quartz; and that up to this temperature the clearness and the brilliancy of the fringes are as great as at the lower temperatures.

II. The expansion of fused quartz is uniform from room temperature to the highest temperature observed. No change in the expansion, either as an increase or a sudden decrease, as was reported by Le Chatelier, was observed.

III. No after effects due to the heating were observed.

IV. The mean value of the coefficient of expansion of fused quartz, as deduced from 43 determinations, is

$$a = 44.9 \times 10^{-8}.$$

In conclusion the writer desires to express his gratitude to Professor Reed, under whose immediate direction this investigation was pursued, for the many suggestions offered, the kindly interest taken during the progress of the work, and for the generous loan of apparatus with which to pursue to completion the work at the University of Rochester. Thanks are also due to Dr. H. M. Randall for valuable suggestion and assistance in overcoming experimental difficulties.

PHYSICAL LABORATORY, UNIVERSITY OF MICHIGAN,
ANN ARBOR, August 10, 1906.

ON THE MAGNETIC SUSCEPTIBILITY AND TEMPERATURE COEFFICIENTS OF Cu_4SO AND $MgSO_4$ SOLUTIONS.

BY CLARENCE K. STUDLEY.

I HAVE made some relative determinations on the solutions of the above named salts by the balance method as used by Professor Stearns in his determination of the magnetic susceptibility of water,[1] and with practically the same apparatus used by him.

The apparatus consists of a very sensitive non-magnetic balance of a special type designed by Helmholtz. This was, of course, enclosed in a glass case and the whole surrounded on all but one side by a zinc box to shield it from such external agencies as light, etc. The open side of the zinc box is provided with a black curtain, which is drawn over the opening when an observation is to be made. A glass tube passes up through a hole in the bottom of the balance case and connects it with the wooden box, *K*, into which the pole pieces, *PP*, of the large electro-magnet, *M*, project. This being almost airtight the apparatus is very effectively protected from external air currents. To the bottom of the balance-pan, *N*, is attached a silk cord which is practically inelastic for small weights so that the bottom of the tube, *T*, is at a constant height when the balance is in equilibrium. This cord passes down through the tube, *Q*, and terminates in a hook made of copper wire, which is inserted into the eyelet in the upper end of the thermometer, *H*. The tube, *T*, which contains the solution is fitted with a rubber cork, *C*, which is perforated so as to allow the thermometer to be passed through it and into the solution. By this means the tube, *T*, is suspended between the pole-pieces, *PP*. The pole-pieces are held apart a distance of 1.6 cm. by a brass block, *V*, which is hollowed out in the center so as to allow the tube, *T*, to pass below the center of the magnet pole. The box, *K*, is fitted with a slide door, *L* (which has a glass window, *W*), which enables one to read the thermometer without opening the box. The electro-magnet rests on a solid bench, *X*, while the balance is supported by a solid plank whose ends, *AA*, rest on masonry piers. At the center of the balance-beam is a mirror

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[1] H. D. Stearns, PHYS. REV., Vol. XVI., No. 1, January, 1902.

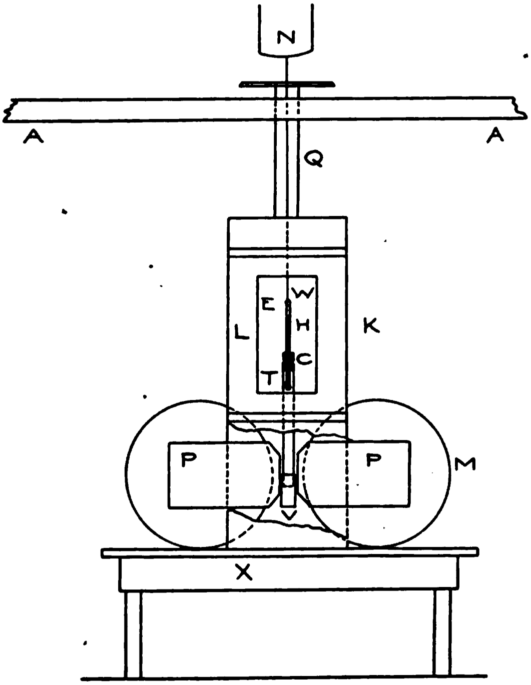

Fig. 1.

which reflects a scale into a reading telescope. The scale and telescope are at a distance of 1.5 meters. By this means it is possible to weigh accurately to $\frac{1}{100}$ of a milligram with a 20 milligram rider.

The magnetic susceptibility (K) is given by the following formula which may be easily derived so it is not deemed necessary to give the theory at this time:

$$K = (2gp/H^2q).$$

Where (g) is the acceleration of gravity, (p) the attractive force in grams exerted by the magnet upon the cylinder in the direction of its axis (repulsion being denoted by a (—) sign before the quantity), (q) is the cross-section of the liquid cylinder, and (H) is the horizontal component of the magnetic field.

Since only a relative determination is desired we have only to measure the attraction (p) with a given tube and constant current. The other quantities remain the same.

I have used the susceptibility of water as $-.73.10^6$ which is the value obtained by Professor Stearns. The reason for my selection is not only because it was determined under the same conditions as those under which I have been working but also because since this is the mean value of the determinations made by the different investigators as shown by the following table I consider it the best determination yet made on water.

TABLE I.

Observer.	K.10^6.	Observer.	K.10^6.
Quincke.[1]	−.84	DuBois.[1]	−.86
Henrichsen.[2]	−.75	Curie.[3]	−.79
Townsend.[4]	−.77	Königsberger.[5]	−.80
Jäger and Meyer.[6]	−.66	Faraday.[7]	−.69
Becquerel.[7]	−.63	Wahner.[7]	−.536
BuBois.[7]	−.837	Stearns.[8]	−.73
			Mean value −.741

The salts used in these determinations were tested for iron but as no perceptible trace of it was found it is safe to say that the per cent. of iron, if there be any present, is so small as to be inappreciable.

The null point was taken both before and after the reading for the excited magnet, and if the variation of these two null points from the mean value is over one per cent. the reading is rejected. The residual magnetism was so slight that it may be easily neglected.

A single rider whose mass was about 20 mg. was used. As only a comparative determination is to be made it is not necessary to know its mass accurately.

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[1] Wied. Ann., Vol. XXXV., p. 137, 1888.

[2] Wied. Ann., Vol. 45, p. 38, 1892.

[3] Journal de Physique, p. 206, 1895.

[4] Proc. Roy. Soc. London, LX., p, 186, 1896–7.

[5] Wied. Ann., Vol. 66, p. 698, 1898; Drude's Ann., Vol. VI., p. 506, 1901.

[6] Wied. Ann., Vol. LXVII., p. 712, 1899; Drude's Ann., Vol. VI., p. 870, 1901.

[7] Pogg. Ann., Vol. XXXV.

[8] PHYS. REV., Vol. XVI., No. 1, January, 1903.

The water used in making the solution was distilled from rain water caught in a glass bottle after copious showers had cleansed the tiled roof.

Readings were taken only every half hour so as to be sure that the solution was in a stable condition.

I have given one set of data in full so as to give some idea of the various features of the work. In this I have used the following notation:

C, the current in the electromagnet.

Rm, the number of scale divisions the rider has been moved along the balance arm from the position with zero current in the electromagnet to the position with (*C*) amperes current in the magnet coil.

No, null point with zero current in the magnet.

Nt, null point with current in the magnet.

Ni, null point after the current has been turned off.

T, the temperature registered by the thermometer (*H*). (See Fig. 1.)

p, the attraction exerted by the magnet on the tube (*T*) and contents.

4.53 *Per Cent.* $CuSO_4$ *Solution.*

C	Rm	No	Nt	Ni	T	p
10	2.5	26.7	29.8	26.6	16.7	−.01044
10	2.5	26.4	31.3	26.9	17.0	−.01065
10	2.5	26.2	31.3	26.8	17.3	−.01065
10	2.5	27.7	31.6	27.7	18.0	−.01051
10	2.5	31.5	36.8	31.8	19.0	−.01065
10	2.5	31.2	36.4	31.2	19.6	−.01057
10	2.5	32.6	39.4	32.8	19.9	−.01086
10	2.8	32.0	34.0	32.1	20.5	−.01082
10	2.8	31.9	34.1	32.2	20.5	−.01084

From the curves for empty tubes (I.) and (II.), Fig. 2, it will be seen that glass has a very high temperature coefficient. This causes a difference in the value of (*p*), for the tube, of .0004 per degree centigrade. Since the limit of possible error does not exceed 2.5 per cent. the variations in the above curves could not be due to this cause.

The curves for the 4.53, 7.16 and the 8.94 per cent. $CuSO_4$ solutions, Fig. 3, lie so near the origin that a small error in taking the data would make a great variation in the final result. In the 7.16 per cent. solution the greatest variation is 5 per cent. and in the 8.94 per cent. solution it is slightly over 3 per cent. but these are easily within the limit of experimental error. Hence I conclude that, between the limits of 16° and 21° C. there is no temperature coefficient in $CuSO_4$ solutions. Whether this is true or not for a wider range of temperatures remains to be seen.

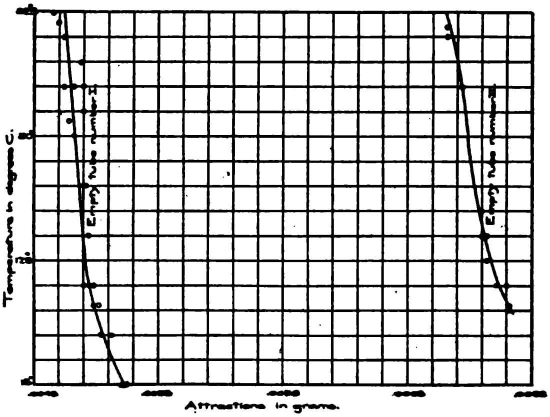

Fig. 2.

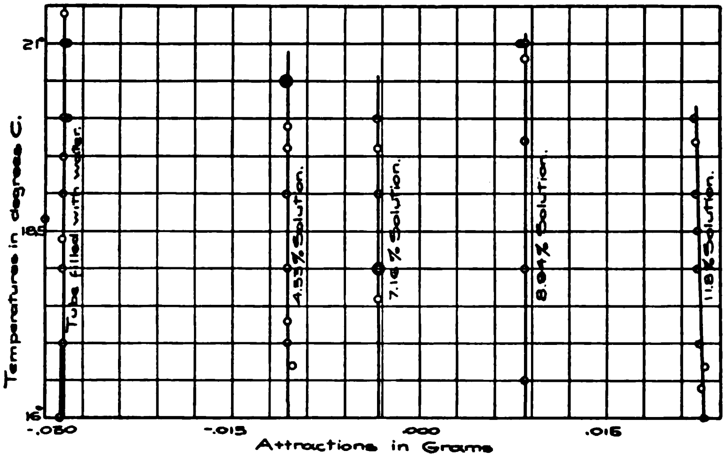

Fig. 3. $CuSO_4$ solutions.

Assuming $K \cdot 10^6$ to be $-.73$ for water, I have calculated the values of this quantity for each of the solutions used.

TABLE II.

Per Cent. Solution.	Specific Gravity.	$K \cdot 10^6$ (Measured).	K' (Calculated).	$K \cdot 10^6$ (Calculated).
7.16	1,071	−.084	.002	.002
11.8	1,117	.537	.476	.532
8.94	1,082	.212	.184	.199
4.53	1,032	−.270	−.266	−.275

The specific magnetism (K') of $CuSO_4$ is given by the formula[1]:

$$K' = (10.4)(P/100) - .80(1 - (P/100)).^2$$

Where P, is the per cent. of water-free salt in the solution, and $K' = K/s$ where K, is the magnetic susceptibility and s, the specific gravity of the solution.

The above computations show a very good agreement except in the case of the 7.16 per cent. solution.

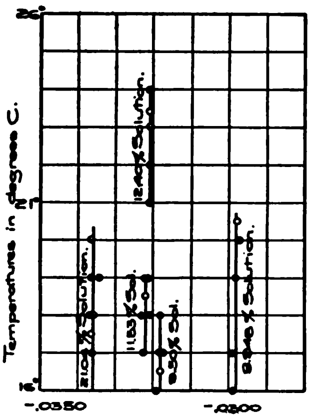

Fig. 4. $MgSO_4$ solutions.

$MgSO_4$ SOLUTIONS.

From the curves for the $MgSO_4$ solutions, Fig. 4, it can be easily seen that the greatest possible variation is much less than 2.5 per cent. (the probable error). Hence I conclude that, $MgSO_4$ solutions have no temperature coefficient for a range of temperatures from 16° to 21°C.

The specific magnetism of $MgSO_4$ is given by the formula:

$$K' = -.46(P/100) - .73(1 - (P/100)).$$

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[1] Wied. Ann., Vol. LXVI., p. 698, 1898.

[2] Where $K \cdot 10^6$ is taken as $-.73$ the formula becomes:

$$K' = 9.49(P/100) - .73(1 - (P/100)).$$

TABLE III.

$MgSO_4$ Solutions.

Per Cent. Solution.	Specific Gravity.	$K \cdot 10^6$ (Measured).	K' (Calculated).	$K \cdot 10^6$ (Calculated).
9.48	1,089	−.698	−.704	−.767
21.04	1,200	−.823	−.672	−.806
12.40	1,118	−.786	−.696	−.778
11.53	1,109	−.789	−.698	−.774
8.95	1,084	−.780	−.706	−.765
4.73	1,038	−.765	−.717	−.744

The results show a very good agreement of experimental and calculated values except for the 9.48 per cent. solution. This is no doubt due to experimental error.

In reducing from the mirror scale to the balance arm scale I have used the relation, 1.5 divisions on the mirror scale equal .1 scale division on the balance arm scale.

What value there may be in this paper is due very largely to the aid and suggestions of Professors Sanford and Stearns, of the Physics Department.

STANFORD UNIVERSITY, 1905.

MAGNETIC SUSCEPTIBILITY OF MIXTURES OF SOLUTIONS.

By H. W. March.

THE following investigation of the magnetic susceptibility of mixtures of solutions of salts of copper, manganese, and aluminium was undertaken at the suggestion of Dr. E. P. Adams, to determine whether the susceptibility of such mixtures is an additive property.

A slight modification of Quincke's manometric method, as employed by Du Bois,[1] was used in measuring the susceptibility. A glass tube bent at right angles, one arm having a diameter of about 2.5 mm. and the other a diameter of about 25 mm., was mounted on a support with the long narrow arm passing between the pole pieces of a powerful electromagnet. The narrow tube was inclined at an angle of about 30° to the horizontal. Its support consisted of a board fastened to two standards embedded in a brick pier. A hole was cut in the board for the admission of the pole pieces. The tube was held in position on the board by two brass straps over the narrow arm through which it slipped, but into which it fitted snugly. There was thus no difficulty in removing the tube and returning it to its former position. A third brass strap over the wide arm of the tube kept it in place. The meniscus between the pole pieces was 25 cm. from the surface of the liquid in the wide arm.

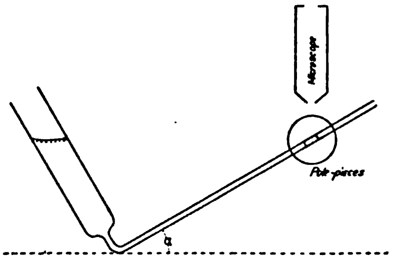

Fig. 1.

The pole pieces had the shape of a frustum of a cone. Their diameter was 3 cm. and they were 1 cm. apart during the experiment. The field strength was measured by means of the change in the resistance of a bismuth spiral placed beside the meniscus. The spiral was made by Hartmann and Braun.

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[1] Wied. Ann., 35, p. 137.

All measurements are relative, water being taken as the standard. Its susceptibility is taken to be -0.8×10^{-6}. As the observations on water and the solutions were made with nearly the same field strengths as indicated by the bismuth spiral, I think any inaccuracy in the calibration of the spiral was practically eliminated Since the measurements were referred to water it was not necessary to know the inclination of the tube nor the ratio of the cross sections of its arms. The displacement of the surface of the liquid was measured by means of a micrometer microscope mounted vertically over the meniscus in the narrow tube.

The susceptibility of a solution was computed by the formula whose derivation follows:

Let a_s = horizontal displacement of the meniscus of a solution.

H_s = corresponding field strength.

D_s = density of the solution.

a_w, H_w, D_w = corresponding quantities for water.

α = inclination of tube to horizon.

n = ratio of the cross section of the narrow arm to the cross section of the wide arm.

K_s = susceptibility of the solution.

If a_v denotes the difference in the heights of the two surfaces of the liquid under the action of the magnetic field, we have the relation,[1]

$$a_v g D = \frac{K}{2} H^2.$$

In this case, $a_v = a \sec \alpha \, (\sin \alpha + n)$.

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[1] Du Bois, Wied. Ann., 35, p. 144.

We have then for the solution,

$$K_s = 2g \sec \alpha\,(\sin \alpha + n)_s D_s \frac{a_s}{H_s^2}.$$

And for water,

$$K_w = 2g \sec \alpha(\sin \alpha + n) D_w \frac{a_w}{H_w^2}.$$

$$\therefore\quad K_s = \frac{D_s}{D_w}\frac{\dfrac{a_s}{H_s^2}}{\dfrac{a_w}{H_w^2}} K_w = \frac{D_s}{D_w}\frac{\dfrac{a_s}{H_s^2}}{\dfrac{a_w}{H_w^2}} (-0.8) \times 10^{-6}.$$

On the assumption that the susceptibility is an additive property, the susceptibility of a mixture of solutions is computed by the formula,

$$K = M_1 K_1' + M_2 K_2'.$$

In this formula

M_1 = mass of first solution per cubic centimeter of the mixture.

K_1' = the specific susceptibility of the first solution, *i. e.*, the susceptibility per gram.

The susceptibility of each solution was found as the result of two sets of observations, with field strengths of approximately 8,000 C.G.S. units and 10,000 C.G.S. units respectively. A set of observations consisted of four separate determinations. Two determinations were made with the storage battery current direct and reversed. Then the connections of the battery used in measuring the resistance of the spiral were reversed to eliminate the effect of thermal currents, and two more determinations were made. Two zero readings were taken with the micrometer, one before closing the storage battery circuit, and one after opening it. I give as an example one of the sets of observations on water.

From a curve supplied with the spiral H is taken as the abscissa of a point whose ordinate Z is given by the formula.

$$Z = \frac{W_f - W_0}{W_0}[1 + 0.014(T - 18)].$$

In this case $H = 9{,}450$ and $(a/H^2)10^{11} = 13{,}800$. Frequent observations were made on water to determine the constancy of the apparatus. The agreement of the values of a/H^2 was very good, showing that the tube maintained its constant inclination to the horizon.

	Initial Resistance. (W_0)	Final Resistance. (W_f)	T	Micrometer Readings.		
				r_0	r_f	Displacement.
D	18.92 ohms	26.53 ohms	21°.5	25.536 25.689	13.349	12.263
R	18.92	26.53	21.6	25.717 25.780	13.443	12.305
Small Battery Reversed.						
D	18.90	26.53	21.7	25.794 25.810	13.418	12.384
R	18.90	26.53	21.7	25.788 25.750	13.423	12.346
	18.91	26.53	21.6			$a = 12.324$
	.08 (lead wires)	.08				
	$W_0 = 18.83$	$W_f = 26.45$				

The results of measurements of simple solutions and of mixtures of solutions are given in Table I. The second column gives, in the case of the original solutions, the mass of anhydrous salt per cubic centimeter. This quantity was obtained by assuming that the salts were pure and that they were represented by the formulas, $CuCl_2 + 2H_2O$, $CuSO_4 + 5H_2O$, $AlCl_3 + 6H_2O$, $Al_2(SO_4)_3 + 18H_2O$, respectively. Solutions No. 2 and No. 13*b* were analyzed by my friend, Mr. L. H. Duschak, Assistant in Chemistry, and the values given are the results of his analysis. In the case of mixtures the composition was determined by using a flask of known volume and weighing the amount of each solution added.

The salts were purchased from Eimer and Amend.

Solutions No. 4, No. 15, No. 16 and No. 17 show considerable difference between the observed and calculated values of K. After No. 4 stood about twenty-four hours a greenish white precipitate formed. This was not noticeable when the observations were made. This precipitate was examined by Mr. Duschak and was found to be a basic chloride of copper. There was a slight trace of aluminium present. It is interesting to note that in the case of solution No. 9 which contained a larger proportion of alu-

TABLE I.

Solution.	Composition.	Density.	Observed. $K.10^6$.	Specific Susceptibility. $K'.10^6$.	Calculated. $K.10^6$.
No. 1.	0.1185 g. $CuCl_2$	1.1012	+0.663	+0.602	
No. 2.	0.0126 g. $MnCl_2$	1.0085	+0.780	+0.773	
No. 3.	0.1732 g. $AlCl_3$	1.1362	−0.862	−0.759	
No. 4.	0.6950 g. No. 1 0.4191 g. No. 3	1.1141	+0.087		+0.418 −0.318 +0.100
No. 5.	0.2684 g. No. 3 0.7715 g. No. 2	1.0399	+0.402		−0.204 +0.596 +0.392
No. 6.	0.8890 g. No. 2 0.1302 g. No. 1	1.0193	+0.766		+0.687 +0.078 +0.765
No. 7.	0.8196 g. No. 2 0.1086 g. No. 1 0.1015 g. No. 3	1.0297	+0.624		+0.633 +0.065 −0.077 +0.621
No. 8.	0.3012 g. No. 2 0.7967 g. No. 3	1.0979	−0.371		+0.233 −0.604 −0.371
No. 9.	0.7112 g. No. 3 0.4099 g. No. 1	1.1211	−0.293		−0.540 +0.247 −0.293
No. 10.	0.5072 g. No. 1 0.5431 g. No. 2	1.0502	+0.730		+0.305 +0.420 +0.725
No. 11.	0.2989 g. No. 1 0.2001 g. No. 2 0.6010 g. No. 3	1.1000	−0.122		+0.180 +0.155 −0.456 −0.121
No. 12.	0.2736 g. $Al_2(SO_4)_3$	1.2297	−0.839	−0.682	
No. 13*b*	0.0174 g. $MnSO_4$	1.0145	+1.016	+1.002	
No. 14.	0.1411 g. $CuSO_4$	1.1336	+0.684	+0.603	
No. 15.	0.2529 g. No. 13*b* 0.9269 g. No. 12	1.1798	−0.396		+0.253 −0.632 −0.379
No. 16.	0.7602 g. No. 12 0.4333 g. No. 14	1.1935	−0.280		−0.519 +0.261 −0.258
No. 17.	0.5621 g. No. 14 0.5123 g. No. 13*b*	1.0744	+0.837		+0.339 +0.514 +0.853
No. 18.	0.3015 g. No. 14 0.1461 g. No. 13*b* 0.7277 g. No. 12	1.1753	−0.199		+0.182 +0.146 −0.497 −0.169

minium chloride solution, no precipitate formed and in this case the observed and calculated values of K agreed very well. The remaining solutions for which there is a considerable difference all contained $Al_2(SO_4)_3$ and in all of them a good deal of aluminium hydroxide was present. It is to be noted that when the relative magnitudes of the quantities added together to give the calculated value of K are taken into account the discrepancies are not so serious as they seem to be. I think the agreement between the observed and calculated values of K for the mixtures is sufficiently good to establish the fact that the magnetic susceptibility is additive for the solutions examined.

I have calculated the specific susceptibility of the salts used and have compared them as far as possible with the values given in Landolt-Börnstein's Tables. The specific susceptibility of a salt was obtained by considering the susceptibility of a solution as the sum of the susceptibilities of the salt and water separately. The method of calculation follows.

Let $K_s' =$ specific susceptibility of anhydrous salt.

$K =$ susceptibility of the solution.

$m_s =$ mass of anhydrous salt per cubic centimeter.

$m_w =$ mass of water per cubic centimeter.

Then $K = m_s K_s' + m_w(-0.8 \times 10^{-6})$.

$$\therefore \quad K_s' = \frac{K + 0.8 \times 10^{-6} m_w}{m_s}.$$

I have also calculated the susceptibility per unit mass of metal in the salt and this quantity I have called K_m'. K_m' is obtained by dividing K_s' by the fraction which expresses the ratio between the mass of metal in the salt and the mass of the salt. I found in the case of $MnCl_2$, $MnSO_4$, $CuCl_2$, and $CuSO_4$ that the susceptibility per unit mass of metal was constant for a given metal. I do not think that this means that the susceptibility of a salt is independent of the acid radical. It seems more probable that the effects upon the susceptibility of the metal, produced by the two acid radicals, are so nearly equal that no difference can be detected by our present methods of measurement. These results agree with those of

Townsend,[1] who found that, for salts of iron, the susceptibility per unit mass of iron was constant for a given state of chemical combination. The metals in the salts I examined were in the same state of chemical combination. The values of K_s' and K_m' are given in Table II. The values of K_m' do not agree in the case of the salts of aluminium. An analysis of the solutions containing these salts may be made later. It is possible the salts were not represented by the formulas, $AlCl_3 + 6H_2O$ and $Al_2(SO_4)_3 + 18H_2O$, respectively.

TABLE II.

Salt.	$K_s' \times 10^6$.	$K_s' \times 10^6$ (Landolt-Börnstein).	$K_m' \times 10^6$.
$CuCl_2$.	12.23	12.3 (L. u. W.)	25.79
		12.6 L. (J. K.)	
$CuSO_4$.	10.47	10.3 (L. u. W.)	26.18
		10.4 L. (J. K.)	
		10.5 (Q.)	
$MnCl_2$.	124.7	117 (J. u. M.)	285.7
		127 (Q.)	
		122 (L. u. W.)	
$MnSO_4$.	104.2	98.4 (J. u. M.)	286.1
		114 (Q.)	
		100 (L. u. W.)	
$AlCl_3$.	−0.531		−2.62
$Al_2(SO_4)_3$.	−0.189		−1.20

(L. u. W), Liebknecht u. Wills; (J. K.), J. Königsberger; (Q.), Quincke; (J. u. M.), Jaeger u. St. Meyer. L standing after a number means that the susceptibility of air instead of that of a vacuum, is taken to be zero.

G. Meslin[2] found a mixture of water and alcohol that seemed to show a greater specific susceptibility than either liquid separately. This mixture contained 4 parts of water in 100. He found that for absolute alcohol, $K' = 1.03 \times K$ for water, and that for the 4 per cent. mixture $K' = 1.045 \times K$ for water. His method was not capable of detecting this difference with certainty but he found it so many times that he believed there was a real increase in the susceptibility. I thought it would be interesting to repeat this experiment. I measured the susceptibility of absolute alcohol and that of mixtures containing 2 per cent., 4 per cent., 6 per cent. and

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[1] Phil. Trans. 1896, p. 545.

[2] Annales de Chimie et de Physique, 1906, p. 188.

8 per cent of water. The alcohol was from E. R. Squibb & Sons and was marked 99.8 per cent. Table III. gives the results of my measurements. In the column headed *calculated K* are given the values of K calculated on the assumption that the susceptibility is additive. The same formula is used as in the case of mixtures of salt solutions. The last column gives the ratio of the specific susceptibility of the mixture to that of water. These values are given to furnish a basis of comparison with Meslin's results. Meslin took the susceptibility of water to be -0.79×10^{-6}.

TABLE III.

	Density (Reduced to T).	T	Observed. $K \times 10^6$	Calculated. $K \times 10^6$	$K' \times 10^6$	$\frac{K'}{K_w}$
Absolute Alcohol (99.8%)	0.7856	24.0°	−0.6536		−0.8320	1.04
2.02% Water	0.7907	24.5	−0.6563	−0.6685	−0.8330	1.0375
4.08% "	0.7942	24.7	−0.6650	−0.6608	−0.8373	1.047
6.21% "	0.8048	20.6	−0.6602	−0.6690	−0.8203	1.025
8.16% "	0.8087	21.0	−0.6731	−0.6718	−0.8323	1.040
Absolute Alcohol (99.8%)	0.7875	21.6	−0.6574		−0.8348	1.044

The water originally in the alcohol is not included in the per cent. of water given in the first column.

It will be seen that K' is greatest for the mixture containing 4 per cent. of water. However the difference between this value of K' and the other values is small and the difference between the observed and calculated values of K is no greater than is to be expected.

In conclusion it appears that the magnetic susceptibility of the liquid mixtures studied is an additive property. The reason for taking solutions of salts of these particular metals was that alloys in varying proportions of aluminium, copper and manganese have been found by Heussler and others to be strongly magnetic, so that the magnetic susceptibility of these alloys is not an additive property.

Throughout the progress of this work I have been indebted to Dr. E. P. Adams for much valuable advice and assistance.

PHYSICAL LABORATORY, PRINCETON.

A NEW TYPE OF ALTERNATING CURRENT GALVANOMETER OF HIGH SENSIBILITY.

By W. S. Franklin and L. A. Freudenberger.

THE alternating current galvanometer to be described is essentially a Kelvin astatic galvanometer modified so as to be suitable for alternating currents.

BB are the two deflecting coils wound in exactly the same manner as the coils of a Kelvin instrument, and having a needle system suspended so as to hang at the centers of the coils.

The needle system differs from the needle system of a Kelvin galvanometer in the fact that the permanent magnets are replaced by unmagnetized soft iron wires inclined 45° to the axis of suspension, *CD*, as shown in Fig. 1. The plane of the magnets lies in the plane of the paper, as shown in the above figure. The two magnets and a light mirror are fastened to a thin glass rod, as shown; and the whole system is suspended at the centers of the coils, *BB*, by a single silk or quartz fiber.

The coils, *AA*, produce a nearly vertical field of about 10 C.G.S. units. This field serves to magnetize the needle system as shown.

The axis of the coils, *AA*, is inclined at an angle of 5° or 10° to the axis of suspension, producing a field in the direction *EF*. This field, *EF*, may be regarded as made up of two components. One, a strong vertical component, *M*, serving to magnetize the needles; the other, a weak horizontal component, *H*, serving to give a directive tendency to the needle system.

In Fig. 1, the arrows, *M* and *H*, indicate the directions of the fields produced by one half cycle of current, (that is, *M* and *H* are the components of the field *EF*). The needle system is then magnetized as shown. If, at this instant, the current to be measured is sent through the coils, *BB*, a torque acts on the needle system tending to turn the magnets to a position at right angles to the plane of the paper. The current for the coils, *BB*, is, of course, obtained from the same source as the current for the coils, *AA*.

During the succeeding half cycle of alternating current, the field, *EF*, is reversed (thus reversing *M* and *H* and the polarity of the needle system); while the current in the coils, *BB*, reverses simultaneously, thus giving a torque in the same direction as before.

It is not necessary that the two currents, — in the coils, *AA* and *BB*, — be strictly in phase with each other. Where the two currents differ in phase, the tangent of the angle of deflection is proportional to the cosine of the angle of phase difference.

It is evident that the galvanometer will indicate equally as well when the coils, *AA* and *BB*, are supplied by direct current, instead of alternating current.

ADJUSTMENTS.

1. The coils, *AA*, rest on leveling screws fastened to the base of the instrument, so that the axis of *each* coil can be placed in any position (not differing greatly from the vertical direction). Thus the angle, *COE* (in the plane of the paper), may be varied, thus adjusting the strength of the horizontal component, *H*, and consequently adjusting the strength of the directive tendency of the needle system. The angle, *COE*, can not be made too small (small value of *H*), as the directive tendency becomes so small that the "zero" position is unstable. Throughout this adjustment the two coils, *AA*, are kept parallel to each other.

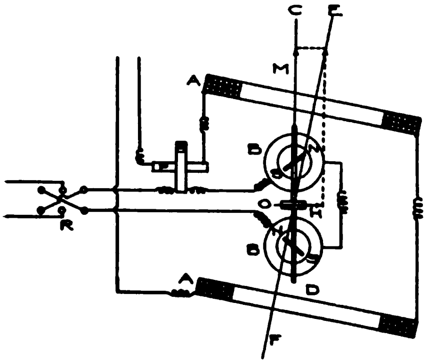

Fig. 1.

2. To adjust the "zero" position of the suspended system, that is to place the magnets in the plane of the paper (Fig. 1), the angle, *COE* (in a plane at right angles to the plane of the paper), is varied — for one coil only, of the two coils, *AA*.

The two adjustments outlined above correspond to the adjustments of the "zero" of an ordinary Kelvin instrument by means of a "controlling" magnet.

3. The field produced by the coils, *AA*, induces small E.M.F.'s in the coils, *BB*. Further the E.M.F.'s induced in the coils, *BB*, are opposite in sign, since the coils, *BB*, are connected in opposition. When the galvanometer is short-circuited at the reversing switch, "*R*," there will usually be sufficient E.M.F. induced in the coils, *BB*, to cause a deflection of a few centimeters (scale divisions). It is possible to place the two coils, *AA*, symmetrically with respect to the field, *EF*, so that no deflection is produced on short circuit, and at the same time satisfy adjustments 1 and 2 above. This adjustment, however, is very tedious and can be eliminated by using a small transformer without iron, with a movable secondary. The primary of the transformer is connected in series with the coils, *AA*, and the movable secondary in series with the coils, *BB*, as shown in Fig. 1. It thus becomes possible to make each of the three adjustments independently of each other.

Three instruments of different resistances, have thus far been built by the writers. The first two instruments built, are not as sensitive, however, as they might be. The design of a sensitive instrument is exactly the same as the design of a Kelvin galvanometer, *i. e.*, the coils (*BB*) must be wound to the theoretical shape,[1] and the best size of wire chosen. This size of wire lies in the vicinity of No. 36 *B* and *S* gauge, single silk covered.

In two of the instruments constructed, the sensibility of the coils (*BB*) was also tried as an ordinary Kelvin astatic galvanometer by replacing the soft iron magnets with a permanent magnet suspension. From the results of these tests, the sensibility of the alternating current instrument varies from one third to one sixth of the sensibility of the same instrument as an ordinary Kelvin galvanometer.

No serious difficulty was experienced from any disturbing effect of the earth's field. The earth's field might be eliminated by surrounding the entire galvanometer by a massive iron cylinder — with perhaps a slight increase in sensibility.

The following are the results of the sensibility tests of the alternating current instruments.[2]

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[1] See Gray's Absolute Measurements.

[2] The sensibility of the best types of telephone receivers is about 10^{-6} amperes. Resistance 75 ohms.

Instrument No. 1.

Coils (*BB*) 1¾″ in diameter.

Needle chamber $\frac{5}{16}$″ in diameter.

Magnets ¼″ long.

Wound with No. 36 *B* and *S* gauge, double silk covered wire.

Resistance, 2,306 ohms.

Sensibility $= 4 \times 10^{-9}$ amperes per mm., with scale at one meter's distance.

Instrument No. 2.

Coils 1¼″ in diameter.

Needle chamber $\frac{17}{32}$″ in diameter.

Magnets ½″ long.

Wound with No. 20 *B* and *S* gauge, single cotton covered wire.

Resistance, 1.75 ohms.

Sensibility $= 6 \times 10^{-7}$ amperes per mm. per meter.

Instrument No. 3.

Coils ⅝″ in diameter.

Needle chamber $\frac{3}{16}$″ in diameter.

Magnets made of a single 10 mil iron wire, ⅛ long.

Wound with No. 33 *B* and *S* gauge, single silk covered wire.

Resistance, 21 ohms.

Sensibility, 6×10^{-8} amperes per mm. per meter.

The sensibilities of the above instruments as ordinary Kelvin galvanometers, are

No. 1. 1.4×10^{-9} amperes per mm. per meter.

No. 2.

No. 3. 1.0×10^{-8} amperes per mm. per meter.

A Kelvin galvanometer has since been built by Mr. McFarlin, a student at Delaware College, with a sensibility of 1.0×10^{-9} amperes per mm. per meter; coils 1″ in diameter, needle chamber $\frac{3}{16}$″ in diameter. Magnets ⅛″ long. Wound with No. 36 *B* and *S* gauge, single silk covered wire. Resistance, 100 ohms. This instrument has the same sensibility as No. 1, with only 100 ohms resistance, as against 2,306 ohms for No. 1. The coils of No. 1 departed rather widely from the theoretical shape.

A slight gain in sensibility may, of course, be obtained by properly grading the wire of the coils; but the gain hardly warrants the difficulties of winding.

The above alternating current galvanometer was developed to meet the demand for a sensitive alternating current instrument required for a new method for measuring electrolytic resistances — the subject of a grant made by the Carnegie Institution of Washington.

SOUTH BETHLEHEM, PA.,
July, 11, 1906.

THE VAPOR PRESSURE OF CARBONIC ACID AT LOW TEMPERATURES.

BY JOHN ZELENY AND ROY H. SMITH.

1. Carbonic acid is remarkable for having a relatively high vapor pressure when in the solid state. At its melting point this pressure is greater than five atmospheres. The boiling or sublimation point, where the vapor pressure is equal to that of one atmosphere, is therefore considerably (over 20° C.) lower than the melting point.

The vapor pressures of carbonic acid, between 0° C. and the boiling point, have been measured by several observers but the results are not in good agreement.[1] For temperatures below the boiling point, we know of but one set of observations.[2]

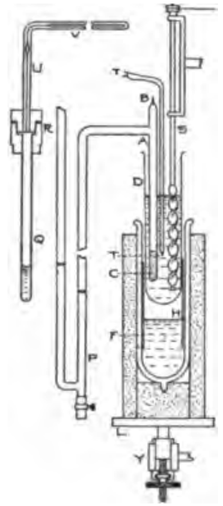

Fig. 1.

The determinations described in this paper extend over the range from — 7° C. to the lowest temperature at which the pressure is appreciable.

2. The apparatus used for making the measurements is represented diagrammatically in Fig. 1.

The solid carbonic acid, *C*, was contained in the glass tube, *A*, whose lower end was immersed in pentane in the glass tube, *D*. The temperatures were measured by the thermoelectric couple, *T*, one of whose junctions was placed near to the carbonic acid tube while the other was kept in ice.

The pentane was agitated vigorously by means of the stirrer, *S*, which was operated by a water motor, and under these conditions it was found that the junction indicated the same temperature in all parts of the bath.

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[1] See M. Faraday, Phil. Trans., 1845, p. 155. Kuenen and Robson, Phil. Mag. (6), 3, p. 149.

[2] Du Bois and Wills, Beibl. zu den Ann. der Physik, 24, p. 428.

The pentane bath was brought to any desired temperature by means of the liquid air contained in the Dewar vessel, *F.* The stand, *L,* which carried this vessel was arranged so that it could be moved vertically by the slow-motion screw, *Y.* By watching the temperature as indicated by the deflection of a galvanometer and carefully moving this screw, the temperature of the pentane could be maintained at a constant value to 0°.1 C. for considerable periods of time, which enabled all of the contents to assume a common temperature before a reading was taken. A metal cylinder, *H,* was fastened to the lower end of the tube, *D,* and dipped into the liquid air. This was a great aid in maintaining some of the temperatures as it made the changes produced by moving the liquid air vessel more gradual and uniform.

Two different arrangements of the apparatus were used, one for the low pressures and one for the high pressures.

3. For the low pressures, the tube, *A,* having a diameter of 1 cm., was connected by a glass tube to the open mercury manometer, *P.* Pressures up to three atmospheres could be measured by this manometer, the readings being taken with the aid of a simple cathetometer together with the reading of the barometer.

The apparatus was filled with pure carbonic acid in the following manner. A small glass tube was introduced into the tube, *A,* through the open end, *B,* and fastened to it by rubber tubing. Carbonic acid from a cylinder of commercial liquid carbonic acid was allowed to flow into *A* through this tube, and passed out through the gauge, *P,* from which nearly all of the mercury had been removed previously. After the air had been displaced the lower end of the tube, *A,* was introduced into a flask of liquid air. The carbonic acid coming from the jet froze at the bottom of the tube while the air mixed with it passed out. The tube, *A,* was lowered little by little until it was filled with solid carbonic acid for about two thirds of its total length. The tube was then partly raised from the liquid air and after the temperature had risen so that the pressure inside was greater than an atmosphere, the end, *B,* was hermetically sealed with a blow-pipe, the auxiliary glass tube having been withdrawn previously.

The solid carbonic acid was then allowed to vaporize from the top and to pass out through the gauge until a length of about 3 cm. only was left. The gas coming out of the open gauge was tested several times and found to contain only about one fortieth of 1 per cent. of impurities, although the original carbonic acid had 1.7 per cent. The tube, *A*, was now introduced into the cooled pentane bath and the taking of observations begun.

4. The tube, *Q*, was used for the higher pressures. This was 25 cm. long and had an outside diameter of 6 mm. and an inside one of 4.5 mm. It was joined to the closed air gauge, *V*, by means of the brass coupling, *R*, into which the two parts were sealed with cement. Apparatus made entirely of glass, with the gauge sealed in as a side joint, was first tried but in all except one case the whole exploded with considerable violence after the pressure had increased to about 60 atmospheres.

The tube, *Q*, was nearly filled with solid carbonic acid in the manner described in the previous section. The gauge, *V*, was then partly screwed on and its mercury thread was forced down· to the open end of its tube. The vaporized carbonic acid escaped through the loose joint of the brass coupling and the apparatus was thus cleared of air. When all but a small portion of the solid carbonic acid had disappeared, the coupling was screwed tight against a leather washer by two powerful wrenches. The joint thus made was practically perfect as no leakage was ever detected after standing several days. The amount of carbonic acid left in the tube was such that liquid began to form in the tube when it was cooled a few degrees below 0° C.

The gauge, *V*, was filled with dry air, having a length of 69 cm. at atmospheric pressure. The tube was 1.2 mm. in diameter and was carefully calibrated. The liquid carbonic acid underwent so much supercooling that it usually froze so explosively as to shatter the mercury thread in the gauge. This difficulty was overcome by having a 15 cm. length of thermometer tubing, *U*, with an enlargement blown at its center, placed between the gauge proper and the brass coupling.

The readings of the gauge were corrected for inequality of the bore, for temperature, and for deviations from Boyle's law. The last corrections were obtained by interpolation from Amagat's values.[1]

5. The thermo-electric couple used for measuring the temperatures was made of nickel and iron. These metals were chosen because their thermo-electric power changes less with temperature than for most metals so that the calibration of the couple does not necessitate the use of so many temperatures. The readings were taken with a d'Arsonval galvanometer, the deflections being brought to zero aperiodically after each reading so as to obtain the correct zero point.

The couple was calibrated by determining the deflections for six known temperatures.

The temperatures used, besides that of 0° C., were obtained as follows:

(*a*) A pentane bath, stirred vigorously, was maintained at a constant temperature whose value was read by a calibrated thermometer as — 29°.2 C.

(*b*) The temperature of a thick mixture of carbonic acid snow and ether was taken as — 78°.77 C.[2] for the barometric pressure 73.5 cm.

(*c*) The boiling point of ethylene, for the barometric pressure 74.0 cm. was taken as — 103°.9 C.[3]

(*d*) The freezing point of ethylene was used as — 169° C.[4]

(*e*) The temperature of liquid air was determined from its density.[5] This was conveniently measured with sufficient accuracy by weighing a small mouthed, 600 c.c. Dewar flask filled with the liquid up to a fixed point for which the volume had been determined and reduced to the temperature of liquid air.

The galvanometer deflections obtained with the above temperatures were plotted on a large scale and the curve, drawn through the points, was then used for getting the temperature corresponding to any deflection.

The galvanometer had a large resistance of manganin wire in

[1] Compt. Rend., 1884, p. 1153.

[2] J. Zeleny and A. Zeleny, this volume.

[3] Travers, Study of Gases, p. 243. (Wroblewski, Witkowski, Ramsey and Travers.)

[4] K. Olzewski, Wied. Annalen (37), p. 337.

[5] Behn and Kiebitz, Annal. der Physik. (4), 12, p. 425.

series with it, in order to diminish the errors, due to changes of the room temperature, to a negligible quantity.

6. The final observations were made on several days and over a hundred readings were taken. The corrected results were then plotted all together on a large scale and those for the lower pressures were also plotted separately on a still larger scale. The results agreed well among themselves not only with either form of apparatus but also where the readings for the two forms overlapped.

The curves drawn through the plotted points were then used to find the vapor pressure corresponding to any temperature.

The results obtained in this way are given in columns II. and III. of the following tables.

TABLE I.

Vapor Pressures of Liquid Carbonic Acid.

Temperature. ° C.	Vapor Pressure in Atmospheres.	Vapor Pressure in Meters of Mercury.	K. and R. Meters of Mercury.
− 7	27.80	21.13	
−10	25.83	19.63	19.8
−12	24.50	18.62	
−15	22.46	17.07	17.0
−18	20.65	15.69	
−20	19.52	14.83	14.7
−22	18.33	13.93	
−25	16.74	12.72	12.5
−28	15.10	11.48	
−30	14.21	10.80	10.6
−32	13.30	10.11	
−35	11.92	9.06	8.97
−38	10.67	8.11	
−40	9.88	7.51	7.46
−42	9.18	6.98	
−44	8.53	6.48	
−46	7.89	6.00	
−48	7.30	5.55	
−50	6.73	5.115	5.02
−52	6.18	4.70	
−54	5.66	4.31	
−56	5.19	3.945	
−56.4	5.11	3.911 Triple Point.	
−58	4.75	3.610 Supercooled.	
−60	4.35	3.306 "	3.27
−62	3.96	3.01 "	
−64	3.58	2.721 "	
−65.5	3.30	2.508 "	

TABLE II.

Vapor Pressures of Solid Carbonic Acid.

Temperature. ° C.	Vapor Pressure in Atmospheres.	Vapor Pressure in Centimeters of Mercury.	K. and R.	
− 56.4	5.11	391		
− 57	4.85	368.5		
− 58	4.51	343		
− 59	4.20	319		
− 60	3.92	298	302	
− 61	3.67	279		
− 62	3.43	260.5		
− 63	3.20	243		
− 64	2.98	226.5		
− 65	2.78	210	211	
− 66	2.59	195.5		
− 68	2.21	168		
− 70	1.88	144	143	
− 72	1.638	124.0		
− 74	1.395	106.0		
− 75	1.283	97.5	97	
			D. and W.	Z. and Z.
− 77	1.102	83.9	88.5	84.0
− 78.2	1.00	76		
− 80		65.7		66.6
− 82		55.8		57.0
− 84		47.3		48.4
− 85		43.5	51.0	44.6
− 86		40.0		41.2
− 88		34.0		35.0
− 90		28.8		29.2
− 92		24.1		24.4
− 94		20.2		20.4
− 96		17.0		16.8
− 98		14.1		13.9
−100		11.9		11.3
−102		9.9	11.0	9.2
−104		8.0		7.4
−106		6.4		6.0
−108		5.3		4.9
−110		4.3		4.0
−112		3.4	4.0	3.25
−114		2.7		2.7
−116		2.2		
−118		1.8		
−120		1.4		
−122		1.1		
−124		.8	.5	
−126		.6		
−128		.45		
−130		.25		
−132		.15		
−134		.1		

The values in the column marked *K.* and *R.* are those given by Kuenen and Robson.[1] Considering the different methods used in making the determinations the results are seen to be in fair agreement with the corresponding ones here obtained. The values under *D.* and *W.* are those obtained by du Bois and Wills[2] for the temperature of carbonic acid snow at different pressures. Their values are in general higher than those obtained by us. The value of the boiling point given by them is — 79°.2 C., which is 1° lower than that given in our tables. By raising all of their temperatures 1°, the two sets of results are brought into much better agreement.

In the last column of Table II., under *Z.* and *Z.*, are added the values recently obtained in this laboratory[3] for the temperatures of carbonic acid snow and its mixtures with ether and alcohol when these are maintained under different pressures. They differ but little from these determinations.

7. It was noticed that the amount of impurity in the low pressure apparatus increased somewhat during a set of observations. This was inferred from the value of the pressure for the temperature of liquid air, which at the start was practically zero, but which towards the end of a set of observations had increased to more than 0.1 cm. The supposed reason for this is that the solid carbonic acid contained a trace of absorbed air. When the pressure of the vapor was increased to two or three atmospheres some of the solid vaporized, freeing the absorbed air. On recooling the tube, the carbonic acid was condensed, but the air was nearly all left free in the tube. By repeated heating and cooling of the tube the amount of impurity was increased in this way. The value of the pressure for the temperature of liquid air was reduced to zero again by allowing the accumulated gas to bubble out through the manometer. In any case the value of the pressure obtained for the temperature of liquid air was subtracted from the other readings taken.

The triple point was determined in the following way. The frozen carbonic acid in the high pressure apparatus was warmed slowly and the pressure noted as it gradually increased. When the carbonic acid began to melt the pressure became constant and remained so for some time, so that its value could be determined with precision. It was found to be 5.11 atmospheres. The temperature of the triple point was determined from the temperature-pressure curve for the liquid carbonic acid, by finding the temperature corresponding to this pressure of 5.11 atmospheres. The value was found to be — 56°.4 C.

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[1] Loc. cit.

[2] Loc. cit.

[3] J. Zeleny and A. Zeleny, loc. cit

The corresponding values obtained by Kuenen and Robson are 5.10 atmospheres and — 56°.2 C. and those obtained by Villard and Jarry[1] are 5.1 atmospheres and — 56°.7 C.

By means of the formula for the latent heat of evaporation,

$$L = (v_s - v_l) T \frac{dp}{dT},$$

Kuenen and Robson[2] calculated the value of the latent heat at the boiling point to be 121 calories, by taking 355 as the specific volume of the vapor of carbonic acid at the boiling point and using for dp/dT, 5.5 cm. per 1° C., the value obtained by du Bois and Wills.[3] This result is much smaller than the experimental values obtained by Behn[4] (142.4 calories) and by Favre and Silbermann[5] (142.2 calories). The value of dp/dT obtained from our curves is 6.35 cm. per 1° C., and using this number the value of L becomes 140 calories. which is in very good agreement with the experimental results.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
June 29, 1906.

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[1] Compt. Rend., 120, p. 1413.

[2] Phil. Mag. (6), 3, p. 627.

[3] Loc. cit.

[4] Annal. der Physik, 1, p. 272.

[5] Ann. de Chim. et de Phys. (3), 37, p. 470.

THE SPARK DISCHARGE IN GASES AND VAPORS.

BY JOHN E. ALMY.

PASCHEN'S law,[1] giving the relation between gas pressure and spark length, is an important advance in the generalization of the phenomena of spark discharge in gases. Carr's[2] verification of the law shows it to hold for a wide range of pressure. Peace[3] first noted the existence of a "minimum" spark potential, and Carr, (loc. cit.), has shown that, — at least for rather low pressures, — there are critical points of spark length and pressure such that if the spark gap is diminished below the critical distance, with pressure constant, or, if the pressure is reduced below the critical value, with constant spark gap, — the spark potential, *increases* with *decreasing* spark length, or *decreasing* pressure as the case may be. The work of Earhart,[4] Kinsley,[5] and Hobbs,[6] seems to have thrown some doubt upon the entire generality of the phenomenon of increasing spark potential with decreasing spark length, however there is evidence suggesting that the phenomena studied by these observers were of the nature of coherer discharge rather than spark discharge.

De la Rue and Müller[7] concluded that the spark discharge passes more readily between electrodes of aluminium than between any other metals; Righi, Peace, and Carr were unable to detect any difference in spark potentials depending upon electrode material. The writer[8] of this paper has pointed out the probability that any such difference due to electrode material would be of the same order as the differences in the "cathode fall" when the same metals are used in the Crookes tube, and consequently would be small beyond the possibility of detection, in most gases. In the case of hydrogen it was found that there is a clearly determined difference in the spark potentials for a given spark length, when different metals are used as electrodes.

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[1] Paschen, Wied. Ann., 37, p. 79, 1889.

[2] Carr, Proc. Roy. Soc., 71, p. 374, 1903.

[3] Peace, Proc. Roy. Soc., 52, p. 99, 1892.

[4] Earhart, Phil. Mag., (6), 1, p. 147, 1901.

[5] Kinsley, Phil. Mag., (6), 9, p. 692, 1905.

[6] Hobbs, Phil. Mag., (6), 10, p. 617, 1905.

[7] De la Rue and Müller, Phil. Trans., 169, pt. 1, p. 93, 1898.

[8] Almy, Phys. Rev., 20, p. 382, 1905.

Roentgen[1] concluded that the minimum potential which will maintain a discharge between a point and a plane, as electrodes, for different gases with same distance between electrodes and same gas pressure, varied inversely as the mean free path of the gas molecules. The insufficiency of the data of spark potentials in various gases and vapors and the fact of the errors due to "lag" of the spark, and differing electrodes, seem to warrant new determinations of spark potentials in various gases and vapors to test with greater care the correctness of Roentgen's hypothesis as applied to the spark discharge. Such was the purpose of the work to be described, the gases and vapors being those which gave as wide a range of mean free molecular path as could be readily obtained.

It has been the usual assumption that spark potentials at a given pressure are a linear function of the spark lengths; this is by no means exactly true; at atmospheric pressure, for any considerable range of spark length it is not even approximately so. Consideration of the data given by Paschen, and by Orgler,[2] shows, however, that the quantity—

$$\frac{1}{P} \cdot \frac{dV}{dx},$$

is a function of the spark potential, alone, being very approximately constant for all pressures and spark lengths, so long as the potential range used is the same.

Consequently this quantity,

$$\frac{1}{P}\frac{dV}{dx} = K,$$

may be taken as characteristic of a given gas or vapor, independent of gas pressure, spark length, or electrode material, if the range of potential used in its determination is the same in all cases. Hence, spark potentials, in various gases and vapors, at various pressures, were determined with potentials varying from 360 to 900 volts, and the values of the gas constant, K, calculated.

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[1] Roentgen, Gottingen Nachr., p. 390, 1878.

[2] Orgler, Ann. der Phys., 1, p. 169, 1900.

The apparatus used and the method were the usual ones; a discharge tube with two spherical electrodes having curvature of 10 cm. radius, was connected with a system of wash flasks and drying tubes for obtaining the pure, dry gas or vapor desired, and to an air-pump to facilitate the filling of the tube. The upper electrode was carried on a screw allowing measurement of the spark gap, to .003 mm. without any rotation of the electrodes. A quartz window, *Q*, Fig. 1, served for the illumination of the light from an electric arc, which is effective in eliminating the "lag" of the spark. A battery of storage cells gave potentials up to 1,000 volts, a Weston voltmeter was used to measure the spark potentials. This voltmeter was calibrated by comparison with a high-accuracy standard Weston voltmeter.

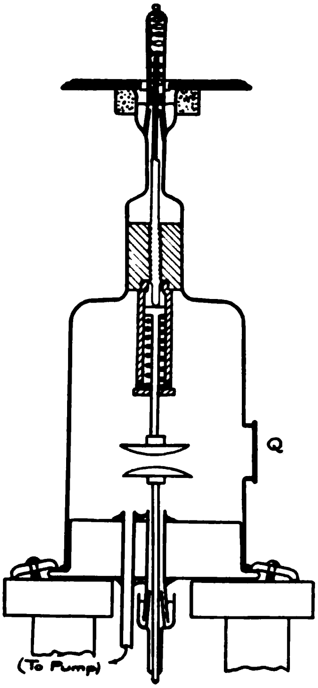

Fig. 1.

A rheostat, consisting of two tubes, one of distilled water, the other of amyl alcohol with a small amount of cadmium iodide in solution, in series with the battery with the voltmeter and spark gap in parallel were shunted off the rheostat, enables one to vary the potential applied to the spark electrodes, at will. The tube of water is of advantage in obtaining almost the whole potential of the battery without the danger of short-circuiting the battery when very short sparks are used.

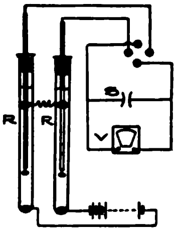

Fig. 2.

The electrical connections are shown, diagrammatically in Fig. 2, *S*, being the spark gap, *V*, the voltmeter, and, *R*, *R*, the rheostat tubes. In taking the observations the spark gap is first closed completely, a small potential (30 to 50 volts), being used to test the short circuit, then the spark electrodes are moved apart, by very minute steps, the potential being applied after each step to detect the point where the electrodes first separate care being taken that the potential was not applied to the spark gap when the electrodes were being moved apart. Having thus obtained the zero point of the spark gap, the electrodes were then separated the desired distance and the potential then gradually increased until a spark discharge occurred, the passage of a spark being evident from the sudden drop in the reading of the voltmeter.

The considerable variation shown by the comparison of the best previous determinations of spark potentials made it advisable to repeat the determinations for some of the permanent gases, so that results obtained under like conditions may be used in comparing the different gases and vapors.

Air which was thoroughly dried from standing in the apparatus with phosphorous pentoxide 48 hours or more, gave the following results:

Air.

Pressure 73.8 cm.			Pressure 30 cm.			Pressure 6 cm.		
x (cm.).	*V*(volts).	*K*	*x* (cm.).	*V*(volts).	*K*	*x* (cm.).	*V*(volts).	*K*
.001	345	1285	.002	322	1033	.01	340	1113
.002	440	1209	.004	384	1067	.02	407	1113
.003	526	1070	.006	448	1033	.03	474	1100
.004	605	954	.008	510	1113	.04	540	1013
.005	677	1030	.010	578	913	.05	601	987
.006	743		.012	633	913	.06	660	1000
			.014	685		.07	720	
Mean values of *K*,		1090			1016			1043

Hydrogen,—generated from zinc with pure, hydrochloric acid, was purified by passing successively through solutions of potassium per-

Hydrogen.

Pressure 73.5 cm.			Pressure 30 cm.			Pressure 6 cm.		
x (cm.).	*V*(volts).	*K*	*x* (cm.).	*V*(volts).	*K*	*x* (cm.).	*V*(volts).	*K*
.001	245	700	.004	300	666	.02	345	700
.002	296	680	.006	340	666	.03	387	813
.003	346	667	.008	380	700	.04	425	750
.004	395	670	.010	422	633	.05	480	600
.006	500	662	.012	460	713	.06	516	543
.008	604		.014	503	700	.08	560	
			.016	545				
Mean values of *K*,		676			680			676

manganate, potassium hydrate, and alkaline pyrogallic acid and dried in tubes of solid KOH and phosphorous pentoxide, giving the following results :

Carbon Di-sulphide Vapor. — Ordinarily pure carbon di-sulphid is by no means a simple substance, but contains various decomposition products. The CS_2 used was some especially cleaned and dried for electro-optical purposes and was almost wholly free from obnoxious odors, a clear, almost colorless, liquid. The vapor of this liquid gave the following results :

Carbon Di-sulphide.

Pressure 30 cm.			Pressure 28.5 cm.			Pressure 16 cm.		
x (cm.).	V(volts).	K	x (cm.).	V(volts).	K	x (cm.).	V(volts).	K
.00166	313	4480	.00166	384	3880	.00166	304	4950
.0025	425	3400	.0025	475	3650	.0025	370	4875
.0033	510	3400	.0033	560	2800	.0033	435	3975
.00417	595	3320	.00417	627	2900	.0050	541	2850
.0050	678		.0050	694		.0067	617	2850
						.0083	694	2780
						.010	769	
Mean values of K,		3650			3320			3 710

Chloroform. — Chemically pure chloroform, dried with fused $CaCl_2$ gave the following results :

Chloroform.

Pressure 13.5 cm.		
x (cm.).	V (volts).	K
.005	510	2370
.010	670	2520
.015	840	2615
.020	1017	
		Mean value of K, 2500

(The data given here are in each case the average of several distinct determinations, the pressure being the same in all.)

Benzol. — In working with benzol, (benzene), it was noticed that the spark potential for a given spark gap, and also the color of the discharge glow, obtained from the first one or two discharges after the discharge tube is filled with fresh vapor is essentially different from that of succeeding sparks. This is probably due to the decomposition of the vapor by the discharge and a consequent change of the ions which carry the discharge, the first spark produces a brilliant green light while later ones give the pinkish tinted glow, the discharge being probably carried by CH_2 or CH_3 ions. A rapid deposition of carbon upon the electrodes as sparks pass seems to confirm the supposition of the decomposition of the benzol.

In the results given below, Set I. are observations taken in each instance, the first spark after the electrode had been freshly polished and the vapor fresh; Set II. are data obtained by taking mean values of the first three sparks in each instance; and Set III. are the results obtained after a number of sparks had passed in each instance; Set I. representing then, the results obtained with the benzol vapor unmixed with impurities while the other sets show the results of increasing amount of impurities introduced.

Benzol.

Set I. Pressure 5 cm.			Set II. Pressure 7 cm.			Set III. Pressure 7.7 cm.		
x (cm.).	V(volts).	K	x (cm.).	V(volts).	K	x (cm.).	V(volts).	K
.010	565	4450	.010	601	3230	.005	520	2600
.015	675	4040	.015	715	3450	.010	620	2210
.020	778	4280	.020	835	3000	.015	705	2210
.025	885		.025	935		.020	790	2340
						.025	880	
Mean values of K,		4240			3230			2340

Ammonia. — Commercial ammonia, taken from a cylinder of the gas was used, being dried by passing through tubes containing solid KOH in small fragments. It is a well known fact that ammonia is decomposed under the action of the electric spark, giving free nitrogen and hydrogen; this was very evident from the observations, the spark potentials dropping rapidly with use of the gas to the values usually obtained for nitrogen, and with extended use falling nearly to the values obtained in hydrogen. A complete set of "first spark" reading was not obtained, the first two readings in each set may be taken as indicative of the results which would be obtained if no decomposition occurred.

Ammonia.

Pressure 32 cm.			Pressure 20 cm.			Pressure 10 cm.		
x (cm.).	V(volts).	K	x (cm.).	V(volts).	K	x (cm.).	V(volts).	K
.002	485	1406	.002	430	1416	.005	400	1400
.005	620	875	.005	515	810	.010	470	900
.010	760	750	.010	596	810	.015	515	800
.015	880		.015	677	1010	.020	555	
			.020	778	1010			
			.025	890				

Mean value of K from first two sparks, 1407.
Mean value of K from all the other observations, 882.

Ether. — The best grade of commercial ether obtainable, dried with calcium chloride, gave the following data:

Ether.

Pressure 20 cm.		
x (cm.).	V (volts).	K
.003	520	3300
.005	620	2500
.010	870	2500
.012	970	
		Mean value of K, 2767

Nitrogen. — From recent, evidently very careful, determinations of spark potentials in nitrogen by Hurst,[1] the values of the gas constant, K are calculated for nitrogen. The results represent averages for various pressures as Hurst gives his spark potentials as function of the product, Px (pressure times spark length).

Nitrogen.

$P.x$ (cm.²).	V (volts).	$\frac{dV}{d(Px)} = K$.
.134	340	1110
.170	380	1140
.240	460	1000
.284	505	730
.420	650	

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[1] Hurst, Phil. Mag. (6), 11, p. 535, 1906.

In comparing the gas constants obtained in the work described above, with the mean free paths of the molecules of the different gases and vapors used it is necessary to deduce the free paths for the particular temperature at which the gas stood during the experiment. According to Meyer (cf. Kinetic Theory of Gases), the viscosity of a gas or vapor depends upon the temperature in the following manner:

$$\eta_t = \eta_0(1 + Bt)^{\frac{1}{2}},$$

where B is the coefficient of cubical expansion of the gas, η, the coefficient of viscosity, and t, the temperature, centigrade.

But we have

$$\eta_0 = .309\ \delta_0\bar{v}_0\lambda_0$$

and

$$\eta_t = .309\ \delta_0\bar{v}_t\lambda_t$$

in which δ, $\bar{v}$, and λ refer to density of the gas, mean square speed, and mean free path of the gas molecules, respectively, and

$$\frac{\bar{v}_0}{\bar{v}_t} = \frac{1}{(1 + Bt)^{\frac{1}{2}}},\ \text{hence}\ \lambda_t = \lambda_0(1 + Bt).$$

The values of mean free path for the different gases and vapors, taken from Landolt and Bornstein's *Tabellen*, at the temperatures at which the substances were used, together with the gas spark-constants determined for spark potentials between 400 and 900 volts are given in the following table. As the values of mean free paths given by different authorities differ considerably in some instances the values given by the most recent determinations are used.

Substance.	Spark Constant K, (volts/cm.^{2})	Mean Free Path, $\lambda \cdot 10^8$ (cm.).	Product $K \cdot \lambda \cdot 10.^2$
Hydrogen	675	1630	1.10
Carbon di-oxide	906	680	.61
Nitrogen	995	990	.985
Air	1050	1025	1.07
Ammonia { first spark	1407	740	1.04
Ammonia { averages	880		.65
Carbon di-sulphide	3560	311 274	1.10 .975
Chloroform	2500	256	.64
Ether	2767	215	.59
Benzol { first spark	4240	235	1.01
Benzol { later ones	2340		.55

It is evident that in the greater number of cases, the product, $K \cdot \lambda$, is approximately constant, or that the function, K, which represents the gas-characteristic in the determination of sparking potentials, is inversely proportional to the mean free path of the gas molecule. The deviations from this law, shown by some of the vapors tested may in all probability be rightly attributed to the decomposition of the gas or vapor so that the medium through which the discharge takes place is not the substance which originally filled the discharge tube. The fact that carbon di-sulphide, of which the decomposition products are solids and hence precipitate out when formed, is the only compound substance which does not show marked difference between first sparks and succeeding ones, obeys the law together with the elemental substances, seems confirmatory to this hypothesis. Again the case of carbon di-oxide is suggestive; it is a well known fact that this gas is dissociated in the electric discharge. Geissler tubes filled with CO_2 under the action of the discharge come ultimately to show almost wholly the spectral lines of the mon-oxide, CO. Now the observations of Orgler and Paschen with carbon di-oxide are such as give a mean value of the constant, K, of 906, (which is obviously smaller than would have been obtained using lower spark potentials), and this quantity, multiplied by the mean free path of the carbon mon-oxide molecule, which is $1040 \cdot 10^{-8}$, gives for the value of the product, $K\lambda$, the number, $.94 \cdot 10^{-2}$ which is approximately the same as that for the permanent gases.

To sum up:

It seems that the assumption of Roentgen is supported by the results of experiment, namely, that the spark potentials in a gas or vapor is a function of the mean free path of the molecule of the gas or vapor, and in fact an inverse function; that is — for a given spark potential, the quantity,

$$\frac{1}{P}\frac{dV}{dx} = K = f\left(\frac{k}{\lambda}\right)$$

the quantity, k, being in turn a function of the spark potential, V, but for a limited range it is approximately constant. And, to the same degree of approximation to which Paschen's law holds, the potential, V, necessary to produce a spark discharge through a distance, x, in any gas is expressed by the equation,

$$V = V_0 + P\left\{f\left(\frac{k}{\lambda}\right).(x - x_0)\right\},$$

in which

V_0 = the minimum spark potential for the gas and electrodes used,
x_0 = the distance of the minimum potential spark at the pressure used,
λ = the mean free path of the molecules of the gas used,
k = a gas spark constant, which has the same value for all gases and vapors, (so long as the spark is not effected by decomposition products of the discharge), but varying with the discharge potential concerned.

THE BRACE LABORATORY OF PHYSICS,
LINCOLN, NEBRASKA, July, 1906.

THE CAPACITY AND RESISTANCE OF ALUMINIUM ANODE FILMS.[1]

BY C. McCHEYNE GORDON.

THERE is formed on aluminium when used as an anode in most solutions, a film which is highly insulating in character so long as the voltage applied is not too high, or the direction of the current is not reversed. These films have been the subject of a large number of investigations ; interest in them having been especially lively during recent years since the possibility of taking advantage of the asymmetric resistance for rectifying alternating currents has been realized. These investigations have proven that the resistance of the films and the voltage at which the resistance begins to decrease rapidly, generally called the critical voltage, vary with the electrolyte used as well as with the temperature. They have shown however considerable conflict of evidence and opinion as to the chemical constitution and reasons for the asymmetry of resistance and its breaking down at higher voltages.

No extensive study of the electric capacity of these films has heretofore been made. Scott[2] made a few measurements on the polarization capacity of aluminium in connection with a number of other metals, while the electrodes were subjected to a definite polarization, but his measurements on aluminium are all taken at low voltages — not greater than 1.75 volts ; and only in one solution — sulphuric acid. A number of others have given capacity values, but without explicit statements as to conditions or method of measurement.

METHOD OF MEASUREMENT.

The method used to measure the capacity is that developed by the writer[3] at the suggestion of Nernst for the measurement of the capacity of polarization. It is the well known wheatstone bridge arrangement for comparing capacities with slight modifications. The connections are shown in the figure. E_1 and E_2 are the aluminium electrodes dipping into the solution *S*; E_1 is made so large in comparison with E_2 that the capacity measured is practically that of the small electrode alone; *K* is a three-point switch by means of which the cell is connected first to the battery *P*, in order to form the film on E_2, and then switched into the bridge for measurement.

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[1] A summary of this paper was given at the Ithaca meeting of the A.A.A.S., July, 1906.

[2] Scott, Wied. Ann., 67, p. 388, 1889.

[3] Gordon, Wied. Ann., 61, p. 1, 1897.

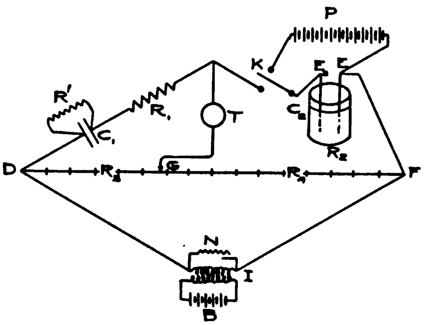

Fig. 1.

The capacity C_1 was a mica condenser of 0.5 Mf., or a paper condenser of 6.8 Mf. This paper condenser was marked by the maker 10 Mf., which was evidently its value by direct current measurement. Such condensers always give much lower values when measured by alternating currents; but their values are always consistent with one another, if a number of such condensers are compared with one another or with mica condensers.

R_1 is a variable non-inductive resistance in series with the condenser C_1, which is necessary to balance the conductivity of the solution. *DGF* is the bridge wire and *T* the telephone receiver. *I* is a small induction coil with automatic breaker, operated by the battery *B*; *N* is a shunt by means of which the current is kept small. Large currents give polarization effects which interfere with the accuracy of the bridge setting.

R′ is a variable resistance which serves to balance the conductivity of the film itself. This resistance was found however to be unnecessary in most of the measurements of the capacity and was generally omitted.

With this arrangement of the bridge three conditions must be satisfied simultaneously in order that there shall be no current in the telephone, namely;

$$C_2 = C_1 \frac{R_3}{R_4}, \quad R_2 = R_1 \frac{R_4}{R_3}, \quad \text{and Film } R = R' \frac{R_4}{R_3}.$$

Using electrodes of a few square centimeters area, the position of the minimum is practically determined by the capacities. If the resistances are too far off from the proper ratios, the minimum is not sharp but with an approximate adjustment of the resistances it is possible to obtain a setting on the bridge certain to one millimeter.

These experiments were planned with the expectation that we could thus readily obtain an accurate measurement of the capacity and the resistance of the films at the same time. It was found however that the films most suitable for capacity measurements had a resistance so large as to have little effect on the sharpness of the telephone minimum, and it was accordingly decided to leave the resistance of the films for a later investigation, which is still in progress. Some interesting preliminary results on the resistance obtained are given further along in this paper.

The electrode E_2 on which the films were formed was a strip of sheet aluminium about 1 cm. wide and dipping into the solution from 1 to 4 cm. The other electrode was a sheet of aluminium about twenty times as large. In preliminary measurements a small electrode was also tried in place of the large one; a large sheet of lead was also substituted for the large aluminium electrode. The capacity calculated per cm.^{2} of one side of the electrode was the same for all three cases.

THE POLISH OF THE SURFACE.

The degree of the polish of the surface makes a considerable difference in the size of the capacity. Surfaces formed by cutting the aluminium under turpentine with a sharp instrument gave the smallest capacities. Sheets polished with emery cloth and sand paper gave larger values. At higher voltages the difference in the polish does not make so much difference. The variations in capacity with the polish of the surface are probably due to the actual differences in surface area. With the higher formation voltages the films are thicker, the uneven places are probably to some extent filled up and the capacity values consequently not so far apart. As the capacity values for surfaces cut under turpentine were practically the same as those of smoothly polished aluminium as obtained from the manufacturers, the latter was used for the rest of the experiments. A series of measurements showing the amount of the variation with the differently polished surfaces is given in Table I.

TABLE I.

$(NH_4)_2CO_3$ Solution.

Preparation.	Capacity in Mf. per cm.^{2}	
	20 Volts.	100 Volts.
Cut under turpentine	0.33	0.0686
Emery cloth under turpentine	0.57	0.111
Emery cloth in air	0.70	0.129
Sand paper in air	0.65	0.129

TUBULAR ELECTRODES.

A number of observers have found that tubular electrodes, especially when water-cooled, are better for rectifying the current than the ordinary sheet electrodes. Fischer[1] found that water-cooled tubular electrodes, in sulphuric acid, insulated well at 100 volts and gave films several tenths of a millimeter in thickness. The tubular electrodes, however, even when cooled with water near 0°, give the same capacity values as the sheet electrodes. The thicker films found for sulphuric acid and sodium sulphate seem to be composed of two layers, the thicker part being a film of good conductivity, concerning which more will be said further on, under the discussion of the thickness of the films. The most noticeable difference between the water-cooled tube electrodes and those without water was in the rapidity with which the capacity increased after the formation current was turned off. In the first five minutes after turning off the current from the water-cooled electrode the capacity increased about ten per cent., while without water the increase was less than one per cent. in the same time.

TEMPERATURE.

A number of observers have found that the voltage at which the films begin to break down — the so-called critical voltage — decreases rapidly with the increase of the temperature. No corresponding variation in the capacity was found. The capacity remained the same from 10° to 25° C., at least within the limits of experimental error, two to three per cent. After this had been established for three or four solutions, the remaining measurements were taken at room temperatures, probably varying between 15° and 25° C.

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[1] Fischer, Zeit. für Eleck. Chem., 10, p. 869, 1904.

Time for the Formation of the Films.

The formation of the film is not by any means instantaneous. The current is at first large, but decreases rapidly, coming to practically constant values after some time. The decrease in the capacity as the current continues to run is likewise at first rapid and then slower, approaching a minimum value. This decrease in the capacity is not proportional with that of the residual current. For example in one case where the residual current was measured, the capacity decreased but six per cent. while the residual current changed in the ratio of six to one. Generally after five seconds application of the direct current the capacity was within a few per cent. of its minimum value. The time required to bring it within one per cent. of its minimum value varied largely with the voltage and somewhat with the solution. For 20 volts it was generally about five minutes and for 100 volts sometimes as much as twenty minutes. The values for the capacity given in this paper are minimum values, assuming that the minimum is attained when a further application of the formation current for five minutes gives a decrease of capacity of less than one per cent.

Capacity Measurements While the Polarizing E.M.F. is Still Applied.

Scott inserted the polarizing battery in the telephone arm of the bridge, and thus was able to take the capacity measurements while the polarizing current was acting. I made a comparison of this method with that as given above and found that the capacity measured with the current on, was the same as after it was off. The method as given above was accordingly used on account of its greater simplicity. The two methods give very different values for the film resistance, but more will be said about this in another paragraph.

FORMATION VOLTAGE AND CAPACITY.

After the above preliminary experiments, the first question considered was as to how the capacity changes with the voltage applied to form the film — which we shall speak of hereafter as the "formation voltage." Scott found that the capacity of aluminium in sulphuric acid decreased rapidly as the formation voltage increased, but as he did not use voltages higher than 1.75, he did not discover any simple relation between capacity and voltage. My first measurements were with a potassium alum solution and it was seen at once that for higher voltage the *capacity varies inversely as the formation voltage.* Measurements for two solutions are given in Table II. Similar results were obtained for a number of other solutions. The areas of the films whose capacities are given in this table were not measured.

TABLE II.

$HNaNH_4PO_4$ Solution.			$K_2Cr_2O_7$ Solution.		
Formation Voltage.	Capacity.	Product.	Formation Voltage.	Capacity.	Product.
I.	II.	I. × II.	I.	II.	I. × II.
20	1.60	32.0	10	3.8	38.0
30	1.11	33.3	20	1.95	39.0
40	0.83	33.2	40	0.95	38.0
50	0.64	32.0	81	0.48	38.9
60	0.54	32.4	90	0.42	37.8
71	0.46	32.7	100	0.38	38.0
80	0.40	32.0			
91	0.35	31.8			
100	0.32	32.0			

The variations of the product from a constant value are well within the experimental error. The natural explanation of this relation between the capacity and the formation voltage is that the thickness of the films is proportional to the formation voltage. During the progress of this investigation an article has been published by Fischer[1] in which he comes to the same conclusion in regard to the films formed in sulphuric acid; but the sulphuric acid films are quite different in character from those of most solutions as will be set forth below.

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[1] Fischer, Zeit. für Elek. Chem., 10, p. 869, 1904.

THE CAPACITY IN DIFFERENT SOLUTIONS.

Experiments were next made to determine how the capacity might vary as the electrolyte was changed. The literature of the subject shows that with a change of the electrolyte there are large variations in the insulating power of the films and in the critical voltages. With sulphuric acid for instance, at room temperatures, the films break down between twenty and thirty volts, while for sodium potassium tartrate the critical voltage has been found to be at least as high as one hundred volts and for citric acid five hundred volts. Where chemical analyses of the films have been made they have been found to contain a considerable portion of the acid radical of the electrolyte. In view of these facts it was expected that each electrolyte would have, for any given formation voltage, its own individual capacity value, differing from that for other electrolytes, because of the difference in thickness and dielectric constant, which a difference in the chemical nature of film would probably cause. This expectation, however, was not realized, as will be seen from the figures given in Table III.

TABLE III.

Electrolyte.	Capacity in Mf. per cm.^{2}	
	Formation Voltage 20.	Formation Voltage 100.
$(NH_4)_2CO_3$	0.34	0.069
$K_2Cr_2O_7$	0.35	0.069
$(CHOH)_2\ (CO_2)_2\ KNa$	0.35	0.069
$NaNH_4HPO_4$	0.34	0.072
$(CH_2)_2\ (COH)\ (CO_2H)_3$	0.33	0.070
Na_2SO_4	0.39	
H_2SO_4	0.50	

The above series of measurements was repeated several times. The variations seldom amounted to as much as three per cent. These small variations are easily accounted for when we take into consideration that with the voltmeter used errors in reading as large as two per cent., at twenty volts, might readily be made. The exact length of the film, determined simply from the depth immersed, could likewise not be measured with any great accuracy on account of capillary irregularities. The above figures show, therefore, that within the experimental error at least, the capacity of the first five solutions, both at twenty and one hundred volts, is the same; and at the same time confirm the relation found before, *i. e.*, that the capacity is inversely proportional to the formation voltage.

The larger values found for sodium sulphate and sulphuric acid may be due to the fact that in these solutions the film dissolves more rapidly than in the others. While for the other solutions there was no decrease of the capacity with the film standing in the solution for five minutes after the formation current was turned off with sodium sulphate there was a small, and with sulphuric acid a large decrease in this time. Two sets of measurements showing the rapidity of this increase are given in Table IV.

TABLE IV.

H_2SO_4 Solution, Dilute.

Seconds after Disconnecting the Formation Current.	Capacity (Size of Electrode not Measured).	
	Series I.	Series II.
20	0.400	0.393
80	0.452	0.427
170	0.65	0.62
230	0.93	0.89
290	1.09	1.07
350	1.20	1.16

FILMS IN FUSED SALTS.

Insulating films are also formed with fused salts as the electrolyte. In a mixture of fused sodium and potassium nitrates, the capacity was found to be smaller than in the water solutions. The capacity increased, however, so rapidly after the formation current was switched off, that no accurate determinations of the minimum value could be made. A series of measurements showing this rapid increase in capacity is given in Table V.

It is evident that the value obtained five seconds after the formation current is switched off is not the minimum value, but even this is only about one-sixth as large as water solutions give at the same voltage. The film evidently dissolves rapidly, making the film thinner and the capacity consequently larger.

Table V.

Fused KNO_3 and $NaNO_3$. Temperature 265° C. Form. Volts 20.	
Seconds after Formation Current was Disconnected.	Capacity in Mf. per $cm.^2$
5	0.06
60	0.16
120	1.05

The resistance of these fused salt films is much less than those of water solutions. While the resistance of the latter was so large that a resistance in parallel with the condenser had little effect upon the minimum, in case of the fused salts this parallel resistance must be adjusted somewhat carefully. On account of the rapid shifting of the position of the minimum no very accurate measurement of this resistance could be made but the largest value obtained was about 400 ohms for a surface of 15 $cm.^2$ or 6000 ohms per $cm.^2$

The Resistance of the Films.

As was stated in a previous paragraph the resistance of the films formed at these voltages is so large as to have little effect on the bridge minimum. As these voltages seemed to be the most suitable for obtaining the comparative values of the compacity, it was decided to carry through the above measurements without attempting to obtain accurate values for the resistance at the same time, and to make an investigation of the resistance of the films later. The latter investigation is not yet completed but a few of the more striking results as determined from the preliminary experiments are included here.

1. The apparent resistance, as calculated from the residual direct current, does not increase at all in proportion with the decrease in capacity. If, as it seems reasonable to suppose, the decrease in the capacity during the formation of the film is proportional to the increased thickness of the film, the increase in apparent resistance is due to something else than mere increase of thickness.

2. The resistance as measured with the small alternating current in the bridge is much smaller than the apparent resistance as calculated from the size of residual current.

3. The resistance as measured in the bridge while the formation

current is still active (the arrangement of Scott) is only about one third as large as when measured after the direct current is taken off (the arrangement of the figure above). A discussion of the probable cause of this striking difference is deferred until some measurements of the resistance under other conditions are completed.

The Thickness of the Films.

If we knew the dielectric constant for these films we could of course calculate their thickness from their capacity. But even without such knowledge we can draw some conclusions as to the maximum and minimum values of the thickness from what we know as to the possible range of the dielectric constants, as well as to the relative thickness of the films under various conditions of formation.

In the first place it is to be noted that the films formed in sulphuric acid and sodium sulphate are very different in character from the other films. In the sulphates the whole film is evidently not insulating in charater; *but it seems as if we have an insulating film covered with a much thicker conducting one.* The reasons for this conclusion are as follows. Mott[1] and others had found that the films in sulphuric acid were much thicker than in other solutions. Mott says in one place that he found the thickness in sulphuric acid at twenty volts greater than that in a phosphate solution at 100 volts. In *apparent* contradiction to this I found the capacity of the sulphuric films to be somewhat larger than in any other solutions at corresponding voltages. If we use water cooled tubular electrodes it is easy to form films in sulphuric acid at formation voltages as high as 100, whose thickness is readily measured as it amounts to several tenths of a millimeter. Fischer[2] found the thickness of such films at 72 volts to be 0.03 cm. and I have found films formed at 100 volts to be between 0.03 and 0.04 cm. thick. The lowest values found for the capacity of such a film was 0.15 Mf. cm.^{2} (Sheet aluminium with same surface polish gave 0.129 Mf.)[3] As the film stood in the solution the capacity increased very rapidly, becoming about ten times as large within a half hour, but there was no corresponding change in the thickness during that time.

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[1] Mott, Elect. Chem. Ind., 2, p. 268, 1904.

[2] Fischer, l. c.

[3] This was in sodium carbonate solution, as given in Table I.

This alone would be enough to cause us to conclude that the visible film is not the same as the insulating film whose capacity is measured. The calculation of the dielectric constant which would be necessary in order that the whole film should have such a large capacity makes this conclusion doubly sure. Using the formula

$$C=\frac{KA}{4\pi d},$$

where C is the capacity, K the dielectric constant, A the area, and d the thickness, all expressed in c.g.s. units, for a capacity of 0.15 Mf. per cm. and a thickness of 0.03 cm., the dielectric constant would be over 50,000, which is of course out of the question.

In all the other solutions the films are not thick enough to be noticeable to the eye, and I know of no reason for supposing them to consist of anything else than a single insulating layer. It is safe to assume that the dielectric constant for this layer is somewhere between 1 and 80. Mott[1] and Zimmerman[2] conclude from their experiments that it is probably about 14. Table VI. gives the thickness of the films calculated from the capacity per cm.^{2} found above, assuming the dielectric constant to be 1, 10 and 100.

TABLE VI.

Calculated Thickness.

	$K=1.$	$K=10.$	$K=100.$
20 volts....................	2.5×10^{-7}	2.5×10^{-6}	2.5×10^{-5}
100 volts..................	12.5×10^{-7}	12.5×10^{-6}	12.5×10^{-5}

Zimmerman and Mott have estimated the thickness of such films from their interference colors. Mott estimates the thickness for 100 volts to be 10×10^{-5}. Zimmerman gives the thickness of a 100 volt film as 7.5×10^{-5}, but in another place speaks of a 100 volt film with a thickness of 2×10^{-5}.

[1] Mott, Elect. Chem. Ind., 2, p. 352, 1904.

[2] Zimmerman, Trans. A. E. S., 5, p. 147, 1904; 7, p. 309, 1905.

THE NATURE OF THE FILMS.

Mott[1] has advanced the hypothesis that the chemical nature and consequent thickness of the films are largely dependent upon the nature of acid radical of the electrolyte. Since, however, the capacity is independent of the electrolyte, this view, at least in so far as it has to do with the insulating film, is no longer tenable. The capacity being the same, we must conclude that the thickness and the dielectric constant are both independent of the electrolyte, or that in all the different solutions tried, the one varies exactly as the other. The latter might accidentally happen for two solutions but that it should be so for so many solutions is too improbable to be considered. If, however, we have the thickness and the dielectric constant both unchanged, it follows that we have practically the same chemical constitution for all these solutions. Of course there must be some chemical or physical differences in the films to account for the differences in conductivity and critical voltages. The results of this paper seem to fit in best with the theory that the films in all the water solutions are made up of aluminium hydroxide full of pores, and that the variations in the behavior of the films as we go from one electrolyte to another are due to the variation in the size or number of such pores.

CENTRAL UNIVERSITY,
DANVILLE, KY., July, 1906.

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[1] Mott, Elect. Chem. Ind., 2, p. 444, 1904.

INFRA-RED EMISSION SPECTRUM OF BURNING CARBON DISULPHIDE.[1]

BY W. W. COBLENTZ.

IT is becoming more and more a recognized fact that the infra-red spectrum is the seat of great disturbances which can be attributed to well-known groups of atoms or "ions." The pioneers in this field of investigation were Ångström and Julius. Their interest in the subject dates back to the time when rock salt first became recognized as a means for producing a heat spectrum. The dispersion of rocksalt was then undetermined beyond 5 μ, and in order to express their emission and absorption bands in wave-lengths they adopted a tentative method of extrapolation, which, since then, has been found to be erroneous. In the meantime, data on infra-red spectra have continued to accumulate, which are often in violent disagreement. For example, carbon disulphide is variously quoted as having a band at 6.7 to 8.4 μ while the true value is about 6.8 μ.

During the past few years the writer has attempted to determine the values of the maxima of absorption and emission as accurately as possible in absolute value of wave-lengths, and thus bring a little harmony out of this chaos. This means repeating part of the work in order to get a check upon the extrapolation. One of the most interesting pieces of work of this type is that of Julius[2] who found the emission spectra of gases during combustion. It contains a very considerable amount of careful work, certain parts of which appear to have gone quite unnoticed by later investigators. It is of no little interest for it contains evidence of emission bands farther in the infra-red than subsequent work, along other lines, has been able to show. These bands belong to the acid elements and appear at low temperatures, *i. e.*, they do not appear in spark (and arc?) spectra. This is just the opposite of what the writer found for the basic elements (metals) in which no emission bands were found beyond 2 μ.

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[1] Extracts from a memoir submitted to the National Bureau of Standards for publication.

[2] Julius, Licht und Wärmestrahlung Verbrannter Gase, Berlin, 1890.

Perhaps this may eventually give us some clue to the mechanism of radiation. We have two sharp distinctions between the acid and the basis elements. The metals have selective absorption in the short wave-lengths, are opaque to infra-red rays, and no emission lines have been found beyond 2 μ. In fact, according to Pflüger's[1] work the maximum of the emitted energy lies in the short wave-lengths beyond the visible spectrum. On the other hand, the acid elements have bands of selective absorption throughout the spectrum and have emission bands extending far into the infra-red, which generally coincide with the marked absorption bands. The question of the coincidence of these emission and absorption bands in CS_2 led the writer to undertake the present work.

A previous examination of the absorption spectra of gases[2] showed very marked bands in SO_2 which were in quite close coincidence with the emission lines of the products of combustion of carbon disulphide, as given by Julius. The transmission spectrum of CS_2, Fig. 1, has a very large absorption band at 6.8 μ (opacity from 6.6 to 7.0 μ) and smaller bands at 4.6 and 11.65 μ, while SO_2, Fig. 2, has a very large band at 8.7 μ, a somewhat narrower band at 7.4 μ

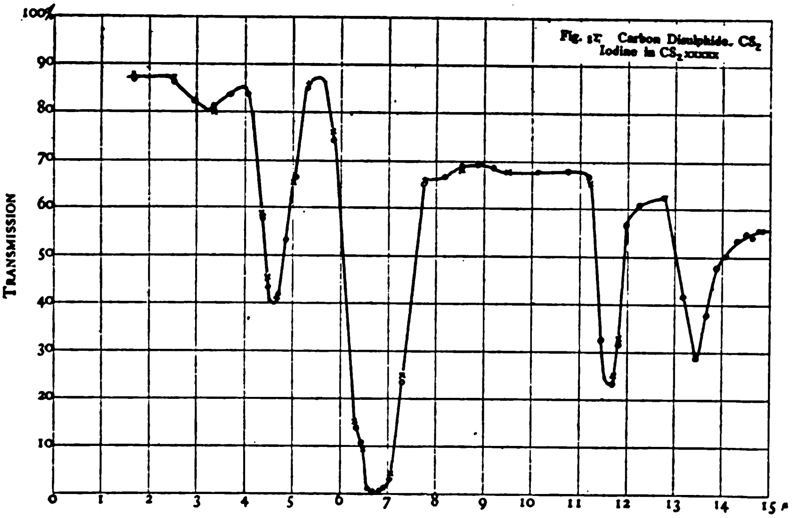

Fig. 1.

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[1] Pflüger, Ann. der Phys. (4), 13, p. 890, 1904.

[2] Physical Review 20, 273, 1905.

and several small bands; H_2S has a series of small bands which need hardly be considered in the question of the combustion products of CS_2. In Fig. 3 is given the emission curve of burning CS_2. In curve *a* the maximum at 4.35 μ is drawn to the same scale as the rest of the curve. In curve *b* the bands at 6.75 and 7.45 μ have been magnified ten times to bring out the sharpness of the maxima. In the present examination the lower part of the flame was examined. The lamp consisted of a tin lubricating-oil can, the neck of which was 5 cm. long, to prevent heating the CS_2 and thus avoid an explosion. Julius (loc. cit.) examined the upper and the central parts of a CS_2 flame and found that for the central region the maximum at 6.75 μ is more intense than the one at 7.45 μ while for the upper region the 7.45 μ band is the more intense. This is to be expected for the 6.75 μ band is evidently due to hot CS_2. In the lower part of the flame the combustion is incomplete and hence the hot vapor ought to be more intense than nearer the top of the flame. In the present examination the lower part of the flame was used, and the ratio of the intensities of these two bands is still greater than that found by Julius; here the ratio is 34 to 73 while Julius found a ratio of 16 to 22 for the central part of the flame and a ratio of 11 to 8 for the top.

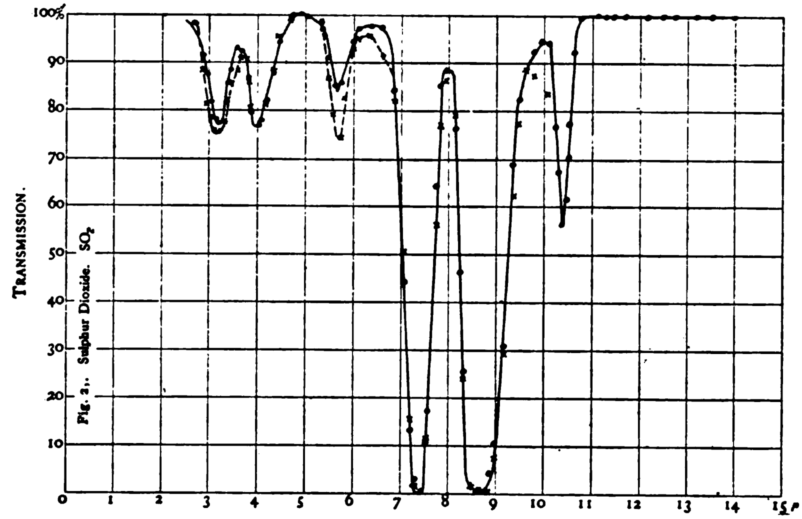

Fig. 2.

The band at 7.4 μ is in common with the absorption band of SO_2 and since Julius found this band also in burning sulphur, it no doubt belongs to this gas. Then the interesting question arises, "Why is there no emission band at 8.7 μ?" The question must be left unanswered. It is interesting to note, however, that the 8.7 μ band is found in H_2SO_4, and in sulphates, and appears to belong to the SO_4 ion, while another band found in SO_2 gas, at 10.37 μ occurs in fuming H_2SO_4 and has been attributed to SO_3, by Pfund.[1]

The bands at 2.7 μ and 4.35 μ are in common with the emission bands of CO_2 and are no doubt due to the combustion of the lamp-wick.[2] This is one of the few examples of the emission of a heated gas during the process of combustion. Water vapor and CO_2 are other examples studied by Rubens and Aschkinass. Kayser in his Spectroscopy very aptly remarks that the emission band of methane, CH_4, at 3.3 μ has never been observed in a flame. This is a very weak absorption band, however, and for this reason might not appear in emission, just as was found for the 4.6 μ band of CS_2. The 6.75 μ band which coincides with that of CS_2 was ascribed to COS by Julius who records its maximum in wave-lengths at 8.48 μ. The 7.45 μ band was thought to be at 10.01 μ. The emission spectra

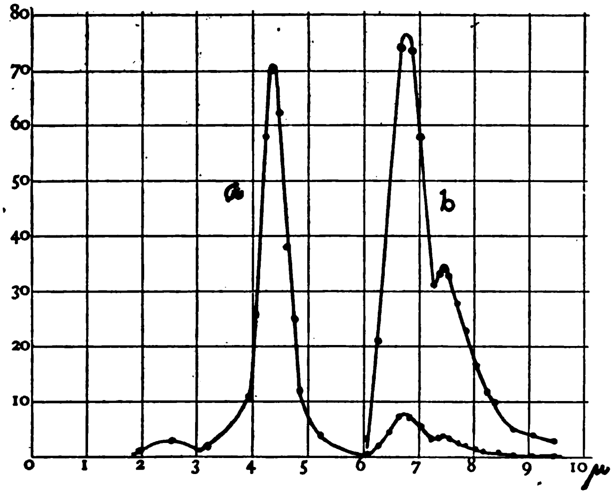

Fig. 3. Emission Spectrum of CS_2.

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[1] Pfund, Astrophys. Jour., 24, p. 19, 1906.

[2] The combustion of CS_2 gives CO_2 and SO_2.

of H_2O and CO_2 have been redetermined by others and need not be mentioned here. It will be sufficient to add that for CO_2 the maximum of emission depends upon the temperature. Whether this is true of the SO_2 bands remains undetermined, but it is hoped to examine this point in the near future. If the shifting of the CO_2 band is really due to dissociation into CO, as experiments seem to indicate,[1] then one would hardly expect a similar shift for SO_2. The only other emission band that needs correction is that of HCl at 3.68 μ. A more probable value is 4.05 μ which is close to the HCl band, found by Ångström and Palmer[2] at 3.41 μ. The corrected value is 3.98 μ.

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[1] Physical Review 221, 1906.

[2] Aengström and Palmer, Ofversigt Kongl. Vet. Akad., No. 6, p. 389, 1893.

ON THE IONIZATION OF FUSED SALTS.[1]

By H. M. Goodwin and H. A. Wentworth.

Contributions from the Research Laboratory of Physical Chemistry, No. XII.

THE following investigation was undertaken in the spring and summer of 1905 in the hope of contributing something to our knowledge of the applicability of the well established laws of dilute solutions to fused mixtures, and possibly of throwing some light on the question of the constitution of salts in the state of fusion.

Investigations bearing upon the determination of the degree of dissociation of fused salts have already been published, but the results obtained are meagre and cannot be regarded as conclusive. This is chiefly due to the peculiar difficulties inherent in the problem which are not encountered in aqueous or other solutions at ordinary temperatures. Aside from the obvious experimental difficulties incident to working at high temperatures and with vessels unattacked by fused salts, are the difficulties arising from the fact that pure fused salts are in general such excellent electrolytic conductors; although their conductivities may now be determined with precision,[2] transference experiments in them are impossible, since no concentration changes occur at the electrodes during electrolysis, and data on the absolute velocity of migration of ions in such conductors are at present entirely wanting. Direct velocity experiments with fused electrolytes have been recently attempted in the electrochemical laboratory of the Institute, but the exceeding difficulty of the problem has prevented as yet the obtaining of any satisfactory results. With solutions of one salt in another the relations are far more difficult of analysis than in aqueous solutions where the conductivity of the solvent may practically be neglected. Here, to be sure, transference experiments are possible and Lorenz[3] has made a first attempt in this direction with solutions of lead chloride in potassium chloride, his results pointing to more complex relationships than had been supposed.

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[1] This research was carried out under a grant from the Wm. E. Hale Research Fund, to the trustees of which grateful acknowledgment for the aid furnished by them is hereby made.

[2] An investigation on this subject is now being carried on by one of us, the results of which will be published shortly.

[3] Zeit. für Electrochemie, 10, 630, 1904.

The most promising method of attacking the problem of the dissociation of one fused salt in another seems at present to lie in the application of some selective ion-phenomenon, *i. e.*, in measuring some quantity which depends, so far as we know, on the presence of a specific molecular or ionic complex. Of the quantities suitable for this purpose the electromotive force of certain voltaic cells composed of fused mixtures lends itself best for determining specific cation concentrations, provided that Nernst's theory of the voltaic cell is applicable to combinations of this type. It is upon the results of electrometric experiments that most previous work on this subject has been based and upon which the present investigation rests.

The first experimental work bearing upon the problem under consideration was carried out by Gordon[1] and consisted of the measurement of the electromotive force of concentration cells of the type

$$\begin{array}{r|c|c|l} & c_1, - \mathrm{AgNO_3} & c_2, - \mathrm{AgNO_3} & \\ \mathrm{Ag} & \text{in} & \text{in} & \mathrm{Ag}, \\ & \mathrm{KNO_3} + \mathrm{NaNO_3} & \mathrm{KNO_3} + \mathrm{NaNO_3} & \end{array}$$

equimolecular mixtures of sodium and potassium nitrates being used to give a solute of low melting point. The concentration of the silver nitrate was varied from 0.1 to 100 per cent. On the assumptions, first, that Nernst's formula applies to cells of this type, second, that the electrolytic solution pressure of silver is independent of the solutions in question, and third, that the dissociation of a 0.1 per cent. solution of silver nitrate may be regarded as complete, Gordon calculated that the dissociation of a 50 per cent. solution of silver nitrate was 69 per cent. at 236° C., and a 100 per cent. solution, *i. e.*, pure silver nitrate, was 58 per cent. dissociated. While the experimental results of this investigation which was carefully carried out may be accepted, certain of the above assumptions on which the computations rest render the conclusions open to question.

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[1] Gordon, Zeit. f. Phys. Chem., **28**, 302, 1896.

More recently the same problem has been taken up from a slightly different standpoint by Lorenz and his pupils at Zurich. Following an extended series of investigations[1] on decomposition potentials and on the electromotive force of cells composed of fused salts, Suchy[2] in 1901, from measurements of cells of the type computed

$$\begin{array}{c|c|c|c} & & \text{AgCl} & \\ \text{Pb} & \text{PbCl}_2 & \text{in} & \text{Ag}, \\ & & \text{LiCl} + \text{KCl} & \end{array}$$

the dissociation of silver chloride in a lithium and potassium chloride mixture as follows:

Two cells of the above type differing only in the concentration of silver chloride were measured at temperatures varying from 500° to 650° C. The concentrations chosen were 1.858 *n* — AgCl and 0.035 *n* — AgCl respectively. If the observed electromotive force of these cells at corresponding temperatures be E_1 and E_2 respectively, and Nernst's formula be assumed to hold for each, then

$$E_1 - E_2 = \frac{RT}{2F} ln \frac{P_{Pb}}{p_{Pb}} - \frac{RT}{F} ln \frac{P_{Ag}}{p_1} - \frac{RT}{2F} ln \frac{P_{Pb}}{p_{Pb}} + \frac{RT}{F} ln \frac{P_{Ag}}{p_2}$$

or

$$E_1 - E_2 = \frac{RT}{F} ln \frac{p_1}{p_2},$$

$$= \frac{RT}{F} ln \frac{c_1}{c_2},$$

in which c_1 and c_2 are the *silver-ion* concentrations at the two electrodes respectively. If $E_1 - E_2$ be measured and the value of c_1 known or assumed, c_2 may be calculated. Suchy assumed that the 0.035-normal solution of silver chloride was completely dissociated, from which it followed that the computed dissociation of the 1.8-normal solution of silver chloride at 600° was 33 per cent. From measurements of a similar cell in which pure silver chloride replaced the silver chloride solution at one electrode, values for the dissociation of the pure salt at various temperatures were also computed. They varied from 17 per cent. at 520° to 60 per cent. at 730°. These results are regarded by the author as only first approximations. They rest on the following assumptions: (1) That a 0.035-normal solution of silver chloride in a chloride mixture is completely dissociated (questioned by Lorenz himself)[1]; (2) that the electrolytic solution pressure of silver may be regarded as independent of the solvent for solvents as widely different as fused pure silver chloride and a mixture of lithium and sodium chlorides (see discussion by Bodlander and Lorenz)[2]; (3) that the potential difference at the liquid junction is negligible — an assumption probably well founded; (4) that the effect of a common anion in the solvent on the dissociation of the solute may be neglected. This last assumption seems to have been entirely disregarded in all previous computations of the kind under discussion.

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[1] Czepinski, Zeit. für anorg. Ch., 19, 208, 1899. Lorenz, ibid., 19, 283, 1899. Weber, ibid., 21, 305, 1899. Lorenz, ibid., 22, 241, 1900.

[2] Suchy, Zeit. für anorg. Ch., 27, 152, 1901.

In concluding this resumé of previous work mention should be made of one other method of computation of the order of magnitude of the dissociation of a fused salt suggested by Abegg,[3] based on the assumption that the decomposition voltage of an ion is independent of the medium in which it is dissolved. Thus, if 0.7 volt be taken as the decomposition voltage of silver chloride, he computes that the silver ions in fused silver chloride must have a concentration of about 0.01 normal.

Beyond these investigations little has been accomplished in determining the degree of dissociation of fused electrolytes. It was with the hope of obtaining some further data which might contribute towards the elucidation of this problem that the following investigation was undertaken, particular attention being directed to the possible effect of the dissociation of the solvent on the dissociation of the solute.

APPARATUS AND METHOD.

Furnaces. — For carrying out the experiments described below, we required two furnaces, both of which were of the electrical resistance type. The principal furnace, in which the measurements were made, was constructed of four porcelain tiles forming the inside lining of a rectangular iron box. Through holes which extended from the top to the bottom edges of the tiles, were threaded about twelve feet of No. 21 (B. & S.) platinum wire. The outside of the furnace was covered with asbestos to prevent radiation as far as possible.

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[1] Zeit. für Anorg. Chem., 32, 246, 1902.

[2] Zeit. für Anorg. Chem., 31, 389, 1902; 32, 235, and 239, 1902.

[3] Zeit. für Chem., 5, 353, 1899.

Within the furnace was placed a box of thin cast iron, the approximate inside dimensions of which were 5.7 cm. × 3.2 cm. × 6.4 cm. deep. This formed the receptacle for a bath consisting of a mixture of fused sodium and potassium nitrates with a small amount of silver nitrate added. The composition of this bath was not determined exactly, a mixture of nitrates being used simply to obtain a low melting point. Gordon found in his experiments that the composition of the intermediate electrolyte, connecting the halves of a concentration cell, had no effect on the results. The bath was well stirred by a glass stirrer driven by a small electric motor, and was kept covered with several sheets of mica to prevent excessive radiation. The surface of the liquid seemed to be the greatest source of heat loss, but for temperatures below 500° C. this was not excessive, when the apparatus was well shielded from air currents. The temperature was regulated by an adjustable rheostat connected in series with the furnace, and could be maintained constant to within 2° C., as long as desired.

Another large platinum resistance furnace was used for melting the salts in the proper cells prior to plunging them into the above described nitrate bath for measurement. This was constructed by winding about three meters of No. 28 platinum wire on a clay cylinder, 25 cm. high by 8 cm. diameter, previously coated with pure magnesium oxide; the cylinder was then heat insulated by packing it with a layer of magnesia ten centimeters thick. It was found convenient to first melt the salts in this furnace, as the glass vessels usually cracked if they were put directly into the fused salt bath. Both furnaces were run on the 110 volt D.C. circuit and the heating current adjusted by rheostats.

Temperature Measurements. — Temperatures were measured by a platinum-rhodium junction. The junction was inserted in the bath between the two parts of the vessel containing the fused salts to be measured. The stirring was, however, so effective that all parts of the bath were always practically at the same temperature.

The temperatures were deduced from deflections of a sensitive d'Arsonval galvanometer of the Le Chatelier type. In order to prevent changes in the sensitiveness of the galvanometer due to temperature changes in the room from time to time, the galvanometer was enclosed in a small air thermostat built up of asbestos and magnesia board, the mirror being observed through a small window. As great precision in temperature regulation was unnecessary in this investigation, the sensitiveness of the galvanometer was so adjusted that one degree temperature change gave a deflection of about one scale division. The instrument could easily be read to one half of a division, hence temperatures could be measured to one half a degree without difficulty, a precision greater than the work really demanded. The junction was calibrated every few days throughout the work, and the calibration was found to remain very constant. The following points were taken for calibration : water, 100°, naphthalene 218°, benzophenone 306°, sulphur 445°. The cold junction was kept always at zero degrees in an ice mixture.

Potential Measurements. — Potentials were measured by the usual Poggendorf compensation method, by balancing them against a fall of potential along a calibrated slide wire bridge. The voltages were repeatedly checked by comparison with a standard Weston cell and a Clark cell standarized at the Reichsanstalt. A Lippmann electrometer sensitive to 0.0003 volt was used as indicating instrument. All circuits were carefully insulated, and to prevent any possible leakage from the 110 volt heating circuit, the latter was opened before each potential measurement.

Form of Cell. — The cells were made of Bohemian glass. As this does not begin to soften until about 800° C., it was sufficiently hard for all temperatures used in our experiments, which did not exceed 450°. For the first portion of the work, H-tubes, 9 cm. in length and 6.4 mm. diameter, having an inclined cross arm of small tubing were used. This cross arm was filled with a plug of prepared asbestos which was found to completely prevent diffusion of the solutions in the two sides. The liquid-liquid contact came well within the boundaries of the plug. The H-tubes were immersed to within 3 cm. of their top, in the fused nitrate bath, their contents being thus entirely insulated from the material of the bath. The greater portion of our work was carried out with cells of this kind.

Since, however, these were found to crack rather easily, the following modified form of cell was used in some of the later work. In this the two electrodes were contained in separate glass tubes drawn down at the end, as in the experiments of Suchy and others, electrolytic connection between them being made by means of the liquid constituting the bath. The potentials obtained in this way were found to check exactly those obtained with the closed tubes. The tubes were suspended in the bath so that the level of the liquid within and without was the same. To prevent convection and diffusion, the ends were closed with asbestos plugs.

The electrodes consisted of fine platinum wires heavily plated with pure silver. These electrodes were scraped and washed perfectly clean before using. Similarly prepared electrodes gave concordant results. The electrodes were carefully suspended in the molten salts and served also the purpose of stirring the latter.

Preparation of Salts.—The following salts were used in our experiments: sodium, potassium, and silver nitrates and silver chlorate. The sodium and potassium nitrates were recrystallized until free from chlorides; "C.P." silver nitrate was recrystallized twice. Silver chlorate was made by precipitating barium sulphate from a hot saturated solution of silver sulphate, with barium chlorate, the latter being first freed from all traces of chloride by crystallization. The silver chlorate, was then separated from the slight excess of barium chlorate in the filtrate, by fractional crystallization. Silver chlorate separates out as beautiful, long, fine, shiny needles, which are easily dried.

Preparation of Mixtures.—All salts were first fused and cooled in a desiccator before the final weighing. The mixtures were then made up in the following manner: The pure fused salts were weighed out in the desired proportions. The sodium or potassium nitrate, as the case might be, was then fused, and while just above its melting point, the silver salt was added. This procedure was followed in order to prevent any reduction of the silver salt by over heating before or during the process of fusion. It was found that the salts of the silver, when once in solution, could be kept fused far above their decomposition points without any apparent reduction. For example, silver chlorate melts at about 230° and decomposes at about 270° ; but, if dissolved in 95 per cent. of sodium nitrate, it can be carried above 350° without any apparent decomposition. After adding the silver salt, the mixture was thoroughly stirred and allowed to cool, vigorous stirring being continued as long as possible. When cold the mixture was broken up and very finely ground in a mortar until different samples gave the same analysis. If the mixtures were allowed to cool without continual stirring, a hard crystalline mass was obtained, which varied in appearance as the composition changed. The mixtures of sodium and silver nitrates solidified in a coarsely crystalline mass which could be broken up easily. With the mixtures of potassium and silver nitrates, however, the texture was fine, and as the concentration of the silver nitrate approached 50 per cent. the mass became extremely hard, resembling marble in appearance. The same was true of silver chlorate, although the mass became very hard with only a small percentage of silver chlorate added.

With the higher concentrations of silver nitrate, the salts were analyzed both before and after the measurements, but with mixtures containing only a very small percentage of silver, this procedure was not practicable. For these mixtures the salts were dried carefully and the proportions made up by weight, the conditions tending to cause a reduction of the silver salt being eliminated so far as possible.

Procedure.—The general procedure for making a potential measurement was as follows: The two mixtures to be investigated were put into their containing vessels or cells and placed in the air-bath furnace to fuse, this bath being kept just above the melting point. As soon as the salts had melted, the vessels were quickly transferred to the nitrate bath of the main furnace, which had previously been brought to the desired temperature. The electrodes were then inserted, the mixtures well stirred, and the measurements begun. As a rule the electromotive force increased rapidly at first, reached a maximum, remained constant for a time and then decreased slowly. The time required for the electromotive force to reach a constant value varied somewhat with each type of cell. The mean of a series of readings taken over an interval of five to ten minutes after the electromotive force had become essentially constant was taken as the observed electromotive force of the cell.

Density Determinations. — In order to calculate the concentration of the various solutions used, in terms of their normality, the density of the salt mixtures had to be determined. The following method was used: a cylinder of Brazilian quartz was cut from a crystal parallel to the principal axis, and used as the sinker of a Westphal balance, a set of weights being adjusted so as to correspond to its volume. A correction for its expansion with the temperature could be made, as the coefficients of expansion along and at right angles to the principal axis of quartz are known. The sinker was suspended in the molten salt the density of which was desired. This was fused in a large platinum crucible placed in the larger electric furnace. In certain experiments where the highest precision attainable was desired the natural quartz sinker was replaced by a fused quartz cylinder, this material being preferable on account of its very small expansion coefficient.

The densities of mixtures of silver and sodium nitrates were all measured, as well as those of a few mixtures of silver and potassium nitrates. For solutions of silver chlorate in sodium nitrate the densities were not measured, but estimated, it being assumed that for a given concentration the same change in density from that of the pure solute (sodium nitrate) occurred, as in the case of silver nitrate solutions. This seemed justified, as the densities of pure silver nitrate and of pure silver chlorate were found to be very nearly the same, and the concentration of all solutions investigated was not greater than 5 per cent. This assumption does not introduce an appreciable error, compared with the precision of other factors of the work.

The data on the density of the fused salts used in this research are not inserted in this paper as they are to be published shortly in connection with the research before mentioned on the conductivity of fused salts.

INVESTIGATIONS OF CELLS OF THE TYPE

$$\mathrm{Ag}\left|\begin{array}{c}\mathrm{AgNO_3\ in\ NaNO_3}\\ c_1\end{array}\right|\begin{array}{c}\mathrm{AgNO_3\ in\ NaNO_3}\\ c_2\end{array}\left|\mathrm{Ag}.\right.$$

The first series of experiments were carried out on cells of the above type which are identical with those investigated by Gordon, except that in his cells a mixture of sodium and potassium nitrate was used as solvent. While Gordon in his experiments chose his solutions of concentrations from 0.1 to 100 per cent. silver nitrate, we have extended the experiments to diluter solutions to which the assumptions underlying the application of Nernst's formula may with more probability be assumed to hold. Experiments have also been repeated with concentrated solutions in order to determine the limits within which Nernst's formula applies.

For dilute solutions the electromotive force of the above cell may be written

$$E = \frac{RT}{F} ln \frac{(\mathrm{Ag})_1}{(\mathrm{Ag})_2}, \qquad (1)$$

where $(\mathrm{Ag})_1$ and $(\mathrm{Ag})_2$ are the concentrations of the silver ions in the two solutions respectively. Now from conductivity measurements of pure fused nitrates, it is certain that both the sodium and silver nitrate must be more or less dissociated. If we assume that dissociation in a fused salt takes place as in aqueous solutions, *i. e.*, in the case of sodium nitrate according to the equation

$$\mathrm{NaNO_3} = \overset{+}{\mathrm{Na}} + \overset{-}{\mathrm{NO_3}},$$

and if the mass action law applies to the equilibrium of this reaction, then at any given temperature

$$\frac{(\mathrm{Na}) \times (\mathrm{NO_3})}{(\mathrm{NaNO_3})} = \text{constant},$$

where the symbols in parenthesis represent the concentrations of the respective ions and undissociated molecules. If to this nitrate as solvent, silver nitrate be added in such quantities that the resulting solutions are so dilute with respect to silver nitrate that the number of $\mathrm{NO_3}$ ions per unit volume arising from its dissociation may be neglected compared with the number of $\mathrm{NO_3}$ ions resulting from the dissociation of the solvent, then, over the range of concentrations to which this assumption applies, the total number of $\mathrm{NO_3}$ ions present may be regarded as practically constant, *i. e.*, we may assume $(\mathrm{NO_3}) = \text{const.}$, and hence also $(\mathrm{Na}) = \text{const.}$ If further the silver nitrate dissolved in sodium nitrate dissociates according to the equation

$$AgNO_3 = \overset{+}{Ag} + \overset{-}{NO_3}$$

we have for a given temperature

$$\frac{(Ag) \times (NO)_3}{(AgNO_3)} = \text{constant.}$$

Since by supposition $(NO_3) =$ constant it follows that

$$\frac{(Ag)}{(AgNO_3)} = \text{constant,}$$

from which by proportion we obtain

$$\frac{(Ag)}{(Ag) + (AgNO_3)} = \text{constant.}$$

But $(Ag) + (AgNO_3) = c$ the total concentration of silver nitrate dissolved in the sodium nitrate; hence for solution (1) we have

$$\frac{(Ag)_1}{c_1} = \text{constant,}$$

and for solution (2)

$$\frac{(Ag)_2}{c_2} = \text{constant,}$$

therefore,

$$\frac{(Ag)_1}{(Ag)_2} = \frac{c_1}{c_2},$$

or in other words as long as the solutions of silver nitrate in a nitrate are dilute, the ratio of the silver ion concentration at the two electrodes is equal to the ratio of the total silver nitrate concentration of the solution. The formula for the electromotive force may therefore be written

$$E = \frac{RT}{F} \ln \frac{c_1}{c_2} \qquad (2)$$

and hence values of E computed on the basis of Nernst's theory and *total* concentrations of silver nitrate should agree with experimental results if the mass action law applies to cases like the above, and this will be true whether the dissociation of the silver nitrate is great or small. To assume complete dissociation for the solute at any given concentration because of a close agreement between observed values and values computed by the above formula, as was done by Gordon in his investigation is, therefore, open to question.

We have tested formula (2) over a range of concentrations with the results given below.

Table I.

$AgNO_3$ in $NaNO_3$.

Equivalent Concentration of $AgNO_3$ at 400° C.		Temperature.	E (Observed).	$E = \frac{RT}{F} ln \frac{c_1}{c_2}$. (Calculated).	Δ Difference.
c_1	c_2				
0.00986	0.06206	350	−0.1014	−0.0986	+0.0028
0.00986	0.06206	350	−0.0996	−0.0986	+0.0010
0.00986	0.1220	337	−0.1335	−0.1320	+0.0015
0.06206	0.1220	349	−0.0370	−0.0362	+0.0008
0.5565	0.1220	340	+0.0781	+0.0800	−0.0019
0.5565	0.00986	340	+0.2094	+0.2130	−0.0036
0.5250	0.04999	329	+0.1230	+0.1220	+0.0010

In Table I. are given the results of experiments made with the diluter solutions. In the first and second columns are given the concentrations of the silver nitrate solutions expressed (throughout) in equivalents per liter and calculated from measurements of their density and percentage composition; in the third column is given the temperature of the cell at the time of the measurement; in the fourth column the observed electromotive force (taken plus from solution (1) to (2)); in the fifth column, the electromotive force calculated by formula (2), and in the last column the difference between the observed and calculated *numerical* values.

It will be seen that up to half normal solutions the agreement is practically complete, the experimental error in cells of this type being from 0.001 to 0.002 volt.

Replacing the sodium nitrate by potassium nitrate as a solute we obtained an equally good agreement between calculated and observed values of the electromotive force. Thus for a cell in which $c_1 = 0.3205$, $c_2 = 0.0480$, $t = 347°$ C., the observed value of $E =$ 0.1003 volt while the computed value was 0.1008 volt. Over this range of concentrations therefore, formula (2) holds valid.

When, however, the concentration of the silver nitrate exceeded about 0.5 normal the deviations between observed and calculated values became increasingly greater and always in the same direction. This will be seen from Table II. These measurements were all carried out at approximately 350°. The concentrations (first and

TABLE II.

$AgNO_3$ in $NaNO_3$. Temperature = 350° *C.*

Equivalent Concentration of $AgNO_3$.		E (Observed).	$E=\frac{RT}{F}\ ln\ \frac{c_2}{0.01}$ (Calculated).	$E=\frac{RT}{F}\ ln\ \frac{c_1}{0.01}$ Reduced.	$\frac{c_1}{0.01}$	$\frac{(Ag)_1}{(Ag)_{.01}}$
c_1	c_2					
0.531	0.0507	0.1230	0.0875	0.2105	53	55
0.531	0.0572	0.1160	0.0908	0.2068	53	53
1.065	0.0421	0.1660	0.0752	0.2412	107	100
1.780	0.1094	0.1412	0.1258	0.2670	178	159
2.345	0.0643	0.1812	0.1000	0.2810	235	199
4.63	0.0418	0.2313	0.0758	0.3071	463	331
7.38	0.684	0.1066	0.2213	0.3279	738	528
13.20	0.0668	0.2502	0.1003	0.3505	1320	764
22.48[1]	0.273	0.1963	0.1957	0.3720	2250	1100

second columns), were as before computed from density determinations of the solutions in question. The third column contains the mean value of the observed electromotive force. In the fourth column are given the computed values of the electromotive force of similar cells, the diluter solutions of which are always 0.01 normal, and the more concentrated solutions of which are c_2 respectively. As c_2 is in every case (except one, 0.68) less than 0.5 normal, these values may be accepted as correct as just shown by the above results in Table I. If therefore these electromotive forces be added to the corresponding observed value of E for cells in which c_2 is the diluter concentration, we obtain the electromotive force of cells in each of which the diluter solution is 0.01 normal and the more concentrated solution c_1 respectively. This assumes that the liquid junction potentials are negligible. All cells may thus be referred to a common concentration, namely 0.01 normal. The computed value of the ratio of the concentration $c_1/0.01$ for each of the cells is given in column six, while the ratio of the silver ion concentration $(Ag)_1/(Ag)_{0.01}$ as computed from formula (1) is given in the last column. It is seen that in passing from solutions 0.5 normal those of greater concentration the computed and observed ratios begin to deviate with increasing rapidity, the ratio of the silver ions being less than the ratio of the total silver nitrate concentrations. This is what theory would lead us to expect as soon as the concentration of the NO_3 ions coming from the silver nitrate itself is no longer negligible compared with the concentration of NO_3 in the solute. Until however, we have further data on the degree of dissociation of the latter and are thereby in a position to compute its effect on the dissociation of an added nitrate, dissociation values computed from the last two columns of Table II., can have but little significance.[1]

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[1] Pure silver nitrate.

CELLS OF THE TYPE

$$\mathrm{Ag}\left|\begin{array}{c|c} \mathrm{AgClO_3\ in\ NaNO_3} & \mathrm{AgClO_3\ in\ NaNO_3} \\ c_1 & c_2 \end{array}\right|\mathrm{Ag.}$$

We next investigated a concentration cell in which the solute and solvent had no ion in common. Silver chlorate was chosen as solute and sodium nitrate as solvent. For this cell, the conclusion arrived at in the discussion of the preceding cells, namely that for dilute solutions the ratio of Ag-ion concentration and total concentration at the two electrodes is the same, no longer holds; for an interchange of ions between solvent and solute probably takes place according to the reaction

$$AgClO_3 + NaNO_3 = AgNO_3 + NaClO_3$$

some undissociated silver nitrate and sodium chlorate being formed. If the amount of undissociated silver nitrate thus formed is appreciable, the concentration of silver ions will not be equal simply to the concentration of $AgClO_3$ multiplied by its percentage dissociation, but, depending upon the quantity of undissociated silver nitrate formed will be less than this. We should expect, therefore, that the computed ratio of the silver ion concentration in a concentration cell of this type would not be equal to the concentration ratio of the dissolved salt even in dilute solutions, but would be less. This was in fact found to be the case as the results in table III. clearly show. Here it will be seen that all values of the electromotive force calculated on the basis of the formula $E = \frac{RT}{F} ln \frac{c_1}{c_2}$, where c_1 and c_2 are the total concentrations of the solutions, are in every case *greater* than the ob-

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[1] Since this paper went to press, the third part of Professor Lorenz's work on the "Elektrolsye Geschmolzener Salze" has reached me and I note that he has come to this same conclusion.

TABLE III.

$AgClO_3$ *in* $NaNO_3$.

Equivalent of Concentration of $AgClO_3$.		Temperature.	E (Observed).	$E=\frac{RT}{F}\ ln\ \frac{c_1}{c_2}$. (Calculated).	Δ Difference.
c_1	c_2				
0.0756	0.0358	349	0.037	0.040	−0.003
0.0756	0.0235	332	0.053	0.060	−0.007
0.0756	0.0235	345	0.053	0.061	−0.008
0.294	0.0235	345	0.116	0.136	−0.020
0.294	0.0235	357	0.116	0.136	−0.020
0.493	0.0756	356	0.095	0.101	−0.006

served values, by amounts considerably in excess of the experimental error. Over this same range of concentration, with silver nitrate as solute, the agreement between observed and calculated values was complete. The difference between observed and computed values indicates either that the dissociations of silver chlorate changes (diminishes) rapidly with increasing concentration, or that the effect of the reaction between solvent and solute results in the formation of a very considerable amount of undissociated silver salt. In either case it seems reasonable to conclude that the dissociation of the chlorate is by no means complete or even approaching 100 per cent. at the concentrations investigated.

In view of the present state of our ignorance of the absolute value of the dissociation of any fused salt in even dilute solutions, it seems to us premature to attempt to conclude more from the above experiments than that Nernst's theory of the cell and the laws underlying the mutual effect of one ionized salt on another hold for salts in the molten state as well as in aqueous solution. These conclusions seem, however, well established by the above experiments, and hence the great importance which must be attached to the effect of the ionized portion of the solvent on the ionization of the solute in cases of fused inorganic mixtures. The analogue in aqueous solutions is to be found in hydrolytic phenomena at very high temperatures when water itself becomes much dissociated. As soon as the degree of dissociation of fused silver nitrate or chlorate has been once independently established, the above measurements will afford data for the computation of their dissociation at various other concentrations.

An attempt is being made at the present time to obtain from electrometric measurements on another type of cell the value of the dissociation constant of one of the salts used in the preceding experiments and it is hoped that shortly we may be able to state more definitely than at present whether the high conductivity of fused electrolytes is due to a great migration velocity of their ions resulting from the high temperatures to which they are subjected, or to a high degree of dissociation as has been usually assumed.

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ENERGY NECESSARY TO IONIZE A MOLECULE BY IMPACT OF NEGATIVE IONS.

By Bergen Davis.

AS is well known, if a sufficient electrical intensity be applied to an ionized gas, all the ions produced by the external ionizing agent will be driven out and the saturation current will be obtained. If now the electrical intensity be greatly increased, the current will also be increased. Prof. J. S. Townsend[1] has shown that this increase of current is due to the production of new ions by collision of the ions already present with the neutral molecules of the gas.

An ion moving freely under a given electrical intensity will have acquired a certain kinetic energy at the end of a given path. If the kinetic energy possessed by an ion at the instant of impact is greater than the least energy necessary to bombard an electron from a molecule, ionization *may* occur.

In an early paper Townsend gives a method of calculating, from the increase of current, the least number of volts through which an ion must freely run in order to acquire sufficient energy to produce ionization by impact. The difficulty of using the method there devised, on account of a series whose terms must be separately calculated for each case, has led the writer to develop a theory that will enable one to calculate the energy of ionization directly and with considerable ease.

1. Professor Townsend gives the following theory for determining the number of new ions which one ion produces in moving one centimeter through a gas:

Assume the original ions to have a volume distribution throughout the gas. Let a force X be applied to N_0 negative ions in a gas at pressure p, and let N be the total number of ions after the N_0 ions have traversed a distance x.

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[1] Townsend, Nature, Vol. LXII., August 9, 1900; Phil. Mag., Feb., 1901; Phil. Mag., June, 1902; Phil. Mag., Nov., 1903; Phil. Mag., March, 1905. Townsend & Kirkby, Phil. Mag., June, 1901.

The number of negative ions produced by N ions through a distance dx will be $\alpha N dx$, where α is the number produced by one ion in going one centimeter.

Then

$$dN = \alpha N dx N$$

and

$$N = N_0 e^{\alpha x}. \tag{1}$$

Let $n_0 l$ ions be distributed between two plates at a distance l apart, and an electrical intensity X be applied to them.

The number of ions C that arrive at the positive plate is

$$C = \int_0^l n_0 e^{\alpha x} dx = \frac{n_0}{\alpha}(e^{\alpha l} - 1)$$

and the ratio of this number to the original number is

$$\frac{C}{n_0 l} = \frac{1}{\alpha l}(e^{\alpha l} - 1).$$

If C_0 denote the original saturation current

$$C = \frac{C_0}{\alpha l}(e^{\alpha l} - 1). \tag{2}$$

By the application of this formula to experimental data Townsend has determined the values of α, in case of air, for a number of pressures and for various electrical intensities.

Some of these values of α are given in the second column of Table II. of this paper.

2. The above investigation considers the ionization to be due to impact of negative ions only. In a later paper (Phil. Mag., November, 1903), Townsend shows that the positive ions may also produce ionization by impact.

In these experiments the original n_0 ions did not have a volume distribution, but were liberated from the negative zinc electrode by the action of ultra-violet light.

By an investigation somewhat similar to the previous one he shows that.

$$\frac{n}{n_0} = \frac{(\alpha - \beta)e^{(\alpha-\beta)a}}{\alpha - \beta e^{(\alpha-\beta)a}}. \tag{3}$$

Where α is the number of ionizing impacts made by a negative ion in going a distance of one centimeter under an electrical intensity X, and β is the number of ionizing impacts made by a positive ion in going the same distance under the same force.

The values of α as calculated from the experimental results in air are given in the second column of Table III. of this paper. Having thus obtained experimental values of α and the corresponding values of the electrical intensity X, a theory will be here developed that will enable one to readily calculate the least ionizing path of an ion when driven by a given electrical intensity. The ionizing path will depend upon the character of the collision between an ion and a neutral molecule. The path λ_0 determined in this investigation will be that ionizing path which an ion has when it makes normal impact with a molecule and loses all its momentum. The value of $X\lambda_0$ will then be the least number of volts through which an ion must move to produce ionization in the most favorable case.

THEORY.

Consider a gas with an ion moving through it under an electrical intensity X. Let α be the number of ionizing collisions which the ion makes in going one centimeter. If l is the mean free path of this ion, then the number of collisions of all kinds that this ion makes will be $1/l$ in going one centimeter. Let λ_0 be the least path that may produce ionization, then according to the kinetic theory of gases.[1]

$$n = \frac{1}{l} e^{-\frac{\lambda_0}{l}}$$

expresses the number of paths that are greater than λ_0. But all paths that are greater than λ_0 will not necessarily produce ionization, as the production of new ions will depend on the nature of the impact as well as the length of path. Some of the impacts will be normal to the surface of a molecule, while in many other cases the ions will strike glancing blows. The actual ionizing impacts will be

$$\alpha = \frac{1}{l} e^{-\frac{\lambda_0 + x}{l}} (f). \tag{4}$$

Where x is any length in addition to λ_0 which may vary from 0 to ∞, and (f) is the fraction of the paths greater than $(\lambda_0 + x)$ that result in ionization.

[1] Meyer, Kinetic Theory of Gases.

Assume a molecule to be spherical in form, and consider that ionization will take place if the kinetic energy due to the normal component of the velocity of the moving ion is equal to or exceeds the minimum energy of ionization $w = Xe\lambda_0$. The ion when it strikes the molecule must move within the cylinder *ABCD* (see Fig. 1). If it moves along some path not in line with the center as *nm* for instance the kinetic energy due to the normal component of the velocity must equal or exceed $Xe\lambda_0$ in order to produce ionization. Its actual energy is $Xe(\lambda_0 + x)$ where x is a path additional to λ_0.

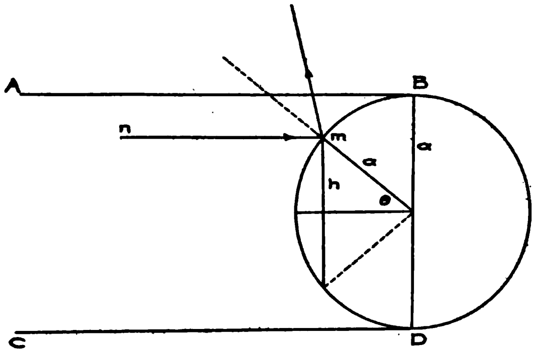

Fig. 1.

Let v be the actual velocity of the ion at end of path $(\lambda_0 + x)$, the normal component of its velocity will be

$$v_n = v \cos\theta.$$

The kinetic energy due to the normal velocity will be

$$\tfrac{1}{2}mv_n^2 = \tfrac{1}{2}mv^2\cos^2\theta. \qquad (5)$$

But

$$\tfrac{1}{2}mv_n^2 = Xe\lambda_0$$

and

$$\tfrac{1}{2}mv^2 = Xe(\lambda_0 + x).$$

By (5)

$$Xe\lambda_0 = Xe(\lambda_0 + x)\cos^2\theta.$$

From which

$$\sin^2\theta = \frac{x}{\lambda_0 + x}. \qquad (6)$$

The ion must strike the molecule within the cylinder *ABCD*. The probability of its striking within any angle θ, is as the ratio of the area of base of cylinder on the segment contained in angle θ to the area of base of large cylinder. The area of base of the small cylinder is $\pi r^2 = \pi a^2 \sin^2 \theta$, and the area of base of large cylinder is πa^2. The probability of the impact occurring within the angle θ is

$$\frac{\pi a^2 \sin^2 \theta}{\pi a^2} = \sin^2 \theta.$$

This angle θ is such that when an ion has traversed a path $(\lambda_0 + x)$ its angle of impact, if it ionizes, must be equal to or less than θ.

The fraction (f) of all the impacts that produce ionization is thus equal to $\sin^2 \theta$ and equation (4) becomes

$$a = \frac{1}{l} e^{-\frac{\lambda_0 + x}{l}} \sin^2 \theta. \tag{7}$$

Introducing (6), the above becomes

$$a = \frac{1}{l} e^{-\frac{\lambda_0 + x}{l}} \frac{x}{\lambda_0 + x}. \tag{8}$$

As the ion traverses a distance dx just beyond a path $(\lambda + x)$ the number of ionizing collisions will be

$$a = \frac{1}{l} e^{-\frac{\lambda_0 + x}{l}} \frac{x}{\lambda_0 + x} \frac{dx}{l}.$$

The number of ionizing collisions which an ion makes in going one centimeter will be

$$a = \frac{1}{l} \int_0^\infty e^{-\frac{\lambda_0 + x}{l}} \frac{x}{\lambda_0 + x} \frac{dx}{l}. \tag{9}$$

This expression is not integrable in finite terms, but is given by De Haan[1] in the following form:

$$a = \frac{1}{l} e^{-\frac{\lambda_0}{l}} \left[1 + \frac{\lambda_0}{l} e^{\frac{\lambda_0}{l}} Ei \left(- \frac{\lambda_0}{l} \right) \right] \tag{10}$$

which may be written

$$al = e^{-\frac{\lambda_0}{l}} + \frac{\lambda_0}{l} Ei \left(- \frac{\lambda_0}{l} \right). \tag{11}$$

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[1] Bierens de Haan, Verk. K. Akad. v-w Dl., II., blad 19.

This is the final equation which is applied to the experimental values of α as obtained by Townsend and others.

The values of the ponential integral $Ei\left(-\frac{\lambda_0}{l}\right)$ were obtained from tables by Dr. W. Láska (*Sammlung von Formeln*). The relation between αl and succeeding values of λ_0/l are given in the table below:

TABLE I.

$\frac{\lambda_0}{l}$	αl	$\frac{\lambda_0}{l}$	αl
.04	.8536	1.5	.0731
.06	.804	2	.0375
.1	.7247	3	.0106
.14	.656	4	.0032
.2	.574	5	.00099
.25	.518	6	.00032
.3	.496	7	.000142
.4	.3893	8	.000034
.6	.2762	9	.0000114
.8	.2008	10	.00000387
1	.1485		

Practical use of equation (11) can be made by plotting Table I. in a curve on a large scale with the abscissæ representing succeeding values of λ_0/l and the ordinates representing the corresponding values of αl. By means of this curve any value of λ_0/l corresponding to any given value of αl may be obtained.

The test of the validity of such a theory is that it should give a constant value of $X\lambda_0$ for all pressures and for all electrical intensities. The equation contains two unknown quantities, the mean free path l of an ion and the least ionizing path λ_0. The mean path l is a constant for any one pressure and varies inversely as the pressure. The path λ_0 depends both on the pressure and on the electrical intensity.

The third columns of Table II. are calculated by means of equation (11) from values of α obtained by Townsend (Phil. Mag., February, 1901) from experiments in which the ionization is considered as due to the impact of negative ions only.

The values of $X\lambda_0$ are as nearly constant as the nature of the experimental determinations of α could lead one to expect. The values

TABLE II.

Air.

Pressure 4.13 mm. $l = .011$ cm.			Pressure 2 12 mm. $l = .0215$ cm.		
X	a	$X\lambda_0$	X	a	$X\lambda_0$
200	.5	7.7	160	.9	8.6
320	2.1	8.45	240	2.35	8.93
480	5.3	8.8	400	6	9.3
640	8.9	9.1	560	9.5	9.46
		Mean = 8.5			Mean = 9.07
Pressure 1.1 mm. $l = .0415$ cm.			Pressure .385 mm. $l = .1184$ cm.		
120	1.1	9.24	120	2	9.9
160	2	9.34	200	3.4	9
320	5.5	9.5	320	4.5	8.96
400	8	9.54	480	5.4	8.63
		Mean = 9.4			Mean = 9.12
Pressure 171 mm. $l = .266$ cm.					
40	.65	9.45			
80	1.35	9.3			
160	2.2	8.25			
320	2.65	9.5			
		Mean = 9.12			

of l given in the above tables were those values that would make $X\lambda_0$ most nearly constant, preserving the relation that l shall vary inversely as the pressure. The mean free path of a molecule of air at room temperature and atmospheric pressure is very closely 10^{-5} cm., and 1.1 mm. pressure it is .007 cm. The value of l that satisfies equation (11) at 1.1 mm. pressure is $l = .0415$ cm. From which it is found that l is *six* times the ordinary mean free path of a molecule at the same pressure. This very large mean free path of the negative electron may be explained by two well known deductions made by Maxwell from the kinetic theory of gases: (*a*) If among molecules having distributed speeds there is one molecule that moves continuously with a speed of higher order than that of the other molecule, this molecule will have a mean free path $\sqrt{2}$ greater than the mean path of the other molecules. (*b*) If among molecules of a given size there is a particle negligibly small in comparison with the other molecules, this particle will have a mean free path *four* times that of the molecule.

It can readily be shown that the negative electron moving in the electric field here applied has a velocity at the end of its free path more than a thousand times the mean speed of a molecule. From which it immediately follows that the mean free path of an electron is $6/\sqrt{2} = 4.25$ times that of a molecule for the same velocity conditions. This ratio of paths (4.25) agrees very well with the theoretical value of *four* referred to above.

III. The values of α given in Table II. were obtained from experiments in which the ionization was considered as due to impact of negative ions only.

In a later paper (Phil. Mag., November, 1903) Townsend describes some experiments from which it is shown that the positive ions may also produce ionization by impact. The values of α as calculated by means of the corrected theory given in his later paper will necessarily be somewhat different from those in which the whole increase of current was considered to be due to the impact of negative ions only. Experimental data are given in the above paper for Air and Hydrogen. The second columns of Table III. give the values of α as obtained in air for various pressures and electrical intensities. The third columns contain the corresponding values of $X\lambda_0$ as calculated by means of equation (11).

Table III.

Air.

Pressure 2 mm. $l = .0228$ cm.			Pressure 1 mm. $l. = 0456$.		
X	α	$X\lambda_0$	X	α	$X\lambda_0$
700	10.5	11.4 volts.	350	5.25	11.4 volts.
			437	6.3	11.45 "
			525	7.27	11.49 "
Pressure .66 mm. $l = 068$ cm.					
350	4.8	11.55 volts.			
437	5.56	12.23 "			
525	5.9	13.6 "			

Table IV. contains similar calculations for the impact of negative electrons in hydrogen.

More recently H. E. Hurst (Phil. Mag., April, 1906) has determined the values of α for nitrogen and carbon-dioxide. I have applied equation (11) to his values of α also. The results are given in the following tables:

In the case of air (Table III.), of hydrogen (Table IV.) and

TABLE IV.

Hydrogen.

Pressure 8 mm. $l = .01047$ cm.			Pressure 4 mm. $l = .0209$ mm.		
X	α	$X\lambda_0$	X	α	$X\lambda_0$
525	5.28	9.4 volts.	350	4.43	10.5 volts.
700	8.86	9.73 "	525	7.4	10.4 "
1050	14.8	10.5 "	700	9.6	11.7 "
Pressure 2 mm. $l = .0419$.					
262	3.7	10.5			
350	4.8	11.7			

TABLE V.

Nitrogen.

Pressure 16 mm. $l = .00285$ cm.			Pressure 8 mm. $l = .0057$.		
X	α	$X\lambda_0$	X	α	$X\lambda_0$
700	.32	10.13 volts.	350	.17	10 volts.
Pressure 4 mm. $l = .0114$.			Pressure 2 mm. $l = .0228$.		
350	1.35	10.8 volts.	350	3.54	10.5 volts.
525	3.83	11.28 "	525	7.19	11 "
700	7.08	11.4 "			
Pressure 1 mm. $l = .0456$.					
350	4.42	10.77 volts.			
525	6.41	13.1 "			

TABLE VI.

Carbon Dioxide.

Pressure 4 mm. $l = .0117$ cm.			Pressure 2 mm. $l = .0234$.		
X	α	$X\lambda_0$	X	α	$X\lambda_0$
350	2.47	9.1 volts.	700	11.3	10.5 volts.
700	9.28	9.9 "	1,400	21.21	9 "
Pressure 1 mm. $l = .0468$.			Pressure .5 mm. $l = .0936$.		
700	10.68	8.8 volts.	525	6.41	9.8 volts.
875	11.83	8.9 "	700	6.74	10.3 "
1,050	12.68	11.1 "	875	7.5	9.3 "

nitrogen (Table V.), the only values of l that would make $X\lambda_0$ nearly constant for the various pressures and electrical intensities were l equal to $6/\sqrt{2}$ times the mean free path of a molecule for any pressure and the same conditions of velocity. In the case of carbon dioxide (Table VI.) the only values of l that would make $X\lambda_0$ constant were l equal to $9/\sqrt{2}$ times the mean free path of a molecule for the same velocity.

CONCLUSION.

The values of $X\lambda_0$ given in Table II. although nearly constant among themselves differ somewhat from those values for air given in Table III.

This disagreement is to be expected, since the values of α for air given in Table II. were obtained by means of a theory that neglected the ionization due to the impact of positive ions. In Table III. the ionization due to the impact of positive ions is allowed for and the values of α there given are probably more nearly the true values for those pressures and electrical intensities.

The mean results obtained are given in the table below:

Gas.	R	$X\lambda_0$	$X\lambda_0 e$
Air.	4.25	11.8 volts.	1.22×10^{-11} ergs.
Nitrogen.	4.25	11 "	1.14×10^{-11}
Hydrogen.	4.25	10.55 "	1.09×10^{-11}
CO_2.	6.38	9.67 "	1.0×10^{-11}

The charge e is here taken as 3.1×10^{-10} E.S. (H. A. Wilson). R is the ratio of the mean free path of an electron to that of a molecule for the same velocity conditions.

PHŒNIX PHYSICAL LABORATORY, COLUMBIA UNIVERSITY,
October 27, 1906.

THREE-COLOR INTERFERENCE PICTURES.

By Herbert E. Ives.

(Preliminary Paper.)

THE subject of the following paper is an application of the Lippmann process to the production of three-color pictures. The method consists in using a set of three-color negatives to make a picture composed of juxtaposed colored lines, red, green and blue, after the manner of the Joly color picture, substituting, however, the laminated structure of the Lippmann film for the pigments which impart color to the true Joly picture. By this combination of the three-color and standing-wave processes pictures are obtained by an indirect though wholly photographic means, possessing the fidelity of the best three-color processes with advantages peculiar to the Lippmann picture. The colors are in the picture itself, so that no viewing device is necessary to render them visible, and from the original three-color negatives an indefinite number of color pictures can be made.

Up to the present Lippmann photographs of natural objects (as distinguished from spectra or monochromatic sources) have been made directly in the camera. A departure from this simple and straightforward procedure of course needs ample justification. This will be found in a consideration of previous attempts to photograph mixed colors, such as those of natural objects, and the conclusions, both theoretical, and practical, to be drawn therefrom.

It has long been recognized that while spectra are easily and satisfactorily photographed by the Lippmann process, such objects as landscapes, still life, etc., are very difficult and rarely reproduce with great fidelity. The principal reasons therefore may be briefly summarized as follows:

1. Complete isochromatism of plates is essential.

In photographing spectra with non-isochromatic plates all the colors may be made to give strong action by mere long exposure.

Additional exposure of the photographically more active colors after their maximum effect is produced causes no increase of action within wide limits. In photographing natural objects recourse to long exposure to produce isochromatic action is obviously destructive of light and shade values, since these are rendered entirely by difference of exposure. The dependence of the best results on careful choosing of exposure has often been noticed.[1]

2. To obtain a close approximation to the color of the object photographed the lamina system must be of considerable depth and must be transparent. Neither of these conditions is fulfilled. The Lippmann film is very thin (not over .01 mm.) and strongly absorbing.

3. It appears from the microsections made by H. Lehmann[2] that the lamina system, owing to the scattering of the incident light and other causes, dies down much more rapidly than that predicted from calculation. The light reflected is therefore always less pure than that acting to produce the laminæ. The frequent resemblance of Lippmann pictures to tinted Ambrotypes and Daguerrotypes may be explained by this general admixture of white.

4. From experiments made by H. Lehmann[3] on photographing two and three colors simultaneously it appears (and theoretical study of the lamina system resulting from mixed colors confirms) that mixed colors are rendered with a loss of luminosity of the components as compared with the rendering of the components alone.

It follows from this that if colors of different degrees of purity occur in an object photographed their reflective luminosities will be wrongly rendered. For instance, an area illuminated by a sodium flame and an area illuminated by a broad spectral region including red and green, while appearing the same color to the eye, and adjusted to be the same apparent brilliancy, would be rendered as of quite different brilliancies. Certain types of colors, such as those of some aniline dyes, in whose spectra two or more maxima occur, would suffer greatly in this respect.

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[1] R. Neuhaus, Die Farbenphotographie nach Lippmanns Verfahren, 1898, p. 45. "Das Bild giebt nur dann die Farben richtig wieder, wenn die Belichtungszeit richtig getroffen wurde. Halbfach Über- odor Unterexposition genugt schon vollständig die Platte zu verderben."

[2] "Beiträge zur Theorie und Praxis der directen Farbenphotographie," 1906, p. 81.

[3] L. c., p. 14.

All of these facts impose obstacles to the complete success of the original Lippmann process. A detailed study of the rendering of mixed colors and the conditions essential for the best results is now being carried on by the writer and will be published in the near future. The above conclusions on the reproduction of colors of all types, have, however, been confirmed and the recourse to the three color process as the means of rendering all kinds of colors faithfully (to the eye), while employing the Lippmann film, is the practical result.

The determining reason for this step is as follows: The Lippmann process is, as seen above, not fitted to render all types of colors faithfully. For the three-color process all that is needed are narrow spectrum bands; these the Lippmann film is thick enough and transparent enough to give.

Proceeding now to the practical details as worked out. It is, of course, necessary to have a set of three-color negatives or color records. Each of these must represent the amount of one of the primary colors, red, green, and blue, necessary to mix with the proper amounts of the others to counterfeit to the eye the color of the object photographed. The three-color records used were positives made for the Krōmskōp.[1]

The first apparatus constructed consisted of an enlarging camera by means of which an image of any one of the three-color positives was projected on the plate; a heliostat to send a beam of sunlight through positive and projecting lens, and an opaque line screen or grating, in contact with the plate, having opaque spaces twice the width of the transparent. By means of this latter one third of the surface of the plate could be exposed at a time. For colored light three color-screens, identical with those used in the Krōmskōp were tried; a ruby-red glass, a pot-green glass, and a combined cobalt-blue and signal-green glass. These give narrow, well separated spectrum bands, quite pure enough to give the best results. The procedure was to make three exposures in succession, between each of which the three-color positive was changed, the color screen changed, and the opaque-line screen moved the width of a transparent space (one three-hundredth of an inch). The method is in all respects similar to that employed by the writer to produce diffraction color pictures.[1]

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[1] The Krōmskōp system is based on Maxwell's color mixture diagram. The three negatives are made through color screens which with the plates used give intensity curves that correspond to the Maxwell color curves representing the three primary sensations. Those curves give the amounts of three narrow spectrum bands (near *C*, between *E* and *F*, between *F* and *G*) to mix to reproduce to the eye both the color and luminosity values of all parts of the solar spectrum. The application of Maxwell's analysis to three-color photography was first published by F. E. Ives in 1888.

It was soon found that much purer colors must be used than those furnished by the color screens, for this reason: The spectrum band by which the plate was illuminated, reproduced, as examination of the photograph in the spectroscope showed, as a much wider one. The triple-line pictures, therefore, instead of showing in the spectroscope three isolated bright bands gave a continuous spectrum with slightly shaded spaces in the yellow and blue-green. This was confirmed by exposures with spectrum bands of varying width from a monochromatic illuminator. In all cases it was found on spectroscopic examination that these bands were rendered by broad, not sharply defined ones of from five to ten times the width of the original. The most monochromatic source then at hand, a sodium flame, was photographed, and the plate showed in the spectroscope a fairly narrow band, although some red and green light was still visible. Colors as pure as that obtained in this way would be quite satisfactory for three-color work. Suitable red, green and blue line sources were not at the time available, so, as the best that could be done, portions of the spectrum of about eighty A.U. width from a monochromatic illuminator were tried. The colored light was thrown on the three color positive, which was placed directly in front of the opaque-line screen and the plate, and so cast a shadow.

With this apparatus three-color standing-wave pictures were obtained which compare favorably with the Krōmskōp and have qualities rarely, if ever, seen in the ordinary Lippmann pictures. The color and luminosity values are faithfully rendered and the peculiar metallic appearance so common in most interference pictures is entirely absent. They are not, of course, color pictures taken directly in the camera, the ideal for which we aim, but they are faithful color pictures produced by entirely photographic means; they are held in the hand to observe, and may be duplicated indefinitely. They do not fulfill the popular conception of a color photograph so well as would Lippmann pictures taken directly in the camera, were the latter free from the unavoidable defects outlined above. They are perhaps the nearest we can come to a true color photograph at present.

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[1] PHYSICAL REVIEW, June, 1906, p. 339.

The defects of these new three-color interference pictures so far made are due to the inadequate means at hand when the work was done. The colored light used was not as monochromatic as was desired. The three bright strips of color seen when one examines the photographs with the spectroscope are as a consequence rather wide and the spaces between not quite black, hence some degradation of color values results. It is intended as soon as possible to make pictures by means of monochromatic light furnished by sources such as the cadium lines, and to make the triple (Joly) lines at least two hundred to the inch, so that they become practically invisible.

With these improvements the pictures should rank as one of the most perfect applications of the three-color process.

The plates used were prepared according to the formula published by Valenta[1] with the exception that it was found necessary to double the quantity of gelatin; this may, however, have been due to the kind of gelatin used.[2] The pictures were mounted under a prism according to the usual method. To destroy all extra reflections possible the back of the glass and the lower surface of the prism were ground with emery before the application of asphaltum varnish to the one and Canada balsam to the other. It was further found advantageous to substitute gum styrax ($n = 1.58$) for Canada balsam in mounting the prism. By this the surface reflection from the film is less than with the balsam, and in addition (if the lower side of the prism is ground) a slight diffusion of light results so that the picture is visible through a wider angle, and the reflected image of the source of light (such as a gas flame) no longer well enough defined to be disturbing.

Physical Laboratory, Johns Hopkins University,
October 7, 1906.

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[1] Die Photographie in natürlichen Farben, 1894, p. 52.

[2] "Gold Label," sold by Eimer & Amend, New York.

THE ABSORPTION OF ALPHA RAYS IN GASES AND VAPORS.

BY E. P. ADAMS.

AT a meeting of the Physical Society held in December, 1905,[1] the writer described some experiments made to determine the maximum distance between a source of alpha rays and a zinc sulphide screen at which the well-known scintillations could be seen. The results arrived at were that this maximum distance was inversely proportional to the pressure of the gas, the temperature remaining the same; and that in different gases the product of the pressure and maximum distance was inversely proportional to the sum of the square roots of the weights of the atoms contained in the molecules of the gas. The latter result is similar to that obtained by Bragg and Kleeman[2] for the so-called "stopping-power" of various gases, vapors, and metals for the alpha rays from radium, the electrical method of measuring the ionization produced by the rays being employed. According to their experiments, the absorption of alpha rays by various substances is not an absorption of the rays in the ordinary sense, but an absorption of a portion of their energy which is spent in ionization, as a result of which the velocity of the alpha rays is continually diminished. When their velocity falls below a certain limiting value the alpha rays lose their ionizing property. These results have been confirmed and extended by Rutherford,[3] who measured the velocity of the alpha particles after having passed through various numbers of layers of aluminium foil. Rutherford also showed that the photographic and phosphorescent effects produced by the alpha rays ceased with their ionizing effects.

The delay in publishing the complete results of these experiments is due to a desire to include the results for the monatomic gases argon and helium which were thought to be of special interest in view of the relation which was found to hold for the different gases and vapors tried.

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[1] PHYSICAL REVIEW, February, 1906, p. 111.

[2] Phil. Mag., 10, p. 318, 1905.

[3] Phil. Mag., 10, p. 163, 1905.

The source of the alpha rays in these experiments was polonium, which is specially suitable as it does not emit any beta-rays. The polonium was separated from the mineral carnotite. As this mineral is comparatively easy to obtain in this country it may be of interest to describe the method of preparing the polonium, the process being very simple. Carnotite, which occurs disseminated through sandstone, is easily soluble in hydrochloric acid. The excess of acid is driven off by heat, water added, and hydrogen sulphide passed into the solution. The precipitate is filtered, thoroughly washed with water, and dissolved in nitric acid. The sulphur which separates is removed by filtration. This nitric acid solution contains the polonium, along with bismuth, lead, copper, etc. It is advisable to add to this solution a small quantity of bismuth, as the polonium is precipitated with the bismuth, and it is desirable to have a rather larger quantity of bismuth precipitated than that originally contained in the carnotite. The nitric acid solution is then evaporated nearly to dryness to remove the excess of free acid, and a large quantity of water added. This precipitates bismuth hydroxide, and with it, the polonium. After filtering this precipitate, it is dissolved in the smallest possible quantity of hydrochloric acid. This last solution contains all the polonium. By placing a polished bismuth plate in this solution, in the course of a few days the polonium is nearly all deposited upon its surface. There is thus obtained, from a pound or two of carnotite, a layer of polonium upon the bismuth plate which is sufficiently active for experiments of this kind.

A description of the apparatus used in these measurements follows: The active bismuth plate, *A*, was carried at the end of a glass rod which floated on a column of mercury whose height could be regulated by raising or lowering a mercury reservoir, *D*. The measuring tube, *F*, which was made of glass, was closed at the top by a glass plate, *B*, on the under side of which the phosphorescent zinc sulphide was cemented. The tube could be exhausted by means of a mercury pump, and the various gases admitted, the pressure being given by a mercury gauge, *E*. The zinc sulphide, in the earlier experiments, was held to the glass plate by a very small amount of Canada balsam; with certain of the vapors used it was found that the Canada balsam was attacked, and water-glass was then used as a cement which proved to be entirely satisfactory. The glass plate was ground to fit accurately the top of the measuring tube; a small amount of water-glass then made the tube perfectly air-tight. The zinc sulphide screen was viewed through a magnifying glass, *C*.

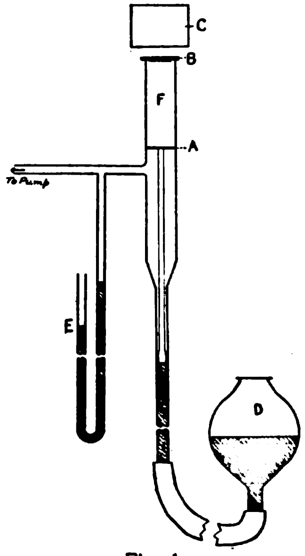

Fig. 1.

The object of the experiment was to measure the maximum distance between the active plate and the zinc sulphide screen at which the scintillations could be seen through the magnifying glass. For this purpose the mercury reservoir was raised or lowered until the scintillations just appeared or just disappeared. A number of settings were thus made at each pressure and the average taken as the proper maximum distance corresponding to that particular pressure. For distances less than about 10 cm. it was found that the disappearance or appearance of the scintillations was very sharp so that the successive settings were in very close agreement with each other. For greater distances, owing mainly to the spreading out of the alpha rays, the number of scintillations seen was much smaller and the successive settings were not in so close agreement. It was frequently observed that scintillations appeared when the polonium was at too great a distance for the alpha rays to produce them, and even when the polonium was taken out from the tube. These scintillations were of a different character than those due to the alpha rays; they appeared to be much brighter. It is probable that these were due to changes in the cementing material, partly temperature changes, and the drying out of the cement which caused a sudden pressure to be exerted upon the zinc sulphide, thus causing light effects. It is known that phosphorescent zinc sulphide is extremely sensitive to mechanical shocks. These scintillations produced no confusion as with a little practice they could easily be distinguished from the scintillations due to the alpha rays. A similar spontaneous scintillation of zinc sulphide has been observed by Schenck and Mihr.[1]

In preparing the gases used only the ordinary precautions were taken to have them pure. They were dried by means of phosphorus pentoxide except in the case of acetone, which was decomposed by the phosphorus pentoxide. The liquids whose vapors were examined were, with one exception, the ordinary "chemically pure" products of commerce. Nickel carbonyl was prepared by the method of Mond, Langer and Quincke.[2] Argon was obtained by passing air over heated copper and magnesium twice, by means of which the oxygen and a large part of the nitrogen was removed. The remaining nitrogen was removed by passing an electric arc between platinum terminals through the gas collected over a solution of caustic potash with the repeated addition of oxygen. Finally the remaining oxygen was removed by means of solid phosphorus. Helium was obtained by heating the mineral fergusonite in a vacuum. This mineral, which came from Llano Co., Texas, gave off, by the application of heat alone, about 0.1 c.c. of helium per gram. The resulting gas was freed from nitrogen and other impurities by means of an electric arc in the same manner as employed for argon. It was necessary to add the oxygen in very small quantities at the beginning as hydrogen is given off by heating this mineral, due to a reduction process, and with too much oxygen present a serious explosion might result on starting the arc. When the helium so prepared was put into the testing tube it was found that the scintillations could not be made to disappear even by removing the polonium altogether. This was due to the fact that fergusonite contains a certain amount of radium, and on heating it the radium emanation was collected along with the helium. It was therefore necessary to wait several months until the emanation had disintegrated sufficiently to allow measurements to be made in the regular way. By means of liquid air it would have been possible to freeze the emanation out of the helium at once.

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[1] Ber. deut. chem. Ges., 37, p. 3464, 1904.

[2] Trans. Chem. Soc. London, 57, p. 749, 1890.

The vapors were introduced into the testing tube as follows. A flask provided with a tap funnel was sealed on to the apparatus and the whole exhausted. A small amount of the liquid could then be introduced into the flask without admitting any air. By measuring the pressure, and knowing the tension of the saturated vapor, an estimate could be formed as to the purity of the vapor.

Observations were always made at night in the dark as then the eye is much more sensitive to effects of this kind than in the daytime. It was found that after thirty to forty minutes of observation the eye became too tired and it became very difficult to get concordant results.

The following tables are characteristic of the results obtained. In each table the first column gives the pressure, *p*, the second the maximum distance between the polonium and the zinc sulphide screen, *d*, and the third the product, *pd*; *d* is measured in centimeters, and *p* in centimeters of mercury.

Air.			**Ethyl Iodide.**		
p	*d*	*pd*	*p*	*d*	*pd*
75.54	3.49	264	9.60	8.74	83.9
69.69	4.20	293	8.65	9.43	81.6
52.86	5.03	266	7.65	10.67	81.6
22.14	12.53	277	7.20	11.33	81.6
15.79	17.72	280			
		Average, 276			Average, 82.2

Oxygen.			**Nickel Carbonyl.**		
p	*d*	*pd*	*p*	*d*	*pd*
70.15	4.03	283	22.8	2.38	54.3
59.30	4.70	279	21.0	2.53	53.1
44.15	6.09	269	17.9	2.97	53.2
34.07	7.87	268	16.6	3.53	58.6
18.60	14.33	267	14.1	4.29	60.5
15.60	16.06	251	9.5	5.97	59.4
			6.4	8.58	54.9
			4.5	11.58	52.1
		Average, 269			Average, 55.8

From these tables it is seen that the product, *pd*, is approximately constant for any one substance. The deviations appear to be within the limits of experimental error. A slight tendency has been noticed for this product, for many substances, to decrease with the pressure, but it seems probable that this is caused by the decreased sensitiveness when the distance between the polonium and the zinc sulphide screen becomes large.

The complete results for all the substances examined are given in the following table. The first column of figures gives the sum of the square roots of the atomic weights of the atoms in the molecules. Thus for carbon dioxide we have:

$$\sum\sqrt{A} = \sqrt{C} + 2\sqrt{O} = \sqrt{12} + 2\sqrt{16} = 11.5.$$

For air:

$$\sum\sqrt{A} = \frac{4 \times 2\sqrt{N} + 2\sqrt{O}}{5} = \frac{4 \times 2\sqrt{14} + 2\sqrt{16}}{5} = 7.6.$$

The second column of figures gives the average value of the product, *pd*, for several different pressures found by experiment; and the last column gives the product of the two preceding ones.

		$\Sigma\sqrt{A}$	pd	$pd\Sigma\sqrt{A}$
Hydrogen,	H_2	2.0	1,108	2,216
Helium,	He	2.0	954	1,908
Argon,	A	6.32	301	1,902
Methane,	CH_4	7.46	292	2,178
Air,		7.60	276	2,098
Oxygen,	O_2	8.00	269	2,152
Carbon dioxide,	CO_2	11.46	186	2,132
Cyanogen,	C_2N_2	14.42	153	2,206
Carbon bisulphide,	CS_2	14.78	125	1,848
Methyl iodide,	CH_3I	17.73	110	1,950
Acetone,	C_3H_6O	20.38	107	2,181
Ethyl bromide,	C_2H_5Br	20.86	98	2,044
Chloroform,	$CHCl_3$	22.31	88	1,963
Ethyl iodide,	C_2H_5I	23.19	80	1,855
Carbon tetrachloride,	CCl_4	27.26	78	2,126
Ether,	$C_4H_{10}O$	27.84	88	2,450
Pentane,	C_5H_{12}	29.30	77	2,256
Nickel carbonyl,	NiC_4O_4	37.50	56	2,100
				Average, 2,081

It is thus seen that the product of the pressure, the maximum distance at which the scintillations can be seen, and the sum of the square roots of the atomic weights is very nearly constant over a wide range. The largest deviation is in the case of ether.

The fact that the absorption of complex molecules for the alpha rays can be expressed as a sum of terms depending only on the atoms in the molecules indicates that the effective collisions of the alpha rays are not with the molecules as a whole but with the atoms in the molecules. The experimental facts can be explained by assuming that the number of collisions that each alpha particle makes is proportional to the total number of atoms present in the gas through which it passes, and that at each collision the alpha particle loses an amount of its energy proportional to the square root of the weight of the atom it collides with. But the reason for the square root is difficult to see. Bragg and Kleeman suggested that the atoms have a disk-like form ; assuming then that ions can be formed only on the circumference of the atom, the circumference being proportional to the square root of the whole, the square root law would seem to be explained, But this seems a very artificial hypothesis ; and moreover, we have no evidence that the atoms have a disk-like form. There is some evidence from the kinetic theory of gases that the cross-section of the molecule is equal to the sum of the cross-sections of the constituent atoms, and hence that the molecules have a disk-like form. But this has no bearing on the present problem as it is evident that molecular structure does not enter into it. In a more recent paper[1] Professor Bragg has again suggested this same explanation, with the extension that some effect is produced on an alpha particle when it strikes the center of an atomic disk. Therefore the stopping power of an atom for an alpha particle should be proportional to

$$a\sqrt{w} + bw$$

where a and b are constants, b being small compared to a, and w is the weight of the atom. The scintillation method is not accurate enough,{however, to make it worth while attempting to confirm this expression.

PRINCETON UNIVERSITY,
PRINCETON, N. J.

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[1] Phil. Mag., May, 1906, p 617.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE THIRTY-FOURTH MEETING.

A REGULAR meeting of the Physical Society was held in the Kent Chemical Laboratory of the University of Chicago on Saturday December 1, 1906. Past President A. A. Michelson presiding.

The following papers were presented:

1. Spark Discharges in Gases and Vapors. JOHN E. ALMY, University of Nebraska.

2. On the Magnetic Behavior of Certain Alloys of Nickel. BRUCE V. HILL, University of Kansas.

3. An Experimental Gyroscope for Quantitative Work. FREDERICK E. KESTER, Ohio State University.

4. Upon the Discharge of Electrons from Ordinary Metals Under the Influence of Ultra-violet light. R. A. MILLIKAN and GEORGE WINCHESTER, University of Chicago.

5. An Inanimate Demon. A. B. PORTER, Chicago.

6. On Multiple Crossed Gratings. A. B. PORTER, Chicago.

7. On the Ruling of Diffraction Gratings. A. A. MICHELSON, University of Chicago.

8. The Effect of Temperature on Metallic Spectra. H. G. GALE, University of Chicago.

9. The Radiation Constants and Temperature of the Nernst Glower. C. E. MENDENHALL and L. R. INGERSOLL, University of Wisconsin.

10. Standard Cells. K. E. GUTHE and C. L. VON ENDE, University of Iowa.

11. Heat of Dilution. F. L. BISHOP, Bradley Polytechnic Institute.

12. The Viscosity of Water at Low Rates of Shear. LAWRENCE E. GURNEY, University of Idaho. Introduced by A. A. Michelson.

13. The Bridge Method for Comparison of Condensers. FREDERICK E. KESTER, Ohio State University.

14. A Method for the Determination of Electrolytic Resistance and Capacity. A. H. TAYLOR, University of Wisconsin.

15. On the Effects of the Electrical Discharge on the Acetylene Flame. C. F. Lorenz, Johns Hopkins University.

16. Change of Phase Due to the Passage of Electric Waves Through Thin Films and the Index of Refraction of Water for Such Waves. Wm. R. Blair, University of Chicago.

17. The Temperature of the Moon. Wm. W. Coblentz, Bureau of Standards, Washington.

18. The Temperature of the Moon. Frank W. Very.

19. Note on the Fluorescence of Sodium Vapor. Ernest Merritt.

20. The Relation between Uranium and Radium. H. N. McCoy and W. H. Ross.

21. Note on the Behavior of Small Particles on the Nernst Glower. C. E. Mendenhall and L. R. Ingersoll.

A resolution was adopted directing the attention of the Council to the desirability of holding a regular yearly meeting in Chicago.

Ernest Merritt,
Secretary.

UPON THE DISCHARGE OF ELECTRONS FROM ORDINARY METALS UNDER THE INFLUENCE OF ULTRA-VIOLET LIGHT.[1]

By R. A. Millikan and George Winchester.

THIS investigation was undertaken (1) to determine whether or not the discharge of negative electricity which takes place from metals illuminated by ultra-violet light is a function of the temperature when all secondary temperature effects due to the surrounding medium are eliminated by placing the metals in a very high vacuum; (2) to determine the order in which the common metals arrange themselves in relation to their power of emitting electrons under the influence of a given source of ultraviolet light: (3) to determine the relative velocities of projection of electrons from the various metals by measuring the positive potentials acquired by them under the influence of ultraviolet light; (4) to determine whether this velocity is a function of the temperature.

The results of the investigation may be summarized as follows:

I. Within the limits of observational error, which amount to but one or two per cent., the photo-electric effect, like radio-activity is wholly independent of temperature for all of the metals experimented upon, namely, copper, gold, nickel, brass, silver, iron, aluminium, magnesium, antimony, zinc and lead.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

II. The relative rates of discharge of electrons from the above metals, under identical conditions, were as follows:

Metal.	**Discharge Rate in Scale Divisions.**	**Melting Point.**
Copper.	25.10	1050° C.
Gold.	24.70	1050 "
Nickel.	24.00	1450 "
Brass.	23.80	——
Silver.	17.16	968 "
Iron.	16.40	1350 "
Aluminium.	14.90	625 "
Magnesium.	11.00	750 "
Antimony.	4.00	632 "
Zinc.	1.20	433 "
Lead.	.90	326 "

This order is not at all in accord with that found in the Volta contact series as previous experiments had indicated might be the case, nor does it agree entirely with the order in which the above metals exhibit any familiar physical property. Its closest relationship seems to be to that of the melting point series which is given the last column in order to facilitate comparison.

III. The positive potentials in volts acquired under identical conditions by the metals experimented upon are given in the following table. If the discharge takes place into a perfect vacuum the maximum velocity of projection of the electrons may be calculated from the formula $v = \sqrt{2e/m \times P}$, in which P is the positive potential acquired and e/m is 1.1×10^7.

Metal.	**Potential in Volts.**
Silver.	1.34
Iron.	1.22
Gold.	1.21
Brass.	1.17
Copper.	1.13
Nickel.	1.12
Magnesium.	.839
Aluminium.	.738
Antimony.	.39
Zinc.	.19
Lead.	.0

The order in which the metals occur in this table is wholly different from that of the preceding table and apparently has no connection either with the atomic weights of the elements or with any other of their known properties.

IV. The potentials acquired were found to be in all cases wholly independent of temperature — a fact which appears to warrant the conclusion that the kinetic energy of an electron within the atom of which it forms a part is not influenced by temperature changes.

THE EFFECT OF TEMPERATURE ON METALLIC SPECTRA.[1]

BY H. G. GALE.

THIS investigation was undertaken in the hope of finding in the laboratory some method of differentiating the spectral lines of Ti, Va, Cr, Mn and Fe which are affected in sun spots. The work was done in the laboratory of the solar observatory on Mt. Wilson, in collaboration with Mr. Hale and Mr. Adams, and was in the nature of a sequel to the important work which had been done by them on the spectra of sun spots.

It was found that when the arc spectrum of one of the elements named was taken with a two ampere current and compared with a spectrum taken on the same plate with a 25 or 30 ampere current, certain lines were strengthened and a few weakened as compared with the rest of the spectrum. Further examination showed that the lines which were strengthened in the two ampere current were in practically all cases, lines which are strengthened in sun spots, and that the lines which are weakened in sun spots are the lines which were weakened in the two ampere arc.

It was found further that in practically all cases lines which were weakened in the two ampere arc as compared with the 30 ampere arc are lines which are enhanced in the spark relatively to the arc.

Of 220 spot lines examined 204 were definitely strengthened in the weak arc and the remaining 16 lines were either unchanged or changed so slightly as to be uncertain.

Of 32 lines weakened in spots 29 were clearly enhanced and 3 were unchanged in the spark. Of these 32 lines 24 were clearly weakened, 7 were not seen and I showed no change in the two ampere arc.

Extending the investigation into the violet beyond the region which is apparently affected in sun spots, of 109 lines enhanced in the spark 84 were weakened, 20 not seen and 5 were unaffected in the weak arc.

Of 152 prominent solar lines which are not affected in spots none showed strengthening in the weak arc, 133 were unchanged and 19 doubtful, possibly slightly weakened.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

The only directly contradictory evidence found was in the case of the two iron lines at λ 5429.91 and λ 5447.13. Both appear to be enhanced in the spark, although they are strengthened in both the weak arc and in spots.

The cause of the variation between the spectra of the 2 ampere and 30 ampere arcs which naturally suggested itself was that the 2 ampere arc was cooler than the other. To test this hypothesis spectra were photographed from the outer portions of the flame coming from a 30 ampere arc and compared with spectra from the middle of the core, between the poles. The agreement between the weak arc and flame was entirely satisfactory, the great majority of the lines being affected not only in the same direction but by the same amounts.

To further test the effect of temperature as distinguished from any possible electrical effect Fe, Mn and Cr were in turn vaporized in an electric furnace and the spectra photographed. All of the lines in the region examined were affected in the same direction as in the two ampere arc and by practically the same amounts.

It seems permissible therefore to conclude that the spark spectrum corresponds to a higher temperature than the arc spectrum, that the variations between the 30 ampere spectrum and that from the weak arc, flame or furnace are due to the lower temperature of the latter and that the differences between sun and spot spectra are due to the same cause.

Mr. Adams has recently found that the spectrum of Arcturus is in remarkable agreement with that of sun spots, and Mr. Hale and Mr. Adams have found that the spectrum of *α* Orionis shows still greater variations in the same direction. It seems probable therefore that the Sun, Arcturus and *α* Orionis are in order of descending temperature.

It does not follow that temperature differences offer the only means of producing changes in the relative intensity of the lines in the metallic spectra, but variations in temperature appear to afford a simple and satisfactory explanation of the changes in the relative intensity of the so-called "spot lines" as observed in the laboratory, in sun spots and in stars.

RYERSON PHYSICAL LABORATORY,
November 9, 1906.

STANDARD CELLS.[1]

BY K. E. GUTHE AND C. L. VON ENDE.

We constructed some standard cells with electrolytic mercurous sulfate, prepared by us, and with sulfate received from Dr. Hulett. The agreement between the electromotive forces for cells of the same series is within 2 in 100,000. Slight variations in the method of construction were made and their influence upon the electromotive force determined.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

Comparison of these new cells with those used last year for the absolute determinations, showed that the Clark cells agreed with the old, but that the cadmium cells had a considerably higher electromotive force than the old, the changes of which have been quite irregular.

While the initial electromotive force of the new cadmium cells agrees very closely with the value found by Hulett for freshly set up cells, it decreases rapidly in a few weeks and has now a value 0.0001 volt lower than at first.

Mercury distilled twice in an ordinary vacuum still gives the same results as mercury distilled by Hulett's method. No difference was noticed between amalgam prepared in the usual way and electrolytic amalgam. Clear and cloudy cadmium sulfate crystals produce no difference in the electromotive force of the cadmium cells.

Clark cells with electrolytic, coarsely grained mercurous sulfate have immediately after construction an electromotive force of 1.4204 volts at 25° C., within a few hundred thousandths of a volt and remain constant in course of time.

Cadmium cells with practically identical electrical behavior cannot be constructed at the present time by different observers.

THE BRIDGE METHOD FOR COMPARISON OF CONDENSERS.[1]

BY FREDERICK E. KESTER.

THEORETICAL considerations show that the two possible arrangements of galvanometer and battery, in the bridge network for the comparison of condensers, give different values for the quantities of electricity which flow through the galvanometer when the conditions of exact balance are disturbed by equal changes in the value of one of the balancing resistances. With the galvanometer connected between the junction of the two condensers and the junction of the two resistances, the expression for the quantity flowing through the galvanometer is

$$\int_0^\infty i \cdot dt = C_2 \cdot E \cdot \frac{\Delta R_2}{R_1 + R_2}.$$

The other connection, with the galvanometer and battery interchanged gives

$$\int_0^\infty i \cdot dt = C_2 \cdot E \cdot \frac{\Delta R_2}{R_1 + R_2 + G}.$$

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

With galvanometers of high resistance the difference between the values of these two expressions is considerable.

Experimental results obtained with various condensers and various values of the balancing resistances verify these formulæ within the limits of observation errors. The verification may be shown by comparison of the product of $(R_1 + R_2)$ and the galvanometer deflection per ohm change in R_2 in the first case, with the product of $(R_1 + R_2 + G)$ and the galvanometer deflection per ohm change in R_2 in the second. The products should be equal since the galvanometer deflections are proportional to $\int^t i \cdot dt$ if a ballistic instrument be used. In the experimental work for the results given below the same value of E was used throughout, the same galvanometer conditions were maintained and the same condenser was used always as C_2; the products given in the last column of the tables should, therefore, be approximately equal.

GALVANOMETER RESISTANCE 1,916 OHMS.

Condensers: C_1, *Elliott Bros.*, ⅓ *m. f.*, *No.* 848; C_2, *Elliott Bros.*, ⅓ *m. f.*, *No.* 847.

Connection.	R_1	R_2	Defl. per Ohm.	Product.
I.	2,000	1,980	0.064_7	258
I.	200	198	0.63_7	254
II.	2,000	1,980	0.039_4	232
II.	200	198	0.10_9	252

Condensers: C_1, *Elliott Bros.*, 1 *m. f.*, *No.* 56; C_2, *Elliott Bros.*, ⅓ *m. f.*, *No.* 847.

Connection.	R_1	R_2	Defl. per Ohm.	Product.
I.	2,000	6,224	0.031_5	259
I.	200	622	0.32_8	270
II.	2,000	6,224	0.025_7	261
II.	200	622	0.09_8	268

THE TEMPERATURE OF THE MOON.[1]

By W. W. COBLENTZ.

AT the Ithaca meeting of the society, June, 1906, the writer offered as a side issue to his main work on reflection spectra, some deductions relating to the temperature of the moon, which subsequent consideration has led him to believe erroneous. At that time the experimental

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

work of Very[1] and the calculations of Poynting[2] were unknown to him, and the conclusions were based upon the original curves published by Langley.

The work of these two investigators is quite convincing that the temperature of the lunar surface must rise to at least 300° abs., and the problem is reduced to the calculation of the relative intensities of the energy emitted and reflected by the moon at 8 to 10 μ. This has been done and the results show that the radiation from the sun which the moon reflects to the earth is only from $\frac{1}{4}$ to $\frac{1}{18}$ the energy which the moon radiates to us, assuming the temperature to be 300° abs.

The writer regrets the fact that the former deductions have gotten into print, for they accentuate the not infrequent occurrence of similar cases where writers have published conclusions before the subject has been thoroughly considered. A fuller discussion of this subject is now in press along side with the work on reflection spectra.

NOTE ON THE TEMPERATURE OF THE MOON.[3]

BY FRANK W. VERY.

THE supposition that the Moon may reflect specularly a portion of the Sun's rays between 8.5 and 10 μ is rendered probable from Professor E. F. Nichols' discovery that quartz has bands of metallic reflection in this region, and from the recent announcement by Dr. W. W. Coblentz that various common silicates possess similar properties.

But this specular reflection has never been observed, and must not be confounded with the maximum in the lunar spectrum of about the same wave-length, since the latter belongs to a continuous spectrum and not to a band. Moreover, the maximum heat in the lunar image is found, not at the position where specular reflection would place it, but near the point of maximum insolation. Consequently, the observed maximum in the extreme infra-red spectrum of the Moon is due to the proper radiation from the heated lunar surface.

During totality of a lunar eclipse of the darker sort, the amount of sunlight refracted into the shadow by the Earth's atmosphere, and reflected from the Moon, becomes vanishingly small; but the true infrared radiation of the Moon may still be as much as one per cent. of its value before the eclipse, and the heat is promptly restored, or very nearly restored, after the eclipse is over, showing that, although a thin superficial layer of the Moon's substance is considerably cooled in shadow, deeper layers continue hot and are competent to restore by conduction the heat lost by radiation during the few hours that the eclipse lasts.

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[1] Astrophys. Jour., 8, p. 199, 1898.

[2] Jahrbuch der Radioaktivität and Elektronik., 2, p. 42, 1905.

[3] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

The superficial layers of rock form a sufficient protection to the deeper ones to produce a large accumulation of thermal energy at the surface of the sunlit Moon after many days of continuous sunshine. Simple reflection will not explain the quality of the lunar radiation, which can be shown to proceed necessarily from a body at a high temperature.

HEAT OF DILUTION.[1]

By F. L. Bishop.

THE apparatus employed was especially designed to insure that the two liquids should be at the same temperature when mixed and to avoid the determination of the specific heat of the solution.

The two calorimeters were so designed that one was inside the other and the liquid of the outer completely surrounded the inner calorimeter. The inner one which was provided with vanes on the inside and outside was suspended and rotated at a constant rate thus agitating the liquids. The mixing was accomplished by having the bottom of the inner one removable and lowering the calorimeter so that the liquids would pass down through the inner up the outer and over the top of the inner calorimeter. The change of temperature thus produced was measured with two Beckman's one in each calorimeter.

A quantity of heat necessary to produce the same change of temperature under the exact conditions existing when the two liquids were mixed, was supplied by an electric current. The coil was placed in the outer calorimeter just below the inner one. The current was adjusted until the rate at which the temperature changed was the same as when the liquids were mixed.

The whole apparatus was placed in a double walled vessel and immersed in a constant temperature bath.

The preliminary results seem to indicate that the values for $NaNO_3$ NH_4NO_3 and $Ba(NO_3)_2$ are within certain limits represented by a linear function of the heat and concentration.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

THE RELATION BETWEEN URANIUM AND RADIUM.[1]

BY HERBERT N. McCOY AND W. H. ROSS.

ONE of us has shown[2] that the radioactivity of any uranium mineral, free from thorium, is directly proportional to its uranium content; and for equal uranium content that minerals are 4.15 times as active as the pure compounds. The value, 4.15, previously found, is subject to three errors: (1) On account of the insufficient ionization space of the electroscope used; (2) on account of the portion of the emanation spontaneously lost; (3) on account of the unequal absorption of the rays by the rims of the shallow vessels containing the active films. New determinations of the composition and activity of uranium minerals have now been made, with all conceivable precautions to insure accuracy. We now find the corrected ratio of the activity of a mineral to that of a pure uranium compound, of equal uranium content, to be 4.58. We have, in addition, compared the activity of a minute quantity of Ra, free from all its products, with the activity of a standard film of U_3O_8; and also the activity of the emanation from this quantity of radium with that of the emanation associated with a known amount of uranium in a mineral. It was found that the activity of a quantity of uranium is 1.87 times that of the equilibrium amount of radium. Radium plus the equilibrium amounts of the emanation, *A* and *C* is 5.64 times as active as radium alone.[3] If we assume that the activity of RaF is proportional to its range, this product is 1.10 times as active as radium. Therefore radium and its products are 6.74 times as active as radium. In consequence, a mineral containing the equilibrium amounts of uranium, radium and products should be 8.61 times as active as the radium, or 4.60 times as active as the uranium it contains. The value found directly is 4.58.

If radium is considered a transformation product of uranium, as these and other experiments indicate, and further if the range of the α particle of uranium is taken as approximately 3.5 cm., that of radium being also 3.5 cm., and if the activity is assumed proportional to the range,[4] for equilibrium amounts of genetically related bodies producing equal numbers of α particles, then since uranium is about twice as active as the equilibrium amount of radium, the former must produce two α particles for every one produced by the equilibrium amount of the latter. This indicates either that each atom of uranium produces, upon disintegration, two α particles, or that there is an α ray product, not yet isolated, between U and UX.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

[2] Ber., *37*, 2,641, 1904; Phil. Mag., *11*, 176, 1906.

[3] Boltwood, Am. J. Sci., *21*, 409, 1906.

[4] Boltwood, loc. cit.

KENT CHEMICAL LABORATORY,
UNIVERSITY OF CHICAGO,
November 23, 1906.

AN INANIMATE DEMON.[1]

BY ALBERT B. PORTER.

CLERK MAXWELL'S intelligent "demon" was created for the purpose of showing the second law of thermodynamics to be a purely statistical result of molecular motion. The following argument avoids the necessity of invoking an animate demon. Imagine a gas-filled non-conducting vessel to be divided into two compartments by means of a conducting partition. Let the partition be pierced by numerous holes of molecular dimensions closed by very light spring trapdoors all opening toward the first compartment. These doors will be opened by impact of the gaseous molecules, thus permitting passage of the molecules from the second compartment into the first, but preventing the passage of any molecules in the other direction. There will thus be an accumulation of gas in the first compartment, and a corresponding rise of pressure which may be utilized as external work by permitting the gas to flow back into the second compartment through a turbine or other pressure engine. Mechanical work would thus be obtained by continuously cooling the working gas far below the temperature of surrounding objects until the molecular momentum became too feeble to force the trapdoors. If the turbine were made to do its work wholly within the containing vessel, this work would all be eventually given back to the gas in the form of heat. The arrangement would then constitute (in the German terminology) a perpetual motion of the second class. If the gas vessel were made with conducting walls, the engine would be capable of utilizing uniformly diffused heat, and would thus be independent of a source and sink.

It is not difficult to imagine a molecular structure having the sort of unilateral porosity required of the partition, but experiment has failed to realize such a septum. The assumption of structure of this sort in certain animal membranes might go far toward explaining such puzzling physiological phenomena as the high thermodynamic efficiency of the animal machine.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

On Multiple Crossed Gratings.[1]

By Albert B. Porter.

These gratings are made by contact printing on bichromated gelatine. By making a half-exposure, then turning the original grating through a right angle and completing the exposure, one secures, on developing the plate, a grating consisting of two sets of lines crossing at right angles. By making a half-exposure of this crossed grating on a second plate, turning it through 45° and completing the exposure, one obtains a set of four gratings crossed at 45° on one film. Continuing this double printing, a succession of crossed gratings are secured having 8, 16, 32, 64, etc., sets of lines. There is of course some loss of intensity at each step, but much less than might be expected. Starting with a grating ruled with 3,000 lines to the inch, a multiple grating has been made in this way having 1,024 sets of lines, each 3,000 to the inch, crossing at symmetrical angles. A gas flame seen through this plate appears to be surrounded by a continuous halo, but around a distant arc lamp the individual spectra can be seen and counted. Under the microscope regularity of structure can be recognized in the films bearing fewer than 32 crossed gratings, but the films which bear from 32 to 1,024 crossed gratings merely show a mosaic of numberless short lines running in apparently random directions.

A Method for the Determination of Electrolytic Resistance and Capacity.[1]

By A. Hoyt Taylor.

For very low voltages a polarizing electrolyte behaves like a capacity in series with a resistance; with possibly a second resistance in parallel.

It is possible to build an A. C. bridge with three non-inductive arms, the fourth arm consisting of a capacity C in series with a resistance r_2 and an inductance L_1 of resistance r_4. The capacity plus r_2 must be shunted with a resistance r_1; the inductance must be shunted with a resistance r_3. Under these conditions a perfect telephonic balance may be obtained providing the following equations hold:

$$C = \frac{r_3^2 L}{(r_3 + r_4)^2 r_1^2}; \quad \frac{r_1^2}{r_3^2} = \frac{r_1 + r_2}{r_3 + r_4}; \quad R = r_1 + \frac{r_3 r_4}{r_3 + r_4}.$$

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

The last equation allows us to calculate r_1, since R is the non-inductive resistance in the third bridge arm, the proportional arms being equal.

The second equations allow us to calculate r_2, while the first one gives C.

An electrolyte, sulphuric acid and water, with platinum electrodes, when substituted for the combination $r_2 + C$ gives very satisfactory balances when the E.M.F. is not over a few tenths of a volt.

AN EXPERIMENTAL GYROSCOPE FOR QUANTITATIVE WORK.[1]

By Frederick E. Kester.

THE fundamental theory of gyroscopic action can be put into such simple form, and the problem is so interesting and so instructive, that it would seem desirable to introduce quantitative work upon a suitably designed piece of apparatus as a regular laboratory experiment in undergraduate courses. The piece described in this paper was designed with the idea in view of allowing all necessary measurements to be made with an accuracy such as to give a sense of reliance in the principle involved in the experiment.

What may be called normal conditions of gyroscopic action exist when the fly-wheel is set spinning and the axis which carries it is turned with constant angular velocity about a second axis perpendicular to the first; in order to maintain this double rotation it is necessary to exert a moment of force about a third axis which is perpendicular to the plane of the first two axes. The magnitude of the moment of force is $I \cdot \omega_1 \cdot \omega_2$, where I is the moment of inertia of the fly-wheel, ω_1 and ω_2 are the angular velocities of the wheel and of its own axis about the second axis. If I be measured in g-cm.^{2} units and ω_1, ω_2 in radians per second, then the moment of force will be in cm.-dynes; if I be in lb.-ft.^{2}, the moment of force will be in poundal-ft.

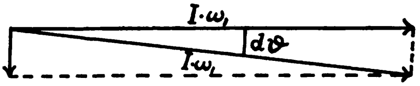

Fig. 1.

In the piece under discussion the fly-wheel is carried on ball-bearings upon a horizontal spindle which is mounted at the end of a tube; the tube is, in reality, a balance beam and is provided with counterpoises on the end opposite the wheel. The balancing shaft (to take the place of the central knife-edge of the ordinary balance-beam) is also mounted on balls; the bearings are at the tops of a vertical Y-piece, the lower end of which is long and constitutes the second axis mentioned above. The lower end of this vertical shaft carries a combination piece which is, essentially, a set of pulleys (graded in size) and a cylindrical surface about 20 cm. in diameter and 3 cm. high.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

The laboratory experiment would involve the measurement of the two angular velocities and of the moment of force necessary to maintain equilibrium; from this data I is then calculated. The angular velocity of the wheel, ω_1, is determined from the clicking of a light spring at each revolution; that of the vertical shaft, ω_2 (a relatively slow motion) from the record, upon the cylindrical surface mentioned above, of a telegraph sounder which is beating seconds and which carries a scriber for tracing upon the whitened surface of the cylinder (a mixture of whiting and alcohol is very convenient here); divisions engraved upon the surface, at one edge, permit considerable accuracy in this measurement. A scale of millimeters along the side of the balance-beam together with the known mass of one of the counterpoises allow the measurement of the moment of force to be made with sufficient accuracy. Stops are provided for holding the balance-beam well within bounds, as is also a device for determining just when balancing conditions are attained.

The value of I thus obtained may be checked by calculation from the known mass and form of the wheel. The piece is so designed as to permit a second check on the value of I by the method described by Glazebrook and Shaw,[1] and by Millikan.[2]

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[1] Practical Physics, 2d ed., p. 166.

[2] Molecular Physics and Mechanics, p. 81.

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THE

PHYSICAL REVIEW.

THE LATENT HEAT OF RECALESCENCE IN IRON AND STEEL.

BY F. K. BAILEY.

THE phenomenon of recalescence in iron and steel has, in recent years, been the subject of considerable study. Apparently, all the properties of iron and steel undergo a change, greater or less, at the temperature at which the magnetic properties are lost. Such a change of properties indicates what is commonly termed a change of state, that is, a change of the physical state or constitution of the substance. As is usual with such changes of state, this change in iron and steel is accompanied by an absorption of heat when passing from the colder to the warmer state. As this heat is liberated at the corresponding change on cooling, and as it seems to be used solely to produce the change, it may correctly be termed latent heat. If the name of the most striking phenomenon connected with the change, that of recalescence, be given to the whole phenomenon, then we may correctly speak of the latent heat of recalescence. Strict analogy, however, would require a name for the phenomenon during rise of temperature, just as we speak of latent heat of fusion or of vaporization. Or, a name might be given to the substance in the heated state, and we should speak of the latent heat of steel in that state as we speak of the latent heat of water or of steam.

The present work was undertaken with a view to determine the value of this latent heat, and to learn in what way, if in any, it depended upon the composition.

Hopkinson[1] has shown that a piece of iron or steel, if heated above the point of recalescence and allowed to cool slowly, pauses in decline of temperature, for several seconds, at the temperature where it becomes magnetic. He also showed graphically that the amount of heat given off in this interval is 175 to 200 times that given off by the same piece of matter, in a fall of temperature of one degree, just above or below this temperature. That is, the piece under observation would give out heat without fall of temperature, for a period long enough to reduce its temperature 175 or 200 degrees if the cooling were perfectly uniform.

These figures, however, give the latent heat in terms of the specific heat of iron at this temperature. The best determinations of this quantity have been made by Pionchon,[2] and more recently by Harker,[3] who has corrected the values of the former, besides making a re-determination. Their figure for the specific heat of iron just above or below this temperature is .20 times that of water. Multiplying by this 175 or 200 degrees, we get 35 or 40 calories per gram, as the latent heat of recalescence in the specimen examined. The very magnitude of this quantity invites further study. For, comparing with other substances, we find it relatively greater than even the latent heat of the ice-water change, which is only about 160 ice gram degrees, or 80 water gram degrees.

The specific heat of a substance is determined from the total quantity of heat given up by a unit of mass in cooling from a given high temperature to 0° C., or to some ordinary temperature. If the upper temperature is above that of the change of state, then the latent heat will be included with the rest. If the heat be expressed by a formula, as is done by Pionchon, the latent heat should be separated from the other and appear as a constant term, in formulas applying to temperatures above the change of state. Pionchon gives several formulas for the total heat between 0° C. and various high temperatures. They contain a constant term, but it is much larger than the value he gives for the latent heat, and it also appears with a negative sign. He finds for the latent heat 5.3 calories.[1] Harker does not give a value for the latent heat, but by plotting his figures[2] one finds the figures 7.5 calories.

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[1] Phil. Trans., 1889, A, pp. 463–465; Proc. Roy. Soc., Vol. 48, p. 442; or Original Papers, Vol. II., Nos. 30, 31, pp. 217–226.

[2] Ann. d. Chim. e. Phys. (6), Vol. 11, p. 33, 1887.

[3] Phil. Mag., Vol. 10, p. 430, 1905.

Hopkinson has also shown[3] that the phenomenon of recalescence is greatly affected by the composition of the steel. This fact and the wide differences of previous results were the incentive for the present work. The object of the work is, therefore, to determine the specific heat and latent heat of several samples of steel and study the results in connection with analyses of the studied samples. Both the calorimetric and the rate of cooling methods have been used. The first part of the research comprised the calorimetric work. Because of certain features of the apparatus, and because the main object of the work was the value of the latent heat, the range of temperature has not been great, *viz.*, 460° to 860 C°.

The Heating Apparatus. — This was an electric resistance furnace constructed by the writer as shown in Fig. 1.

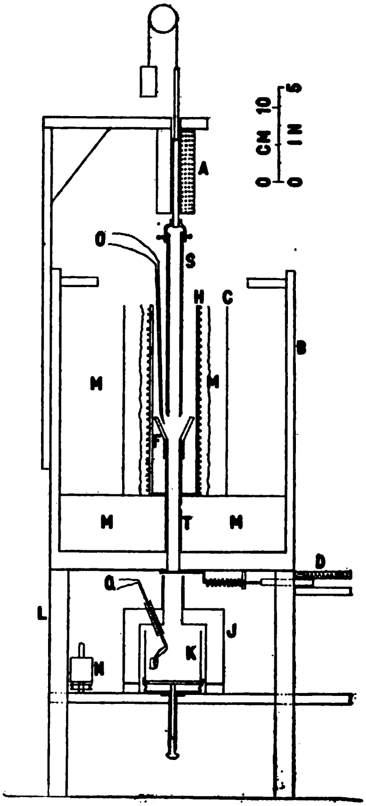

Fig. 1.

A square wooden box, *B*, was made, whose inside measure was 30 cm. side and 38 cm. deep. It was open at the top. Through the center of the bottom was a hole 2 cm. in diameter. A porcelain tube, *T*, 15 cm. long, and having an internal diameter of 1.5 cm., was placed vertically over this hole. The whole bottom of the box, around the tube, was then covered to a depth of 8 cm. with magnesia, *M*, firmly packed. On top of this was placed a sheet of asbestos board on which was stood a sheet iron cylinder, *C*, 18 or 20 cm. in diameter, and 25 cm. high. (A piece of stove pipe would be suitable.) This cylinder was placed with its axis coinciding with that of the box. The space between the cylinder and the box was also tightly packed with magnesia as far up as the top of the cylinder. The object of this cylinder was to allow the heating coil to be removed without disturbing the packing. The heating coil, *H*, was placed in the center of the cylinder. The heating coil was a clay cylinder wound with nickel wire. Two battery porous cups were used, placed end to end. The bottom of one cup was cut off and the end ground to fit neatly the other cup. The bottom of the second cup was pierced with a hole large enough to admit the tube, *T*. The cylinders were wound separately. Nickel wire No. 20 B. & S. gauge was used. The cups were first wrapped with two layers of asbestos paper, then wound with wire, and then covered with a paste made of magnesia and water. The wire, after being annealed, was wound on by hand, the turns being about 3 mm. apart. The paste was spread on smoothly to a thickness of half a centimeter, the ends of the wires, in the meantime, being held by clamps. When the paste had dried, the coils were placed in the center of the space within the cylinder, *C*, the tube, *T*, projecting up into the lower cup some 8 cm. The lead wires were brought to the top of the box through glass tubes, and the remaining space between the coil, *H*, and cylinder, *C*, was tightly packed with dry magnesia. A layer about 2 cm. thick of the magnesia paste, nearly dry, was then spread over the top and firmly pounded down, so that the top was even and hard. The shell of magnesia on the heating coils held the wires in place and remained intact even after many heatings. When, after long use, the wires had to be renewed and the clay cups were withdrawn, this shell came out whole and was firm enough to require a metal tool for its removal. The heating coil having two parts was a decided advantage, for it not only allowed greater variation in heating, but also made the process of packing easier. Moreover, if one coil were destroyed, it alone need be rewound. This completed the furnace, it consisting essentially of a clay cylinder wound with wire, and surrounded with a thick packing of magnesia, there being a tube connecting the space inside the cylinder with the space outside beneath the furnace.

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[1] Loc. cit , pp. 77, 106.

[2] Loc. cit., p. 437.

[3] Loc. cit., p. 463.

The furnace constructed in this manner proved both durable and efficient. Temperatures above 1,000° C. were not attempted, but the furnace was heated many times to somewhat lower temperatures, 700° to 900° C., the period of the high temperature being four or five hours. Even when the temperature had been maintained near to 1,000° C. for several hours, the outside of the box could be touched with the hand with no discomfort. The inside of the box may have been as warm as 100° C.; hence, the magnesia would maintain a gradient of 80° C. per cm. The heating current was alternating current taken from the lighting circuit at 110 volts. The current was regulated with a rheostat. To maintain a temperature of 1,000° C. between 5 and 6 amperes were required. The resistance at 1,000° was about 10 ohms; hence, the energy radiated by the furnace when the interior was at 1,000°, must have been about 360 watts.

Calorimetry. — Two methods of calorimetry were tried. The first method was that of Hesehus, successfully used and described by Waterman.[1] While the method will give accurate results if sufficient care is taken, it has still one serious defect in that two temperatures must be measured with great precision on two different thermometers. Even if these are carefully calibrated and compared, an extra source of error is introduced at a vital part of the process. In addition to this, the increased difficulty of manipulation is an objection to the method. The results not being sufficiently accurate, the method was abandoned and the ordinary method of mixtures used.

The furnace was mounted on four legs, L, 30 cm. long. Connecting the legs on each side were cross sticks on which was placed a board 65 cm. long and 25 cm. wide. On this board was placed a small turntable consisting of a disk of hard rubber, 7 cm. in diameter, grooved on the edge, and supported by its center on the upper end of a small shaft. The lower end of the shaft rested on the conical end of a screw, by which the height of the disk could be adjusted. A small electric motor, *N*, was also mounted on the board and connected to the turntable by a small belt.

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[1] Physical Review, Vol. 4, p. 161, 1897.

The calorimeter, *K*, was a polished silver cup 7 cm. high and 7 cm. in diameter, and had a weight of 50 grams. Its water equivalent was, therefore, 2.8 grams. The calorimeter was carried on the turntable, resting on three bits of cork. The turntable was so placed that when the board was pushed in to a stop, the calorimeter was directly under the center of the furnace (this is the position shown in the figure), and when drawn out to another stop, the calorimeter was in a convenient position to remove and change. The wires leading to the motor were so arranged that the circuit closed automatically when the board was pushed in.

To reduce radiation, a water jacket, *J*, was made, which completely covered the calorimeter except for a circle 2.5 cm. in diameter above the center. The water jacket was bright on the inside and had a water thickness of 2 cm. The outside of the jacket was covered with two thicknesses of asbestos paper. Through the top of the jacket passed two tubes. One of these was 2.5 cm. in diameter and was in the center of the top. This opening served to admit the hot body to the calorimeter and was lined on the inside with asbestos paper, the tube of lining extending above the top of the jacket so that it came within .5 cm. of the bottom of the furnace. The other hole had a diameter of 1 cm. and had an oblique position. The purpose of this tube was to admit the thermometer. The thermometer, *Q*, consisted of a thermopile of four elements of copper-german silver. The wires in the elements were 60 cm. long, except the terminal copper wires, which were long enough to reach the galvanometer. The pile was connected directly to a d'Arsonval galvanometer having a resistance of 114 ohms. The pile was carefully calibrated with mercury thermometers and found to give a deflection of 23.3 mm. (scale distance, 50 cm.) per degree Centigrade difference in temperature of the junctions. The deflection was found to be a linear function of the difference in temperature, which fact permitted the rise in temperature in the calorimeter to be derived directly from the difference in deflection, without first reducing to initial and final temperatures. The constant temperature junctions were fixed into a glass tube containing oil, and this tube placed in water contained in a jar insulated with a thick covering of magnesia. The temperature of this bath was determined to a tenth of a degree by a mercury thermometer. This temperature was very constant, not changing by a degree in several hours. Since, in any one operation, the pile was in use only two or three minutes, the constant temperature ends could not have changed by one one-hundredth degree during the operation. The wires of the pile were bound together in a rubber tube and one end passed through the small hole in the water jacket. It was securely fastened in this hole with the ends projecting inward in such a position that when the jacket was placed over the calorimeter, the ends dipped therein. The ends were spread apart and small pieces of mica (1 cm. square) attached to them. The ends and mica were covered with collodion to give insulation and also to hold on the mica vanes. These vanes served to stir the water when the calorimeter was rotated. This method of stirring proved very effective and convenient. (If the axis of the calorimeter were made to describe a cone, the water could be rotated without rotating the calorimeter. This might be desirable in some cases.)

Manipulation. — The specimen whose specific heat was to be determined was made into a sphere small enough to pass through the tube, *T.* This tube projected some 8 cm. into the chamber of the furnace, and carried on its upper end a small funnel, *F*, made of cast iron. (For temperatures above 800° the iron funnel is unsatisfactory by reason of its oxidation, the oxide sticking to the rod, *S*, and preventing its rising. A clay funnel would, no doubt, solve the difficulty.) The funnel was closed by a rod of clay or porcelain, *S*, larger than the hole, resting in it. This rod extended through the cover of the furnace and carried at its upper end a short iron rod which extended into a solenoid, *A*, rigidly fixed above the furnace. The rod could be lifted by passing a current through the solenoid. The iron rod was continued in a smaller brass rod, to upper end of which was attached a counterbalance. A notch in the brass rod, and a detent (not shown) served to hold up the rod when once lifted. This was to prevent the rod from falling back with a jar into the funnel. The lower end of the tube, *T*, was covered by a slide which could be drawn aside by a magnet, *D*, and closed by a spring. The wires were so connected that the circuit through the upper solenoid was closed by the opening of the slide. This insured the funnel opening immediately after the slide.

When the specimen was introduced into the furnace, it rested in the funnel beside the rod, *S*, and when this was lifted, the ball rolled down the funnel and fell directly into the calorimeter. Its total fall was 28 cm., of which 10 cm. was through hot tube in the furnace, 8 cm. through tube varying from the furnace temperature to atmospheric temperature, and the remainder, 10 cm., through the water jacket at atmospheric temperature. There was, therefore, very little opportunity for cooling before the calorimeter was reached.

The size of the specimen and of the calorimeter was so proportioned that the rise in temperature of the latter was small. This reduced the radiation to negligible proportions. The calorimeter temperature would rise quickly and remain stationary for a much longer period. Moreover, the initial temperature of the calorimeter could be varied several degrees without appreciable change in the results. The greatest error observed which could be assigned to this cause, was about two per cent., and this was a very unfavorable case. The temperature of the calorimeter was usually such that the final temperature was not more than one or two degrees above that of the water jacket. (The errors due to radiation might be wholly eliminated if the water in the jacket were heated at the same time and to the same degree as that in the calorimeter. By making the chamber of the furnace double and providing a piece whose heat capacity should bear the same ratio to that of the calorimetric piece that the heat capacity of the jacket bears to that of the calorimeter, and then dropping the two pieces at once into the jacket and calorimeter respectively, with proper stirring, the desired result should be obtained. Or the jacket might be heated electrically and equality of temperature maintained by the use of a second thermo-couple.)

The temperature of the furnace was determined by a platinum-platinum-rhodium couple, *O*. The E.M.F. was measured on a Wolff potentiometer, adjusted to read directly in hundredths of a millivolt. The temperature of the hot junction was then found from a table made for the purpose. The element was compared with a standard element made of wires from the same pieces and calibrated at the National Bureau of Standards in Washington, District of Columbia. The formula given for the standard element was:

$$E = -0.280 + 0.00788t + 0.00000182t^2.$$

The element used in these experiments had a slightly higher E.M.F. and the values for temperature and E.M.F. in the table used were as follows, the cold junction being at 0° C.

Temp.	300	400	500	600	700	800	900	1,000	degrees C.
E.M.F.	2.14	3.08	4.06	5.05	6.08	7.14	8.24	9.37	millivolts.

The cold junctions were fixed in a glass tube in a manner quite similar to that in case of the pile Q, and they were kept in the same water bath. The hot junction was inserted in the furnace bare, the wires passing through a length of two-way clay tubing. The junction was very close to the funnel and could almost be touched by the ball. In some later experiments the junction was broken and the wires reunited. This new junction was then compared with the standard and found to agree exactly with the former calibration. The furnace temperature rose slowly during any series of experiments (see below), but the rate was so slow that it was assumed that the ball and element were at the same temperature. This assumption was justified by the fact that when the ball had been in the furnace for fifteen minutes it became perfectly indistinguishable from its surroundings. This was repeatedly observed. A difference of a single degree would have rendered it visible. The ball was left in the furnace, as a rule for twenty minutes.

A single experiment comprised the following operations: (1) Placing specimen in furnace, (2) filling and weighing calorimeter, (3) placing same in jacket under furnace, (4) setting and reading potentiometer, (5) reading galvanometer for initial temperature of calorimeter, (6) dropping specimen into calorimeter, (7) reading galvanometer for final temperature of calorimeter, (8) reading thermometer in water bath, (9) removing calorimeter and specimen.

The time between operations 3 and 4 was sufficient to reduce the previous set of readings. Below is given a specimen set of observations as taken from the note book.

Specimen No. 7.

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
8	227.2 grams. 180.0 "	650° 740°	400 *R*	482 *B* 2.1°	882 3.79°	16.2° 18.3°	748° 730°	682.2 .1513	7.9

Weight of ball, 6.103 grams; weight of covering, 0.327 gram.
Heat capacity of covering per 100°, 1.09 calories.

Headings of columns, first row.	Second row, calculations.
A. Number of experiment.	
B. Weight of water and calorimeter (obs.).	Water equivalent of same.
C. Reading of potentiometer (obs.).	Temperature corresponding in table.
D. Reading of galvanometer before drop (obs.).	
E. Reading of galvanometer after drop (obs.).	Diff. temperature bet. cal. & bath.
F. Swing of galvanometer $=E-D$ (calc.).	Rise in cal. temp.
G. Temperature of water bath (obs.).	Final temp. of cal. $= G + E_1$.
H. True temp. of furnace $= C_1 + \frac{G}{2}$.	Fall in temp. of specimen $= H - G_1$.
I. Heat given to calorimeter $= B_1 F_1$.	Mean specific heat $= \frac{I-J}{6.103 H_1}$.
J. Heat given to calorimeter by covering $= \frac{H_1 1.09}{100}$	

The first part of the research consisted in the determination of the specific heat of several samples of steel having different composition. In addition to this, experiments were made on platinum, nickel, aluminum and copper. The results of these experiments follow:

Temp.	Platinum.		Nickel.		Aluminum.		Copper.	
	Total.	Specific.	Total.	Specific.	Total.	Specific.	Total.	Specific.
400–20			42.4	.1115	86.6	.2280		
500–20			55.0	.1145	113.8	.2370		
550–20	17.6	.03331			128.0	.2415	51.6	.0975
600–20	19.4	.03344	68.0	.1173	142.7	.2460	57.2	.0987
700–20	22.9	.03370	81.6	.1200			68.7	.1010
800–20	26.4	.03395	.95.7	.1227				
900–20	30.1	.03422						

Only a few experiments were made on aluminum and copper; hence the figures given have little value. The pieces were taken from commercial stock found in the shop of the laboratory. The platinum and nickel specimens were obtained from Baker & Company, of Newark, New Jersey. No analyses were made of these pieces. The figures given for nickel and platinum are derived from the means of about 40 experiments in each case.

The steel specimens are numbered consecutively from 1 to 9. Nos. 1 and 2 are from pieces of machinery steel or rolled iron found in the shop of the laboratory. No. 3 is a piece of Stubbs tool steel. Nos. 4 to 9 were supplied by the firm of Pratt & Inman, of Worcester, Massachusetts, from their regular stock. Nos. 4 to 7, inclusive, were made by the Gautier Department of the Cambria Steel Company, of Johnstown, Pennsylvania, and Nos. 8 and 9 were made by the Crucible Steel Company and the Colonial Steel Company, respectively.

All the samples were analyzed in the chemical laboratory of Clark University, by Mr. E. A. Harrington, a student in the university.

The results were as follows, in fractions of 1 per cent:

No.	*C*	*P*	*S*	Mn.
1	.09			
2	.05			
3	.97	.07	.01	
4	.08	.08	.01	.82
5	.18	.13	.03	.77
6	.26	.16	.01	.42
7	.28	.19	.04	.83
8	.40	.02	.02	.49
9	.79	.03	.02	.40

As already stated, the specimens were used in the shape of a ball. These balls were about 11 mm. in diameter and were covered with platinum foil. The balls weighed between 5 and 6 grams each. They were first given a thin nickel plating. Two circular disks of foil were cut, each large enough to cover more than half of the ball. These were annealed by heating to redness in a gas flame, and pressed into shape by placing them on a cake of wax and forcing the ball and foil into the wax. The two cups thus formed covered the ball and overlapped by one or two millimeters. The ball was then seized in tongs, in such a way as to press the two caps together, and heated to redness in a flame. By quickly hammering the joint, the two halves of the covering could be welded together. By reheating and rolling the ball between two smooth pieces of iron, the covering was made to fit the ball closely. No covering, however, remained intact through a whole series of experiments. It was usually punctured by the sixth or eighth heating. This gave no trouble, for the piece was frequently weighed and the change in weight was very small, even when quite a crack appeared. The foil used was 0.03 mm. thick.

Results. — The data obtained were reduced to mean specific heat for each experiment, and the results plotted on cross-section paper. The mean was then found by drawing a curve through the points plotted. Something over 30 points were found for each curve and nearly all were within one per cent. of the mean line. The final temperature of the calorimeter was always very near 20° C. and this temperature is taken as the lower temperature in subsequent calculations. The total heat per gram was then calculated from the mean specific heat curve. Although this is a round-about method, since the total heat is directly determined, it has some advantages in finding the mean. For, if $q_0{}^T$ be the total heat required to heat the piece from 0° to $T°$, and c the specific heat at any temperature, then

$$q_0{}^T = \int_0^T c\,dT$$

where c is a function of T. If c can be expressed as a very rapidly converging power series, as

$$c = \alpha + 2\beta T + 3\gamma T^2 + \cdots$$

where α, β, γ, etc., are constants, then

$$q_0{}^T = \alpha\int_0^T dT + \beta\int_0^T 2T\,dT + \gamma\int_0^T 3T^2 dT + \cdots$$

$$= \alpha T + \beta T^2 + \gamma T^3 + \cdots$$

and

$$\frac{q_0{}^T}{T} = \alpha + \beta T + \gamma T^2 + \cdots = S_0{}^T.$$

If γT^2, etc., are small in comparison with α and βT, as is usually the case, then S becomes a linear function of T.

If the latent heat L appears then the formulas become:

$$q_0^T = L + \int c dT = L + \alpha T + \beta T^2 + \gamma T^3 + \cdots$$

and

$$S_0^T = \frac{L}{T} + \alpha + \beta T + \gamma T^2 + \cdots$$

For the curve beyond the point where the latent heat appears, the latent heat should be subtracted before dividing by T, giving:

$$S = \frac{q_0^T - L}{T} = \alpha + \beta T + \gamma T^2 + \cdots$$

Table I.

Total Heat $= q_{20}^T$.

Temp.	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
470	57.6	55.0	54.8	51.8	57.5	56.0	55.3	55.6	55.0
520	64.8	62.6	63.2	59.3	65.0	63.5	63.0	63.6	63.0
570	72.4	70.5	72.3	67.1	72.6	71.1	71.3	72.0	71.3
620	80.8	78.6	81.7	75.9	80.7	79.6	80.2	80.6	80.2
670	90.8	87.1	91.4	86.0	90.1	89.3	89.9	89.7	89.6
720	103.6	97.5	101.6	97.0	101.0	99.5	99.8	99.4	99.5
770	117.7	110.8	126.5	111.1	118.2	117.0	119.0	125.4	124.9
820	128.4	120.8	134.8	127.1	129.6	128.5	128.0	134.6	134.9
860	136.1	128.5	139.8	135.2	136.2	134.0	134.1	142.0	143.0

Table II.

Mean Specific Heat $= S_{20}^T = \frac{q_{20}^T}{T - 20}$.

Temp.	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
470	.1281	.1223	.1220	.1150	.1280	.1242	.1220	.1231	.1218
520	.1296	.1252	.1268	.1182	.1300	.1269	.1260	.1270	.1260
570	.1316	.1281	.1313	.1218	.1319	.1293	.1298	.1308	.1300
620	.1346	.1310	.1359	.1262	.1347	.1328	.1338	.1344	.1338
670	.1397	.1340	.1405	.1322	.1386	.1372	.1380	.1381	.1377
720	.1480	.1393	.1451	.1385	.1444	.1419	.1421	.1420	.1420
770	.1570	.1477	.1684	.1485	.1577	.1559	.1590	.1670	.1665
820	.1605	.1510	.1683	.1590	.1619	.1606	.1600	.1680	.1685
860	.1620	.1530	.1682	.1610	.1623	.1595	.1597	.1687	.1702

The values of q and S are given in Tables I. and II., respectively, and these values are plotted in Figs. 2 and 3. Plots are given only for samples 3 to 9 inclusive. In the plots the abscissæ represent temperature and the ordinates, in Fig. 2, represent $q_{20}{}^T$ and in Fig. 3, $S_{20}{}^T$. For the sake of comparison, the curves are placed one above another, the initial and final values being given at the left and right respectively. The scale of ordinates is also given at the right.

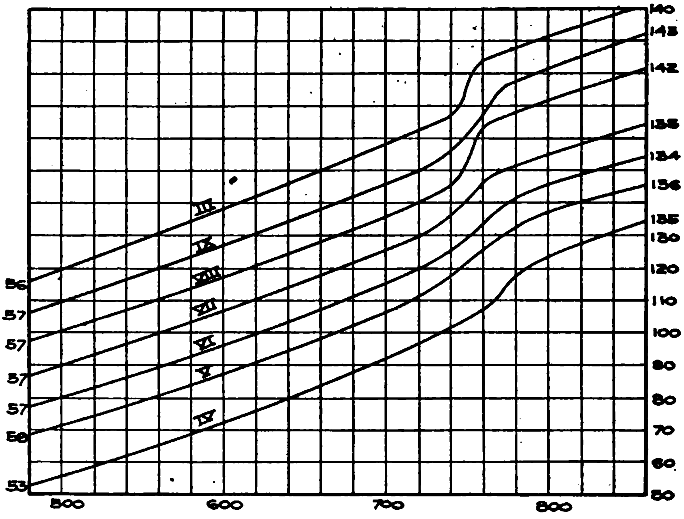

Fig. 2

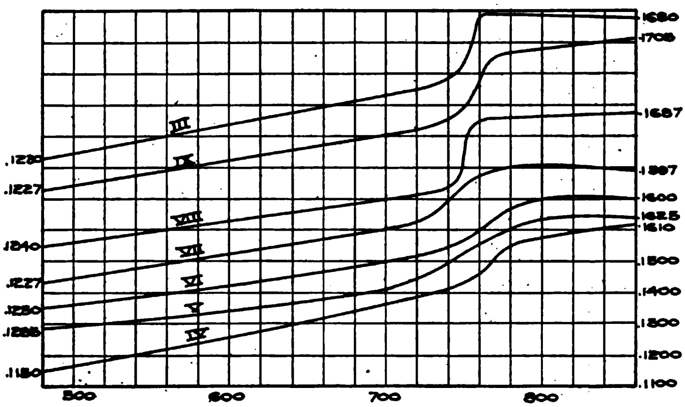

Fig. 3.

Inspection of the curves shows that the values of both q and S increase with the temperature. The curves for S between 500° and

700° are nearly straight and rise at a rate varying between .004 and .009 per 100°. This gives the value of β (γ and higher coefficients being neglected). The greatest difference between values of q for different samples are between 1 and 4 at 470°, 5.8 = 10 per cent.; between 3 and 4 at 620°, 5.8 = 6 per cent.; between 2 and 9 at 820°, 14.1 = 11 per cent.

Taking sample 7 for example, we find, between 470 and 720 β = .00008. Between 720 and 780 latent heat appears to the amount 9.7 cal. Subtracting this from values of q in the table, we have:

T	770	820	860
Q_{20}^T	108.3	118.3	124.4
S_{20}^T	.1444	.1479	.1481

These values are not sufficient to calculate β, but show that it may even become negative. The figures and the curve here indicate that it is very nearly zero. By extrapolation, we find $\alpha = S_{20} =$ .0860, whence

$$S_{20}^T = .0860 + .00008\ T$$

and

$Q_{20}^T = .0860\ T + .00008\ T^2$ between $T = 470°$ and $T = 720°$.

The value .0860 for α being too low, shows that another coefficient is needed. Above 820° the formulas become

$$S_{20}^T = \frac{9.7}{T} + .1480 \pm$$

and

$$Q_{20}^T = 9.7 + .1480\ T.$$

Table III.

Specific Heat = c.

Temp.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
470–520	.150	.150	.150	.154	.160	.160
520–570	.156	.152	.152	.174	.168	.166
570–620	.176	.162	.170	.178	.172	.178
620–670	.182	.188	.194	.194	.182	.188
670–720	.220	.218	.204	.198	.194	.198
720–770	.282	.344	.350	.384	.520	.508
770–820	.320	.228	.230	.180	.184	.200
820–860	.202	.165	.137	.152	.185	.202

The specific heat c_T at the temperature T is given by the slope of the curve q. These can be calculated from Table I. and are given in Table III.

Table IV. gives the figures obtained for a single sample, No. 7, as recorded in the notebook.

TABLE IV.

No.	Temp., Deg. C.	Total Heat $= q_{20}^T$ Calories Per Gram.	Mean Sp. Heat $= S_{20}^T$
1	520	62.7	.1254
2	552	68.3	.1282
3	575	72.4	.1304
4	612	78.8	.1331
5	618	81.2	.1358
6	631	83.5	.1366
7	634	82.6	.1346
8	636	83.5	.1355
9	656	86.5	.1359
10	664	88.1	.1367
11	665	88.9	.1376
12	672	90.4	.1386
13	673	89.5	.1370
14	687	92.7	.1389
15	692	93.6	.1390
16	692	95.0	.1414
17	700	96.1	.1413
18	708	96.4	.1401
19	721	100.6	.1435
20	728	102.3	.1444
21	732	102.5	.1439
22	745	112.2	.1546
23	750	110.5	.1513
24	764	118.4	.1591
25	767	118.3	.1583
26	782	122.4	.1604
27	790	120.8	.1568
28	813	125.0	.1575
29	817	127.5	.1600
30	825	126.9	.1575

The second part of the research consisted in studying the manner in which the various kinds of steel increased or decreased in temperature. A cylinder of each of Nos. 4 to 9 was cut, having a length of 5.1 cm. and a diameter of 2 cm. A hole large enough to admit the platinum-platinum-rhodium element was drilled in the axis to a depth of 2.5 cm. so that the end of the hole was near the center of the piece. These cylinders were covered with mica and one or two layers of asbestos paper, then wound with wire having a resistance of about 6 ohms. The whole was then thickly covered with a paste of magnesia wet with water, the thermo-element previously used in the furnace being inserted in the hole in the iron. The paste was dried and the whole thing packed in dry magnesia so that there was 5 to 8 cm. of magnesia between the cylinder and the open air. The cylinder was then heated by passing a current through the wire. Three amperes was sufficient to carry the piece slowly to a temperature of 850° C. In order to read the temperature directly on the galvanometer, a resistance was put in series with the galvanometer such that one millivolt on the potentiometer produced a deflection of 10 cm. on the scale of the galvanometer. The potentiometer was then set at 5.50 millivolts which corresponded to a temperature of the thermo-junction of 652° C. The range of the galvanometer scale used was from — 10.00 cm. to + 20.00 cm., or from 552° C. to 841° C., when the cold junction was at 16° C. Thus one millimeter on the scale corresponded very nearly to one degree Centigrade.

When the cylinder had reached a temperature which could be read on the galvanometer, the reading of the galvanometer was noted every minute, and when passing the critical points, readings were made every half minute. The rate of rise or fall varied, but seldom exceeded 20° C. per minute. The current was allowed to pass until the piece had reached a temperature of 800° to 850° C., when the circuit was opened and the cylinder allowed to cool. The cooling was allowed to proceed until a temperature of about 600° C. was reached, when the circuit was again closed and the piece reheated. In general, three heating and three cooling curves were made with each piece.

In every sample there was a point of inflection in the heating curves at a temperature of about 730° C. In piece No. 4 the inflection was slight, the temperature rising continuously but more slowly as this temperature was passed. With the higher carbon steels, however, the temperature at a certain point remained constant, to a fifth of a degree, for a minute or more. Once the temperature appeared to fall about a degree while heating, but the cause of this is uncertain.

In cooling the temperature fell steadily to a temperature of 670° to 710° C., when, except in the case of No. 4, the fall ceased, the temperature rose, remained constant for a period, and then fell steadily. The temperature of recalescence was higher with the higher carbon steels, and the time spent in the process was also greater. In No. 4 there was no recalescence, but the temperature fell decidly less rapidly for a minute or more at about 670° C. The phenomenon observed by Hopkinson[1] and Osmond was also observed, that the temperature of recalescence was lower if the heating had been higher. Some of the cooling curves show a decided bend at the temperature where absorption occurs on heating. There appears to be some change at this point, but it has not been given any special investigation. The heating and cooling curves are given in Figs. 4 and 5.

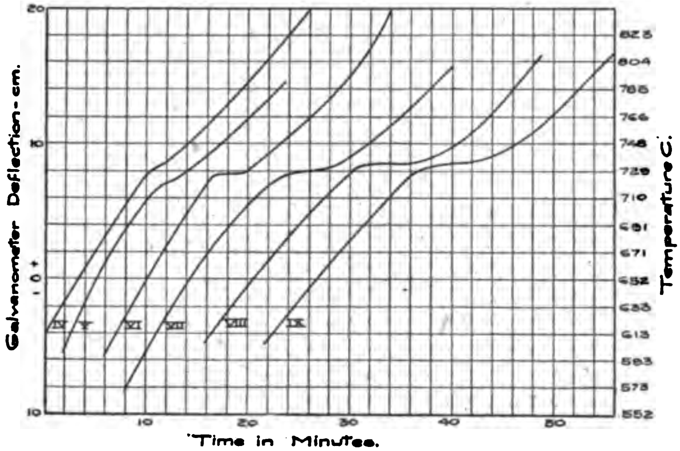

Fig. 4.

To see if tempering the steel had any effect, cylinder No. 8 was five times heated to bright redness and plunged into water. It was then rewound and heated as before. No decided change in behavior could be noticed, and the amount of heat absorbed and liberated in the change of state was, within the limits of error, the same. No. 8 was then heated until its temperature became constant, and just as it began to rise again the current was turned off and the piece began to cool. There was no recalescence, but the temperature

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[1] Proc. Roy. Soc., Vol. 48, p. 442.

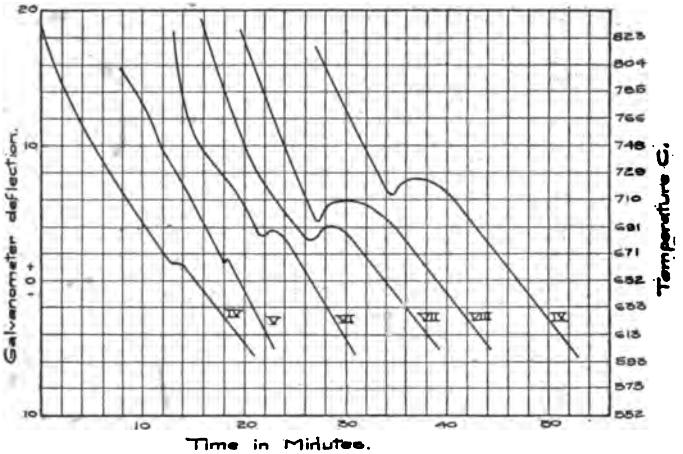

Fig. 5.[1]

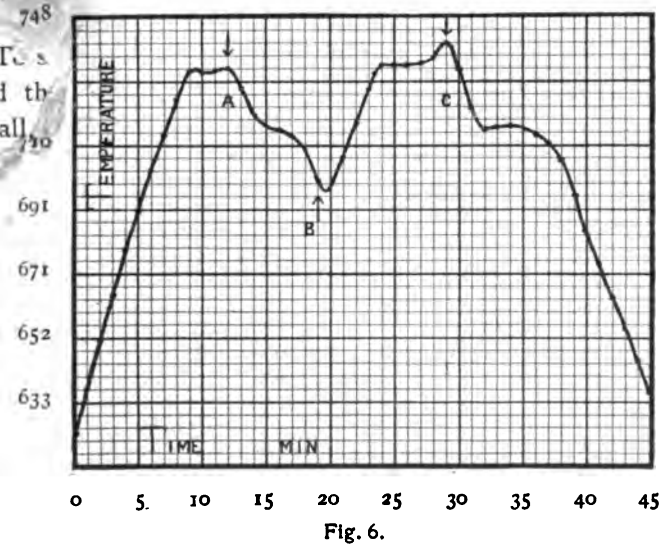

Fig. 6.

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[1] By an error in drawing Curve VII has been slightly shifted upward. The form of the curve is correct.

fell rapidly at first, then slowly, then rapidly again. The current was then again turned on and the piece carried to a temperature some 8° higher than before. At this temperature the second rise was well begun. The current was then turned off and the piece cooled rapidly for about 28°, then rose in temperature 1° and finally declined again at nearly the same rate as in the beginning. (Fig. 6.)

The amount of heat absorbed and liberated in the change of state was deduced from the heating and cooling curves by multiplying the mean of the rate before and after the change by the duration of the change and by the specific heat at that temperature as found from the other experiments. In the heating curves, it was hard to decide what tangent to use for the calculation; hence, the values differ widely. In many cases, a tangent was taken parallel to the one previous to the change. The following is a comparative table of the results:

TABLE V.

No. of Piece.	Latent Heat by Calorimeter. gm. cals.	From Heating and Cooling Curves.		Temperatures of—		Temperature Rise in Recalescence.
		Heat Absorbed.	Heat Liberated.	Absorption.	Liberation.	
4	8.6		2.8	729	669	0
5	11.4	3.6	3.6	720	663	[illegible]
6	9.7	8.2	6.5	728	691	
7	9.7	7.8	8.5	727	661	[illegible]
8	16.0	10.9	18.0	733	705	[illegible]
9	15.4	10.3	16.0	735	713	[illegible]
8 after hardening.			18.2	729, 733	708, 698	9

Comparing the curves for heating and the curves for mean specific heat (Fig. 3), it is seen that the absorption begins in the former very nearly at the temperature at which the latter begin to curve upward. It is seen, also, that the second bend in the specific heat curve is at the temperature where the tangent to the heating curve is parallel to the tangent below the absorption point. These facts, in connection with those of Fig. 6, indicate that the change of state does not all take place at one definite temperature, even if this temperature is maintained for some time. In other words, the substance seems to exist in approximate equilibrium in two states, the proportion of each state being a function of the temperature. Above the point of absorption, the heating curve gets steeper and steeper, showing that the specific heat in the upper state is lower than that in the lower state. This was indicated by the calorimetric work, but this work was not carried to temperatures sufficiently high to give figures for the specific heat in the upper state.

The lack of agreement in the values of the latent heat, as given by the two methods, is due to the inaccuracy of the method of cooling. In this method the specimen is surrounded by a mass of matter having a heat capacity considerable in comparison with that of the specimen. If this matter could be done away with and the heating and cooling performed with no covering, the results would, no doubt, agree better. To do this, the piece should be wound only with mica and platinum foil and suspended by platinum wires in a receiver exhausted of air. The agreement between the two methods is best where the latent heat is great.

The effect of recalescence in tempering was not investigated, but it is interesting to compare samples 6 and 7. The two have nearly the same percentage of carbon and the same latent heat, but No. 6 recalesces only 4 degrees, while No. 7 goes up 10 degrees. Neither of these are tool steels, but No. 7 can be hardened more than No. 6.

To show the relation between the temperature of a piece of steel and the temperature of its surroundings, when the difference is small, a piece of No. 7 was turned into a ball having a diameter of nearly 2 cm. A hole about 1 mm. in diameter was drilled through the center of this, and the wires of the thermo-element passed through this hole. Care was taken to place the junction of the wires at the center of the ball. The ball was then suspended at the center of the furnace. The standard element above mentioned, was placed close to the side of the furnace. The furnace was then heated and the temperatures of each element noted at regular intervals. In general, the two temperatures rose or fell at the same rate, but at the temperature where absorption occurs, the steel heated less rapidly, and on cooling, the phenomenon of recalescence occurred in the steel while the temperature of the furnace declined uniformly. The two elements were then connected in opposition, and in series with the galvanometer. The readings of the galvanometer were then proportional to differences of temperature between the ball and furnace. The galvanometer deflection was noted every minute while the furnace heated and cooled as before. The readings are shown in Fig. 7, where the ordinates represent galvanometer deflections and the abscissæ represent time. The curve needs no further explanation.

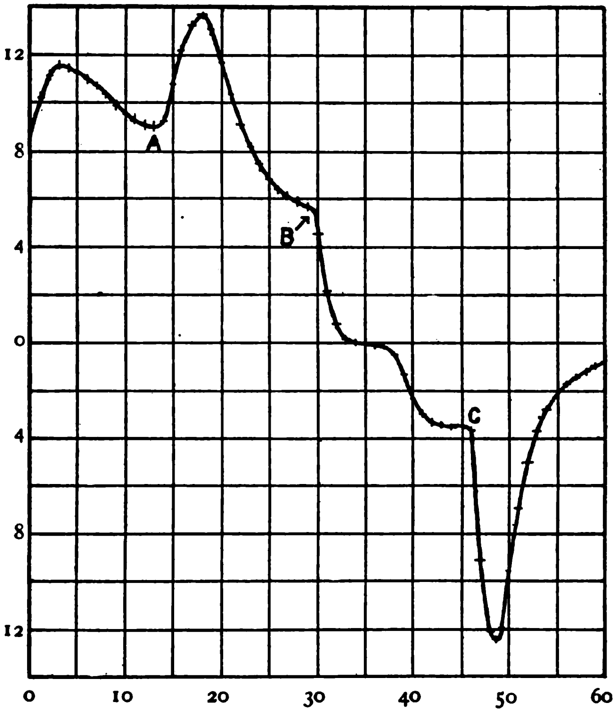

Fig. 7. Abscissæ represent time in minutes. Ordinates represent deflection of galvanometer in cm. and also approximately difference in temperature in Centigrade degree. Positive ordinates represent furnace hotter than iron.

Finally to show the manner in which the furnace heated and cooled, one of the elements was placed in the center of the furnace and connected to the galvanometer as described on page 145. Readings were made every minute, and the results plotted as shown in Fig. 8.

In conclusion, I wish to thank all who by friendly word or generous deed have assisted me during the progress of the work. Special thanks are due to the trustees and faculty of Clark University for providing instruments and means, to Professor A. G. Webster, for continual interest and suggestions, to Mr. A. H. Inman, of

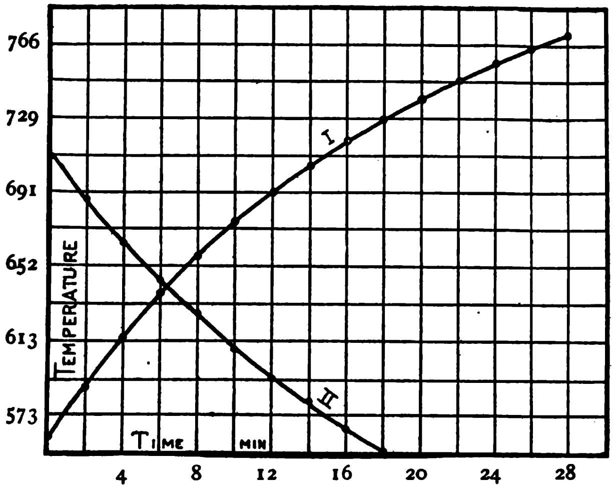

Fig. 8.

the firm of Pratt & Inman for supplying me with samples of steel, and to my fellow student, Mr. E. A. Harrington, for analyzing the samples.

Clark University,
Worcester, Massachusetts,
May, 1906.

EXPERIMENTS ON RESONANCE IN WIRELESS TELEGRAPH CIRCUITS. PART V.[1]

XIII. The Electrical Oscillations in Connected Systems of Circuits.

By George W. Pierce.

As a contribution toward the study of the nature of the oscillation at the sending station used in wireless telegraphy, the following account is given of a series of experiments on the wave-lengths obtained when two condenser circuits inductively or directly connected together are set into oscillation.

The problem here undertaken is the measurement of the wave-length of each of two condenser circuits with each circuit standing alone, and then a measurement of the resultant wave-lengths when the two circuits are coupled together after the manner of the electromagnetically coupled and the direct coupled sending stations of wireless telegraphy. This problem has already had ample theoretical study, but I am not aware of its having been given extended experimental examination. So the present experiments were undertaken, in part, as a test of the applicability of the theory, and, in part, for the benefit to be derived from an experimental study of specific cases.

The dimensions of the condensers and inductances were chosen to have such values as would bring the wave-lengths within the range ordinarily used in wireless telegraphy (200 meters to 2,000 meters).

Forms of Circuit Studied. — The two types of circuits in which the oscillations were studied are shown in Fig. 49 and Fig. 50. In the *Electromagnetically Connected System* shown in Fig. 49 two condensers C_1 and C_2 are connected to two coils L_1 and L_2 which are inductively related but insulated from each other. The number of active turns of wire on each of the coils may be varied. Each of the condenser circuits is provided with a spark-gap, so that either circuit, when connected to a step-up transformer, may be used as the discharge circuit. The other circuit may then be looked upon as a secondary circuit. When the spark-gap of the secondary is opened too wide to permit the passage of a spark, or better when the secondary is removed, the period of oscillation is the period of the primary alone. When, on the other hand, the secondary is left in place and the spark-gap of the secondary is closed, the oscillations of the discharge circuit are modified by the presence of the closed secondary. It is the purpose of the experiment to measure the wave-lengths produced under these several conditions and to compare the measured values with values computed from theoretical considerations.

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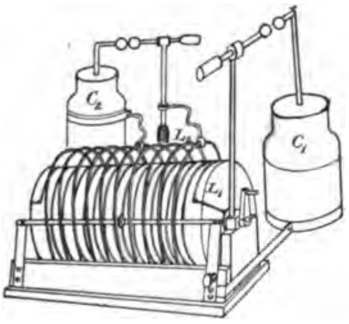
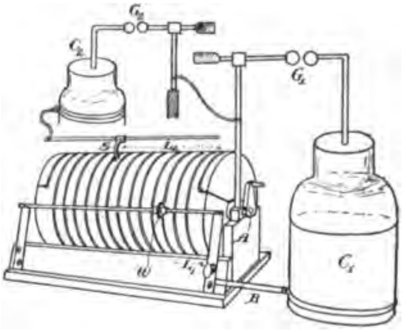
[1] The previous parts were published in the Physical Review, as follows: Part I., Vol. 19, p. 196, 1904; part II., Vol. 20, p. 220, 1905; part III., Vol. 21, p. 367, 1905; part IV., Vol. 22, p. 159, 1906.



INDUCTIVE

Fig. 49.



DIRECT

Fig. 50.

In the *Direct Coupled System*, represented in Fig. 50, which was also studied, the transformer of the inductive coupling is replaced by an auto-transformer; that is the two condensers C_1 and C_2 are made to discharge through parts of the same coil. In this case also, both the inductances L_1 and L_2 can be varied independently by the motion of the contacts W and S. Also both the condenser circuits are provided with spark-gaps so that either circuit may be caused to oscillate alone or to constitute the discharge circuit in a connected system with closed secondary.

These two forms of circuits, Figs. 49 and 50, are derived from the ordinary wireless telegraph circuits by replacing the antenna and ground of the wireless telegraph station by the two coatings of a condenser respectively. The circuits were thus simplified for the purpose of the present experiments because in the simplified form the theoretical formulas ought to apply exactly. In a subsequent investigation it is proposed to study also the less simple cases.

Dimensions of the Inductances.—The coils employed in the apparatus shown in Figs. 49 and 50 had the following dimensions.

Coil.	No. Turns.	Diam. Wire.	Coil Diam.	Pitch.
Outer of Fig. 49	24	.208 cm.	18 cm.	.81 cm.
Inner of Fig. 49	51.5	.208	13	.42
Coil of Fig. 50	51.5	.208	13	.42

The inductances of various numbers of turns of these several coils was measured on a Rayleigh's bridge, and these values are recorded in the tables containing the wave-length measurements given below.

Measurement of Wave-Length.—The measurement of wave-length was made by a resonance method, in which an auxiliary circuit consisting of a variable condenser in series with an inductance was attuned to resonance with the oscillations to be determined. The adjustable auxiliary circuit is called the *wave-meter circuit.*

A wave measuring apparatus based on this principle was described by Drude in 1902 in a paper entitled "Resonanz-methode zur Bestimmung der oscillatorischen Condensatorentladung."[1] In Drude's apparatus the wave-meter circuit consisted of a condenser connected in series with a rectangular loop of wire, whose inductance could be varied by a bridge across the loop. The position of the bridge when the wave-meter circuit was in resonance with the oscillation to be determined gave the wave-length in the oscillation circuit, which could be read off on a scale attached to the wave-meter circuit. Drude ascertained when the wave-meter circuit was in resonance with the oscillation circuit by observing the glow in a Geissler tube attached to one plate of the condenser. The glow is a maximum at resonance.

Several commercial wave-meters based on the principle of resonance have been constructed, prominent among which are the wave-meter of Doenitz and the wave-meter of Fleming.

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[1] P. Drude, Ann. d. Phys., 9, p. 611, 1902.

In both of these wave-meters, as in the apparatus of Drude, the essential parts are a condenser and inductance in series, one or both of which may be varied in a practically continuous manner, some means of ascertaining when the wave-meter circuit is in resonance with the oscillations to be determined and a scale from which the wave-length corresponding to the resonant adjustment may be read.

In the Doenitz wave-meter[1] the adjustment of the wave-meter circuit is made chiefly by the variation of the capacity of the condenser, and the resonant condition is ascertained by observing the deflection of a Riess hot-wire air thermometer.

In Fleming's apparatus,[2] both the capacity and the inductance are varied in a practically continuous manner, and the resonant condition is determined by observing the glow in a Geissler tube.

On account of the feebleness of some of the maxima to be investigated in the present experiments it seemed doubtful if either of these wave-measuring instruments is sufficiently sensitive for the measurements required, so that an apparatus was constructed for the purpose.

The Wave-Meter. — One form given to the wave-metric apparatus is shown in Fig. 51. *C* is a variable air condenser of two sets of semi-circular plates, one of which may be rotated to vary the capacity. The position or the movable plates is given by the reading of a pointer *P* passing over a semi-circular scale. This

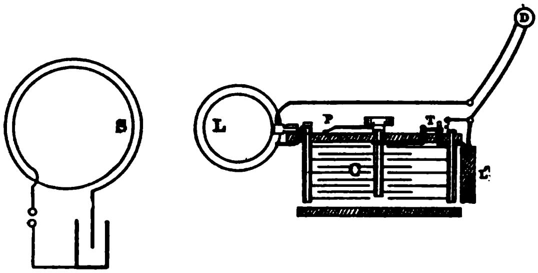

Fig. 51.

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[1] Doenitz, E. T. Z., 24, pp. 920-925, 1903.

[2] J. A. Fleming, Phil. Mag., 9, p. 758, 1905. See also Principles of Electric Wave Telegraphy, London, 1906.

scale is divided in circular degrees, and is also calibrated in wave-lengths by the method described below. In series with the variable condenser is a loop of inductance L (8 turns 13.5 cm. in diameter) and the high-frequency dynamometer with variable sensitiveness D described in Parts I. and II. The loop L, is the "receiving" loop to be acted on inductively by the oscillations to be measured, and the dynamometer is for the purpose of determining when the wave-meter circuit is in resonance with the required oscillations.

The wave-meter circuit is also provided with a switch T, by means of which a second inductance L', may be thrown in or out of the circuit. For short waves, between 200 meters and 650 meters, L' may be thrown out, while for waves between 550 meters and 1,800 meters, L' may be thrown in. This gives an easy method of extending the range of the instrument, which must, of course, bear a double calibration giving two wave-length scales, one to be used with L' in, the other with L' out.[1]

The manner of using the apparatus is as follows: If, for example, it was desired to determine the wave-length of the oscillation in the circuit S, Fig. 51, the loop L was placed in such a position that the currents in S acted inductively on the loop L, and gave a deflection of the dynamometer. By observing the deflection of the dynamometer for different adjustments of the variable capacity C, the value of the capacity that put the wave-meter circuit in resonance with the required oscillations could be determined, and from the calibration on the scale of the capacity, the wave-length of the required oscillation could be read off directly in meters.

The apparatus in this form may be extremely sensitive, or its sensitiveness may be decreased to any degree by one of two methods: first, by decreasing the sensitiveness of the dynamometer by drawing the coil of the dynamometer away from the suspended disc, or second, by rotating the receiving loop L about a horizontal axis so that the inductive action upon it is diminished.

Only one precaution is necessary in the use of such a resonance method of determining wave-lengths, namely, care must be taken that the inductive coupling between the wave-meter circuit and the discharge circuit be so " loose " that the reaction of the wave-meter circuit on the oscillation circuit is negligible. On account of the high sensitiveness of the present apparatus, there was never any temptation to bring the wave-meter circuit too near to the oscillation circuit.

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[1] Doenitz's wave meter has a triple range, attained by replacing the loop L by loops of higher or lower inductance.

In some of the experiments described below the dynamometer of the apparatus above described was replaced by other instruments for detecting the resonant condition, but the description of the other instruments is for the present deferred.

Experiment XIX.: Calibration of the Wave-Meter. — The wave-meter was calibrated by an experimental method as follows: A glass-plate condenser in series with a variable inductance and spark-gap was set up in such a position that the spark across the spark-gap could be photographed with the aid of a revolving mirror after the manner of Feddersen's original experiment. The speed of revolution of the mirror was kept constant by the aid of a stroboscope[1] operated by a tuning fork of known period. From the speed of revolution of the mirror and the distance apart of the impressions on the photographic plate the time of one oscillation was determined. This time multiplied by the velocity of light gave the wave-length of the oscillation. The wave-meter circuit was then brought up to within 20 or 30 cm. of the oscillation circuit, adjusted to resonance with it, and the resonant point of the scale of the wave-meter was marked with the wave-length determined by the photograph. In this way a large number of points on the wave-meter scale were determined, and from these points, with the aid of a curve, the wave-length corresponding to any value of the wave-meter scale could be obtained. It was found convenient to write these wave-lengths upon the scale of the wave-meter and to make subsequent readings directly in wave-lengths.

To show the probable accuracy of this method of calibrating the wave-meter some of the data from the measurements of the photographs are given in Table XVI.

In these experiments the speed of the mirror was 41.3 revolu-

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[1] G. W. Pierce, On the Cooper-Hewitt Mercury Interrupter, Proc. Am. Acad., XXXIX., No. 18, Feb., 1904.

Table XVI.

Photographic Determination of Wave-Length. Capacity Four Boxes of Flat Plates.

No. Turns Inductance.	.5	2.5	4.5	6.5	8.5	10.5	13.5
Inductance in 10^{-5} Henrys.	.50	1.12	2.20	3.43	5.05	6.66	9.38
Distance in cm. between oscillations on plate.	.1096	.164	.227	.301	.340	.404	.483
	.1080	.165	.224	.290	.352	.402	.481
	.1094	.164	.230	.290	.343	.403	.488
	.1103	.160	.238	.292	.343	.404	.484
	.1094		.230	.292	.342		.475
	.1091		.229		.350		
	.1108		.230		.353		
	.1097				.342		
					.346		
Average.	.1095	.163	.230	.293	.344	.403	.482
Wave-Length Meters.	416	630	875	1115	1320	1530	1835
Mean Error per cent.	.5	1.0	1.0	1.0	1.0	.2	.7

tions per second. The distance from the mirror to the sensitive plate was 153 cm., so that the image of the spark travelled across the photographic plate with a speed of $41.3 \times 4\pi \times 153$ cm./sec. = $.792 \times 10^6$ mm./sec. Multiplying the reciprocal of this quantity by the velocity of light in meters/sec., we have

1 mm. between oscillations = 380 meters wave-length.

In determining the distance between oscillations on the photographic plate the length of a whole series of oscillations was measured and divided by the number of oscillations. The table shows that this part of the work was done with a mean error of about 1 per cent.

Besides the values given in the table, the wave-lengths 463 meters and 350 meters were also determined photographically.

Experiment XX.: Extending the Calibration to Shorter Wave-Lengths. — With the particular apparatus at my disposal it was difficult to photograph oscillations giving rise to wave-lengths shorter than 350 meters, so that wave-lengths below this value were obtained by partitioning the condenser. This was done as follows: By the aid of the photographs the wave meter has already been calibrated from 350 meters to 1,835 meters. Using such values of the inductance in the discharge circuit as will bring the wave-length always within this range the following table (Table XVII.) was obtained by discharging each of four condensers and then the four together through various inductances.

TABLE XVII.

Wave-Lengths with Different Condensers Separate and Together in Discharge Circuit.

Turns of Inductance.	Wave-Length in Meters Given with Condenser.				
	1.	2.	3.	4.	All Together.
4.5	430	415	450	440	875
6.5	555	533	575	570	1115
8.5	675	640	700	685	1320
10.5	790	750	815	800	1530
12.5	895	850	905	870	1780
13.5	950	895	970	935	1835

If now we divide the wave-lengths in columns marked Condenser 1, 2, 3, 4 respectively by the wave-length in the last column of Table XVII., we obtain the ratio of the wave-lengths with the separate condensers in the discharge circuit to the wave-length with the four condensers together in parallel in the discharge circuit. These relative values are compiled in Table XVIII.

TABLE XVIII.

Wave-Lengths with Separate Condensers Relative to Wave-Length with Aggregate.

Turns Inductance.	Condenser.			
	1. λ_1/λ	2. λ_2/λ	3. λ_3/λ	4. λ_4/λ
4.5	.492	.474	.514	.502
6.5	.492	.477	.515	.511
8.5	.502	.477	.521	.510
10.5	.517	.490	.532	.522
12.5	.501	.477	.508	.490
13.5	.517	.488	.528	.508
Mean	.507	.480	.520	.507
Mean error	2 per cent.	1 per cent.	1.4 per cent.	1.4 per cent.

The ratio in each column of Table XVIII. is approximately constant whatever the inductance in the discharge circuit. On the assumption that these ratios are also constant when still smaller inductances are used, as they should be from Thomson's formula, we can extend the calibration to the necessary shorter waves. For example, the four condensers discharging together through 2.5 turns of inductance gave wave-lengths 630 meters; condenser 1 discharging through the same inductance should give .507 × 630 = 320 meters. Condensers 2, 3 and 4 respectively give wave-lengths 303, 327 and 320 meters, each of which was used to determine a point on the wave-meter scale. In a similar manner the wave-lengths 200, 211, 217, 245, 259, 266, etc., were found and used in the calibration of the scale of the wave-meter.

Experiment XXI.: Resonance Curves Taken in Calibration of Wave-Meter. — In the preceding paragraphs, it was shown how certain standard discharge circuits were made up, and their wave-lengths determined by revolving mirror photographs of the spark. These standard circuits were used in the calibration of the circuit of the wave-meter. The curves of Fig. 52 show how the standard wave-lengths were transferred to the wave-meter scale. Here it should be recalled that there are two scales on the wave-meter, one for use when the inductance L' of Fig. 51 was thrown into the wave-meter circuit thus lengthening the period of the wave-meter, the other when the inductance L' was thrown out. The curves of Fig. 52 were taken with the lengthening coil in. For example, with the discharge circuit arranged to give out wave-length 625 meters, the first curve at the left was constructed by plotting the deflections of the dynamometer against the readings of

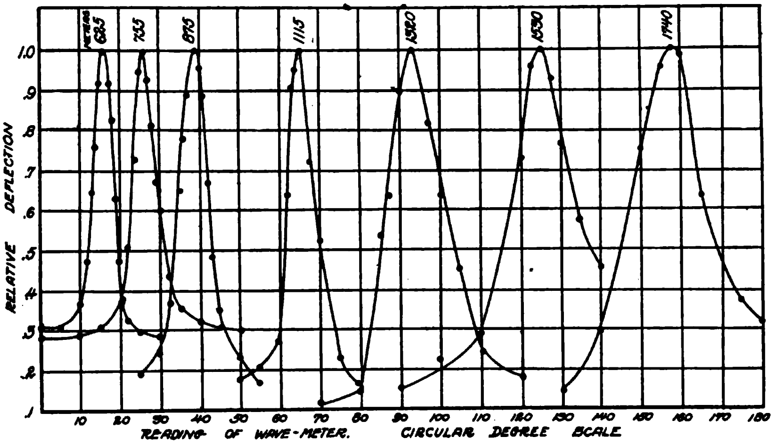

Fig. 52.

the circular degree scale attached to the variable condenser of the wave-meter. It is seen that resonance is obtained when the condenser is set at 17 degrees of the circular scale. Since the wave-meter circuit was at a distance sufficiently great from the discharge circuit so that the reaction of the wave-meter on the discharge had no appreciable effect in modifying its period, the period of the wave-meter when set at 17 degrees is the period corresponding to the wave-length 625 meters. In a similar manner from the other curves of the series the number of circular degrees corresponding to the other known wave-lengths was obtained. It is interesting to note that while the resonance curves widen as the wave-length is increased the interval on the scale comprehended within a given number of meters also widens, so that the percentage accuracy of the settings of the wave-meter is about the same throughout the scale. An idea of the accuracy of the settings will be had from an examination of the results obtained in the applications that come later in the paper.

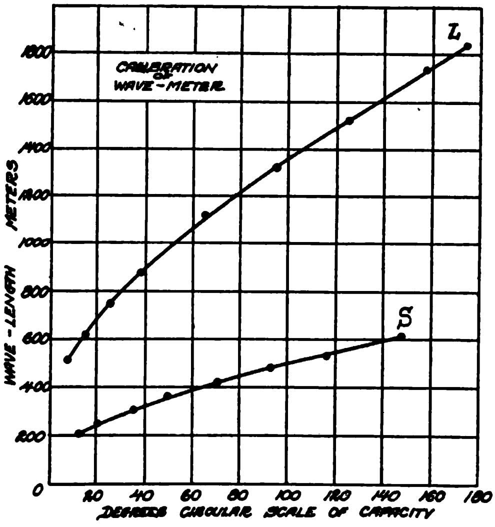

Fig. 53.

If we plot the wave-length at the vertices of Fig. 52 against the reasonant value in circular degrees on the scale of the condenser of the wave-meter, the curve marked *L* in Fig. 53 is obtained. In like manner when the switch on the wave-meter is thrown so as to cut out the lengthening coil L', and the discharge circuit is made to produce the known shorter waves, the curve marked S in Fig. 53 is obtained. From these curves respectively two scales were constructed and attached to the wave-meter condenser so that the readings could be made directly in wave-lengths.

Experiment XXII.: Use of the Wave-Meter in the Determination of the Capacity of the Discharge Condenser. — Fleming has pointed out the utility of the waver-meter in the determination of the capacity of a condenser. His method consists of discharging the condenser across a spark-gap through a known inductance and measuring the wave-length by the wave-meter and calculating the capacity from the formula

$$\lambda = v \cdot 2\pi\sqrt{LC}.$$

This method was utilized in the determination of the capacity of the several condensers used in the present experiments. To show the probable accuracy of the method, the following table (Table XIX.) is given showing the results obtained for Leyden jar No. 45 used in the later experiments. The values of the inductances were measured on a Rayleigh's bridge.

TABLE XIX.

Determination of Capacity by the Wave-Meter. Leyden Jar No. 45.

Inductance in Discharge Circuit in Henrys.	Wave-Length in Meters.	Capacity in Farads Computed by Thomson's Formula.
3.10×10^{-5}	690	$.00432 \times 10^{-6}$
4.90	865	.00432
6.61	1005	.00430
8.35	1130	.00432
10.0	1235	.00430
12.0	1345	.00427
14.05	1450	.00418
16.1	1560	.00423
		Mean $.00428 \times 10^{-6} \pm 1$ per cent.

The mean error in the measurement of the capacity recorded in Table XIX. is about 1 per cent. This is perhaps better than we ought to expect of the method because in the calculation of the capacity the error in the measurement of the wave-length has been doubled.

Turning our attention now to the main problems of the investigation let us study the oscillations in, first, The Electromagnetically Connected System of Circuits and, second, The Direct Connected System of Circuits.

XIV. WAVE-METRICAL STUDY OF THE OSCILLATIONS IN THE ELECTROMAGNETICALLY CONNECTED SYSTEM OF CIRCUITS.

This form of oscillation system is represented in Fig. 49. The theory[1] of this system has been the subject of numerous mathematical researches, so that only so much of the theory will be given here as is necessary in order to enable us to examine the results of the experiments in the light of the theory.

Brief Sketch of the Theory of the Electromagnetically Connected System. — Let the secondary circuit C_2L_2, Fig. 54 be closed, while the primary circuit C_1L_1 is charged and allowed to discharge across a spark-gap.

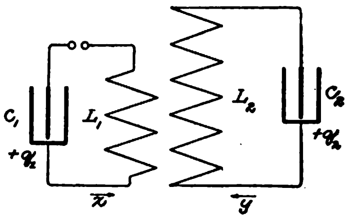

Fig. 54.

Let q_1 be the quantity of positive electricity on the outer plate of the condenser C_1 at any time t during the discharge; let x be the current in the primary circuit at that time; let q_2 be the quantity of positive electricity on the outer coating of C_2, y the current in the secondary circuit: then taking the electromotive force around each of the circuits we have

$$(1) \qquad L_1\frac{dx}{dt} + R_1x + M\frac{dy}{dt} = \frac{q_1}{C_1},$$

$$(2) \qquad L_2\frac{dy}{dt} + R_2y + M\frac{dx}{dt} = \frac{q_2}{C_2},$$

in which L_1 and R_1 are the self-inductance and resistance of the primary, L_2 and R_2 the self-inductance and resistance of the secondary, and M the mutual inductance between the two circuits.

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[1] Lord Rayleigh, Theory of Sound; J. v. Geitler, Sitz. d. k. Akad. d. Wiss. z. Wien, February and October, 1905; B. Galitzine, Petersb. Ber., May and June, 1895; V. Bjerknes, W. A., 55, p. 120, 1895; Oberbeck, Wied. Ann., 55, p. 623, 1895; Domalip and Koláček, W. A., 57, p. 731, 1896; M. Wien, W. A., 61, p. 151, 1897, and D. A., 8 p. 686, 1902; Compare also Webster, Theory of Electricity and Magnetism, p. 499, 1897, and Fleming, The Principles of Electric Wave Telegraphy, p. 209, 1906.

Also

$$x = -\frac{dq_1}{dt}\,;\; y = \frac{dq_2}{dt}. \tag{3}$$

Differentiating equations (1) and (2) and substituting from equation (3), we have

$$L_1\frac{d^2x}{dt^2} + R_1\frac{dx}{dt} + M\frac{d^2y}{dt^2} + \frac{x}{C_1} = 0, \tag{4}$$

$$L_2\frac{d^2y}{dt^2} + R_2\frac{dy}{dt} + M\frac{d^2x}{dt^2} + \frac{y}{C_2} = 0. \tag{5}$$

If we eliminate y from these two equations we have

$$\begin{aligned}(L_2L_1 - M^2)\frac{d^4x}{dt^4} + (R_1L_2 + R_2L_1)\frac{d^3x}{dt^3} + \left(R_1R_2 + \frac{L_1}{C_2} + \frac{L_2}{C_1}\right)\frac{d^2x}{dt^2}\\ + \left(\frac{R_1}{C_2} + \frac{R_2}{C_1}\right)\frac{dx}{dt} + \frac{1}{C_1C_2} = 0.\end{aligned} \tag{6}$$

If we eliminate x we get the same equation with x replaced by y. The equation (6) solved for period of oscillation, on the assumption that the resistance terms have no effect on the period, give the result that the current in the primary oscillates with double periodicity, the current in the secondary also oscillates with double periodicity, and the two periods of the primary are the same as the two periods of the secondary.

The two periods obtained by the solution of equation (6) are

$$T_1' = \sqrt{\frac{T_1^2 + T_2^2 + \sqrt{(T_1^2 - T_2^2)^2 + 4\tau^2T_1^2T_2^2}}{2}}, \tag{7}$$

$$T_2' = \sqrt{\frac{T_1^2 + T_2^2 - \sqrt{(T_1^2 - T_2^2)^2 + 4\tau^2T_1^2T_2^2}}{2}}, \tag{8}$$

in which

T_1 = period of the primary when standing alone,

T_2 = period of the secondary when standing alone, and

$$\tau^2 = \frac{M^2}{L_1L_2}. \tag{9}$$

τ is called the "coefficient of coupling."

If equations (7) and (8) are each multiplied by the velocity of light, we have, remembering that $vT =$ wave length,

$$(10) \qquad \lambda_1' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 + \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}},$$

$$(11) \qquad \lambda_2' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 - \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}}.$$

In these equations

$\lambda_1 =$ the natural wave-length of the primary alone,
$\lambda_2 =$ the natural wave length of the secondary alone,
$\tau =$ the coefficient of coupling, and

λ_1' and λ_2' are the resultant wave lengths in both primary and secondary when the circuits are coupled together.

Measurement of Wave-Lengths Produced by the Electro-magnetically Connected System. — With the aid of the apparatus described above the oscillations in the electromagnetically connected system, Fig. 49, were studied, by measuring the wave-length of the primary circuit and of the secondary circuit when each stands alone and then the wave-length in each circuit when they are coupled together. The results obtained are compared with computations by the aid of the equations (10) and (11). For the computations we need also to know the self inductance of each circuit and their mutual inductance. These quantities were measured on a Rayleigh's bridge.

Experiment XXIII.: E.M.C. System. $L_2 = 24$ *Turns of Outer Coil.* $\lambda_2 = 1{,}060$ *Meters.* — The results obtained in this experiment are plotted in the curves of Fig. 55. The method of taking the observations is as follows: First the condenser $C_2 (= .00482$ m.f.) was connected about 24 turns of the outer coil (Fig. 49) and was provided with a spark-gap. In this position, with the inner coil thrown out of circuit by disconnecting both plates of its condenser, the wave-length λ_2 was found to be 1,060 meters. Next, with the secondary condenser disconnected, the wave-length of the primary (inner) circuit was determined with its condenser $C_1 (= .00432$ m.f.) connected about 50 turns of the inner coil. This wave-length λ_1 was 1,560 meters. Next, with the primary left unaltered, the secondary was closed by attaching its condensers without spark-gap to the 24 turns of the outer coil. This is the case of the closed secondary, and when the discharge was established in the primary, the wave-lengths were found to be $\lambda_2' = 710$ meters and λ_1' too great for the wave-meter scale. The value $\lambda_2' = 710$ was plotted against $\lambda_1 = 1,560$, Fig 55. Now decreasing the primary inductance to 45 turns, the values $\lambda_1 = 1,460$, $\lambda_1' = 1,650$ and $\lambda_2' = 680$ were obtained, and the last two values were plotted against the value of λ_1, and so forth.

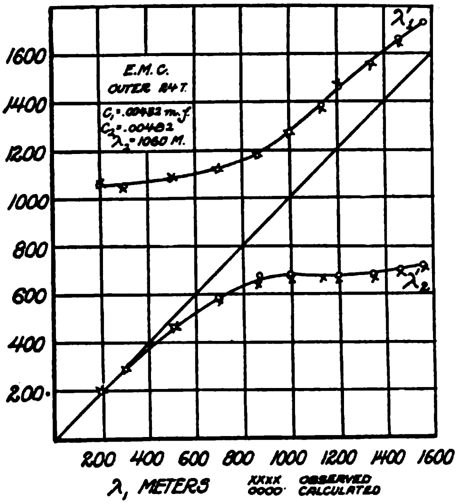

Fig. 55.

The complete record of the observed and calculated values is given in Table XX. In the curves of Fig. 55 the *crosses* are the observed values and the *circles* are the corresponding calculated values. When the observed and calculated values fall together the point is indicated by a *combination* of cross and circle. The 45° line between the two curves may be looked upon as λ_1 plotted against itself, while a horizontal line across the figure at 1,060 meters (not shown) would represent the graph of λ_2. With this in mind it will be seen that the two derived wave-lengths λ_1' and λ_2' are asymptotic toward the origin to λ_2 and λ_1 respectively. The observed and the calculated values are in satisfactory agreement, and their departure one from the other may be due as much to the inaccuracies of the data for the calculated values as to the errors in wave-length measurements.

The formulas for the calculation of λ_1' and λ_2' are the formulas (10) and (11) of page 19, which involve merely the independent periods

TABLE XX.

Electromagnetically Connected System.

Primary capacity .00432 microfarad.
Primary inductance varied.
Secondary capacity .00482.
Secondary inductance 24 turns outer coil, $L_2 = 6.60 \times 10^{-6}$ Henry.
Wave-length of secondary $\lambda_2 = 1060$ meters.

Turns Primary.	L_1 Primary Inductance. Henry.	M Henry.	τ^2
50	15.85×10^{-6}	6.52×10^{-6}	.412
45	13.9	6.14	.421
40	11.8	5.80	.430
35	10.0	5.12	.397
30	8.20	4.45	.360
25	6.50	3.56	.295
20	4.82	2.70	.228
15	3.15	1.95	.183
10	1.72	1.20	.128
5	.69	.47	.048
3	.32	.23	.0277

Turns Primary.	λ_1 Meters.	Calculated.		Observed.	
		λ_1' Meters.	λ_2' Meters.	λ_1' Meters.	λ_2' Meters.
50	1560	1740	727		710
45	1460	1670	712	1650	685
40	1350	1567	686	1570	665
35	1230	1462	680	1480	660
30	1130	1390	660	1370	660
25	1000	1273	685	1280	660
20	870	1185	680	1185	630
15	700	1127	595	1125	565
10	510	1080	467	1090	460
5	300	1060	292	1040	285
3	210	1062	193	1075	210

of the two circuits and their coefficient of coupling. The latter quantity was obtained by the measurement on a Rayleigh's bridge of L_1, L_2 and M for each setting of the oscillation circuit. The values of these inductances and of the values of τ calculated from them is also included in Table XX.

The intensity of the various periods of the circuits under the different conditions of the experiment varies greatly. No attempt was made to record these intensities, since it was found that their relative values depended on the position of the receiving loop of the wave-meter circuit with respect to its relative distance from the primary and secondary circuits. It is proposed, however, to examine these intensities in a later investigation in which the wave-meter circuit is to be placed at a great distance from the discharge circuit.

Experiment XXIV.: E.M.C. System. $L_2 = 15$ *Turns of Outer Coils.* $\lambda_2 = 775$ *Meters.* In a similar manner an experiment was performed with λ_2 constantly equal to 775 meters, obtained by connecting C_2 about 15 turns of the outer coil. The results are plotted in Fig. 56, from which a comparison of the observed and calculated values may be had. Here also the agreement is within the limit of error of the method. An abbreviated record of the experiment is given in Table XXI.

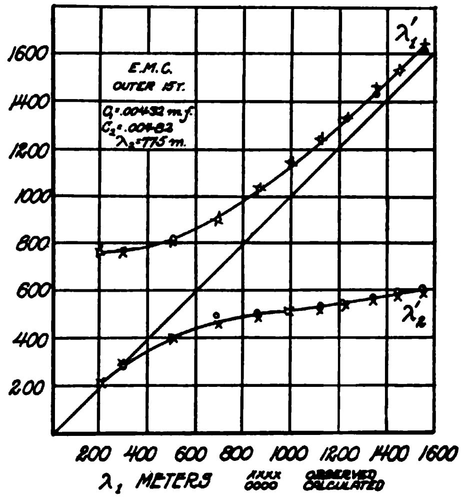

Fig. 56.

Experiment XXV.: E.M.C. System. Special Case $\lambda_2 = \lambda_1$. — A case of especial interest is the case in which the primary and secondary have the same independent periods. This is the case of so-called "resonance" between the two circuits. It is of interest because in this case the wave-length formulas (10) and (11) become greatly simplified, as may be seen by substituting $\lambda_2 = \lambda_1$ in these equations, which under this condition become

$$(\lambda_1')^2 = \lambda_1^2(1 + \tau) \tag{12}$$

$$(\lambda_2')^2 = \lambda_1^2(1 - \tau). \tag{13}$$

In the present experiment the two independent wave-lengths λ_1 and λ_2 were made equal, and the wave-lengths produced by the compound system were then measured and compared with calculations from the formulas (12) and (13). To make the two wave-lengths of the separate circuits the same, the wave-lengths obtained with the capacity C_1 discharging through various turns of the inner coil were measured, with the secondary removed, and the curve of wave-length against turns was plotted. In a similar manner the curve of wave-length against turns was obtained for the outer coil with the inner coil disconnected from C_1. From these two wave-length curves, the number of turns of the inner coil that with its condenser acting alone gave the same wave-length as a given number of turns of the outer coil with its condenser acting alone could be selected. The two circuits were now allowed to oscillate together. That is, the primary condenser C_1 was allowed to discharge

TABLE XXI.

Electromagnetically Connected System.

Primary capacity .00432 microfarad.
Primary inductance varied.
Second capacity .00482.
Secondary inductance 15 turns outer coil, $L_2 = 3.53 \times 10^{-5}$ Henry.
Wave-length of secondary $\lambda_2 = 775$ meters.

Turns Primary.	τ^2	λ_2 Meters.	Calculated.		Observed.	
			λ_1'	λ_2'	λ_1'	λ_2'
50	.295	1560	1610	627	1650	600
45	.317	1460	1530	612	1540	580
40	.345	1350	1440	590	1460	560
35	.377	1230	1337	560	1345	550
30	.396	1130	1255	545	1240	525
25	.390	1000	1152	522	1150	510
20	.354	870	1040	518	1050	490
15	.305	700	918	491	900	465
10	.252	510	830	410	800	410
5	.107	300	780	281	760	300
3	.062	210	774	206	775	210

$\tau^2 = \left(\frac{M^2}{L_1 L_2}\right)$ was obtained by measurements of L_1, L_2 and M.

through L_1 and the spark gap, while the gap in the secondary circuit was closed, so that oscillations were also set up in the secondary. Two wave-lengths λ_1' and λ_2' were obtained. The results are recorded in Table XXII., and plotted in the curves of Fig. 57.

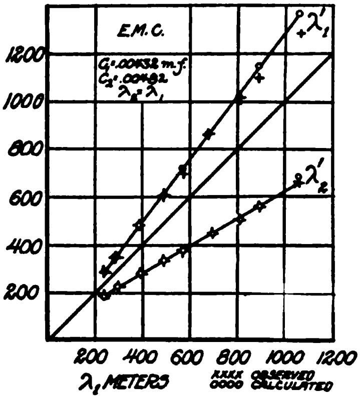

Fig. 57.

Table XXII.

Electromagnetically Connected System. Special Case $\lambda_1 = \lambda_2$.

Turns Inner Coil.	Turns Outer.	$\lambda_1 = \lambda_2$ Meters.	Observed.		Calculated.	
			λ_1'	λ_2'	λ_1'	λ_2'
27	24	1060	1290	655	1335	680
19.2	19	900	1095	555	1150	555
18	16	810	1025	503	1032	507
14.9	13	690	860	450	870	440
11.5	10	570	700	380	714	374
9.5	8	487	600	330	609	322
7.3	6	395	480	280	485	278
4.9	4	290	345	224	352	210
3.6	3	252	275	204	294	174

The values of the inductances, used in computations, were

L_1 Henry.	L_2 Henry.	M Henry.	τ^2
6.70×10^{-5}	6.95×10^{-5}	4.02×10^{-5}	.348
4.90	4.45	2.90	.386
3.87	4.10	2.30	.374
2.87	3.00	1.72	.343
1.95	2.00	1.14	.323
1.40	1.55	.83	.316
.92	1.10	.50	.253
.45	.64	.257	.227
.21	.40	.14	.230

In this case also the agreement is satisfactory with the exception of the two largest values of λ_1'. No significance is attributed to the departure of these two observations from the theoretical values, as no tendency to a departure of this character was noted in the more general cases described above. It may be that errors were made in reading the wave-meter for these values or, as is more probable, the selection of the turns to make λ_1 and λ_2 equal was erroneous.

The two curves of Fig. 57 are approximately straight lines, which do not pass through the origin. This is due to the particular manner in which τ happens to vary. Constant τ would make the lines pass through the origin.

From Experiments XXIII., XXIV., XXV., with this system of circuits, it is seen that the theoretical formulas give accurately values of the wave-lengths in the electromagnetically connected system, and that the values of the coefficient of coupling required in the equations are accurately enough given by measurements of self and mutual inductance with the comparatively slow frequencies (500 to 1,200 per second) used with an ordinary induction bridge. The theoretical and experimental investigations given below show that the same formulas apply also to the Direct Connected System of circuits.

XV. Wavemetrical Study of the Oscillation in the Direct Connected System of Circuits.

This form of oscillation system is represented in Fig. 50 and in diagram in Fig. 58. The experiments given below show that so far as concerns the wave-length of oscillations exactly the same formulas apply as have been found to be applicable to the electromagnetically connected system. The following sketch of the theory of this system of circuits leads also to this result.

Sketch of the Theory of the Direct Coupled System of Circuits.— In Fig. 58, let the circuit C_1L_1 be looked upon as the discharge circuit, and the circuit C_2L_2 be looked upon as a secondary circuit.

At any time t, let the charge on the outer plate of the condenser C_1 be q_1, let the current flowing from this plate at this time be i, and let the current in the coil L_1 be x. Let q_2 be the charge on the outer plate of C_2, and y the current flowing towards this plate: then

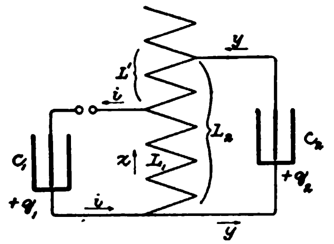

Fig. 58.

$$(14) \qquad i = x + y,$$

$$(15) \qquad q_1 = -\int idt,$$

$$= -\int (x+y)dt,$$

$$(16) \qquad q_2 = \int ydt.$$

Let L_1 be the self-inductance of the discharge circuit, L_2 the self-inductance of the secondary circuit, and L' the self-inductance in that part of L_2 that is not common to L_1, and let M' be the mutual inductance between L_1 and L'.

Let us suppose that the inductances are localized in the coils, and the capacities localized in the condensers.

If we neglect the resistances of the two circuits, which have but small influence on the period of the circuits, and take the electromotive force around the circuits C_1 L_1 and C_2 L_2 we have

$$(17) \qquad L_1\frac{dx}{dt} - M'\frac{dy}{dt} - \frac{q_1}{C_1} = 0,$$

$$(18) \qquad L_1\frac{dx}{dt} + M'\frac{dx}{dt} - L'\frac{dy}{dt} - M'\frac{dy}{dt} - \frac{q_2}{C_2} = 0.$$

Replacing q_1 and q_2 by their values from (15) and (16), we have

$$(19) \qquad L_1\frac{dx}{dt} - M'\frac{dy}{dt} + \frac{1}{C_1}\int (x+y)dt = 0,$$

$$(20) \qquad (M' + L_1)\frac{dx}{dt} - (L' + M')\frac{dv}{dt} - \frac{1}{C_2}\int ydt = 0.$$

In order to eliminate x from these two equations let us take $(M'$ $+ L_1)$ × equation (19) and add it to $(-L_1)$ × equation (20), giving

$$(21) \quad (L'L_1 - M'^2)\frac{dy}{dt} + \frac{M'+L_1}{C_1}\int xdt + \left(\frac{M'+L_1}{C_1} + \frac{L_1}{C_2}\right)\int ydt = 0.$$

The second derivative of (21) with respect to t, added to $-1/C_1$, × equation (20), gives

$$(22)\quad (L'L_1 - M'^2)\frac{d^3y}{dt^3} + \left(\frac{L_1}{C_2} + \frac{L' + M_1 + L_1}{C_1}\right)\frac{dy}{dt} + \frac{1}{C_1C_2}\int y\,dt = 0,$$

which differentiated again gives

$$(23)\quad (L'L_1 - M'^2)\frac{d^4y}{dt^4} + \left(\frac{L_1}{C_2} + \frac{L' + 2M' + L_1}{C_1}\right)\frac{d^2y}{dt^2} + \frac{y}{C_1C_2} = 0.$$

This equation may be somewhat simplified by introducing the total inductance of the secondary, L_2, and by expressing the mutual induction in terms of the total mutual inductance between L_1 and L_2 instead of in terms of M' which is the mutual inductance between L_1 and L'. Let the mutual inductance between L_1 and L_2 be M, then by reference to the diagram, Fig. 58, it may be seen that

$$(24)\qquad M = M' + L_1$$

and

$$(25)\qquad L_2 = L_1 + L' + 2M'.$$

From these equations it follows that

$$(26)\qquad L' = L_2 - L_1 - 2M' = L_2 + L_1 - 2M,$$

and

$$(27)\quad L'L_1 - M'^2 = L_1(L_2 + L_1 - 2M) - (M - L_1)^2 = L_1L_2 - M^2.$$

Equations (25) and (27) substituted in equation (23) gives

$$(28)\qquad (L_1L_2 - M^2)\frac{d^4y}{dt^4} + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1}\right)\frac{d^2y}{dt^2} + \frac{y}{C_1C_2} = 0.$$

In like manner, if instead of eliminating x from equations (19) and (20), we eliminate y, we get

$$(29)\qquad (L_1L_2 - M^2)\frac{d^4x}{dt^4} + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1}\right)\frac{d^2x}{dt^2} + \frac{x}{C_1C_2} = 0.$$

This equation is identical with the equation used for the determination of the period of oscillation with the electromagnetically connected system; namely equation (6) with the resistances pu-equal to zero. Whence it follows that the formulas for determining the wave-lengths of the direct coupled system are identical with those for the electro-magnetically coupled system, which are given as equations (10) and (11) on page 19. These formulas are found to apply to the experimental cases that follow.

Experiments with the Direct Coupled System. — Several experiments were made with the direct connected system of circuits. In all these experiments the primary capacity was kept the same, $C_1 = .00432$ m.f. Two different values of the secondary capacity were used, $C_2 = .00178$ m.f. and $C_2 = .00445$ m.f. In each experiment the secondary inductance was set at some fixed value L_2, thus making λ_2 constant, and the primary inductance L_1 was varied from 50 turns to 3 turns, so that the two wave-lengths λ_1' and λ_2' derived when the secondary circuit is closed are functions of the variable primary wave-length λ_1.

Experiment XXVI.: D. C., $C_2 = .00178$, $L_2 = 50.5$ Turns, $\lambda_2 = 1{,}010$ Meters. — The apparatus for the experiments with the direct circuit is shown in Fig. 50. The steps of the experiment are similar to those with the other system of circuits. Table XXIII. contains a record of this experiment. The observed and calculated values of the wave-lengths in the compound oscillating system are plotted in Fig. 59. The formulas of calculation are the formulas (10) and (11), and the only difference between the calculations in the present case and those of the former case is in the method of obtaining M. In the previous case M had to be meas-

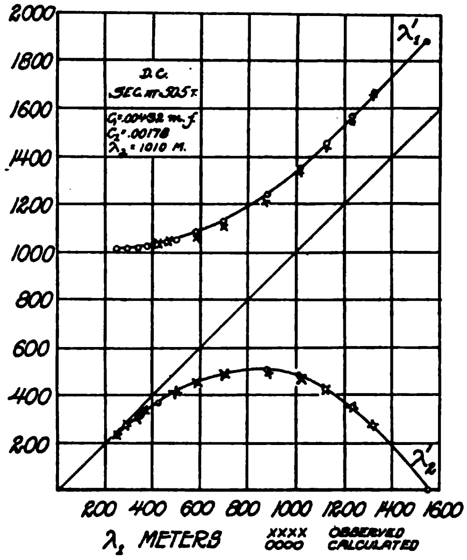

Fig. 59.

TABLE XXIII.

Direct Coupled System.

Primary capacity .00432 microfarad.
Primary inductance varied.
Secondary capacity .00178 microfarad.
Secondary inductance 50.5 turns of coil $= 16.0 \times 10^{-5}$ Henrys.
Wave-length of secondary 1010 meters.

Turns Primary.	L_1 Primary Inductance. Henry.	L'	M	τ^2
50	15.85×10^{-5}	$.05 \times 10^{-5}$	15.90×10^{-6}	1.00
45	13.9	.78	14.56	.95
40	11.8	1.85	12.97	.89
35	10.0	3.30	11.35	.805
30	8.20	5.00	9.60	.702
25	6.50	6.66	7.92	.601
20	4.82	8.40	6.21	.498
15	3.15	10.20	4.47	.396
12	2.30	11.30	3.50	.332
8	1.30	12.80	2.22	.237
7	1.05	13.25	1.89	.212
6	.88	13.63	1.62	.113
5	.69	14.00	1.35	.145
4	.50	14.40	1.05	.138
10	1.72	12.00	2.86	.296

Turns Primary.	λ_1 Meters.	Calculated.		Observed.	
		λ_1' Meters.	λ_2' Meters.	λ_1' Meters.	λ_2' Meters.
50		1870			
45	1450	1770	175		
40	1325	1665	272	1650	265
35	1235	1563	355	1540	342
30	1125	1450	427	1435	415
25	1020	1350	477	1325	465
20	880	1240	500	1200	485
15	695	1125	485	1100	485
12	585	1072	450	1050	450
8	425	1030	368	1000	360
7	375	1025	330		335
6	340	1018	317		300
5	290	1018	270		270
4	250	1015	234		235
10	500	1050	405		415

ured on an inductance bridge for every setting of the inductances in the experiments. With the direct coupled system the determination of M is somewhat simpler. It may be had by calculation from the calibration curve of the inductance coil used, for by equation (26) on page 93 we have

$$M = \frac{L_1 + L_2 - L'}{2}. \tag{30}$$

In this equation L_1 is the inductance of the primary, which may be obtained from the calibration curve of the coil and the number of turns of the primary; L_2 is the inductance of the secondary, had from the number of turns of the secondary; and L', see Fig. 58, is the inductance of that part of L_2 which is not common to L_1. This L' may be obtained as the inductance on the calibration curve belonging to the difference between the number of turns on L_2 and the number on L_1.

The value of M by the use of equation (30) found for each setting of the primary inductance is recorded in the fourth column of Table XXIII. From this value and the corresponding values of L_1 and L_2 a value of $\tau^2 = M^2/L_1L_2$ is obtained, and recorded in the fifth column. λ_1 was measured by the wave-meter for several values of the primary inductance. λ_2 was measured and left constant throughout the experiment. From this value of λ_2 and the several values of λ_1 and τ^2, the values of λ_1' and λ_2' were computed by the formulas (10) and (11). The corresponding values of λ_1' and λ_2' were also observed directly with the wave-meter. The observed values are represented by *crosses* in Fig. 59, and the calculated values by *circles*.

The agreement between the observed and calculated values is within the limit of accuracy of the measurement of the wave-lengths.

Experiment XXVII.: Same as XXVI. Except that $C_2 = .00445$ m.f., and $\lambda_2 = 1,575$ Meters.—The capacity about the secondary was replaced by a larger jar giving a longer secondary wave-length of 1,575 meters and observations like those of the preceding experiment were made. The results are plotted in Fig. 60. The curves in this case bear a marked resemblance to those of Fig. 59. The chief differences are that in the latter case with the longer secondary wave-length, λ_1' begins higher up, tangential to 1,575 meters instead of to 1,010 meters as in the preceding case; and the arch of the curve λ_2' is a little higher than in Fig. 59.

In both experiments the curve λ_2' comes down to the horizontal axis in the neighborhood of λ_1 equal 1,575 meters. This point, which was calculated since the wave-meter scale does not extend below 200 meters, is the point of *perfect coupling* indicated by the theory. This value occurred in the experiments when the primary and secondary condensers were both connected about the *same inductance*, 50.5 turns of the coil, so that from the standpoint of the experiment the two condensers may be looked upon as discharging in parallel through the same inductance, and producing, therefore, only one wave of wave-length

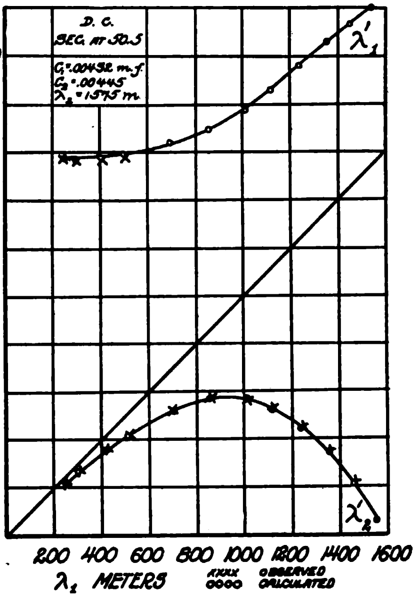

Fig. 60.

$$\lambda_1' = 2\pi \cdot v \cdot \sqrt{L_1(C_1 + C_2)} = \sqrt{\lambda_1^2 + \lambda_2^2}.$$

This result is also obtainable from the theoretical equations (10) and (11)

$$\lambda_1' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 + \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}},$$

$$\lambda_2' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 - \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}}.$$

For when the primary and secondary condensers are connected about the same inductance

$$L_1 = L_2 = M, \text{ therefore}$$

$$\tau^2 = \frac{M^2}{L_1 L_2} = 1.$$

When τ is equal to unity *the coupling is said to be perfect* and the equations given above for the derived wave-lengths become

$$\lambda_1' = \sqrt{\lambda^2_1 + \lambda^2_2}; \text{ and}$$

$$\lambda_2' = 0.$$

That is to say, the oscillation becomes single valued.

The case of perfect coupling was not observed in the experiments with the *electromagnetically coupled system*, because for perfect coupling the primary and secondary coils must have the same number of windings and the two coils must be so close together as to be practically coincident, conditions that could not be realized with the apparatus used in the experiments with the electromagnetic coupling

Other experiments made with the Direct Coupled System of circuits may be epitomized as follows.

	Experiment XXVIII.	Experiment XXIX.	Experiment XXX.
Secondary at	25.5 turns.	25.5 turns.	15.5 turns.
Secondary inductance L_2	6.7×10^{-6} Henry.	6.7×10^{-6} Henry.	3.33×10^{-6} Henry.
Secondary wave-length λ_2	645 meters.	1020 meters.	735 meters.
Secondary capacity C_2	.00178 m.f.	.00445 m.f.	.00445 m.f.
Primary capacity C_1	.00432 m.f.	.00432 m.f.	.00432 m.f.
Primary inductance L_1	varied.	varied.	varied.
Carves.	Fig. 61	Fig. 62	Fig. 63

In all of these experiments the agreement between the observed and calculated values is satisfactory. In the experiments that gave Fig. 61 and Fig. 62 the *point of perfect coupling* occurs at the same value of primary wave-length, about 1,010 meters. This is because the secondaries had the same number of turns in both cases. In Fig. 63 the point of perfect coupling occurs nearer the origin at wave-length $\lambda_1 = 650$ meters. In each of the figures, the curve of λ_2' after passing the zero point again rises toward the right giving again the two periods of oscillation. Lengthy comment on these curves is unnecessary,. since on account of the agreement of the observations with the theory the complete trend of the curves can be obtained from the theoretical equations.

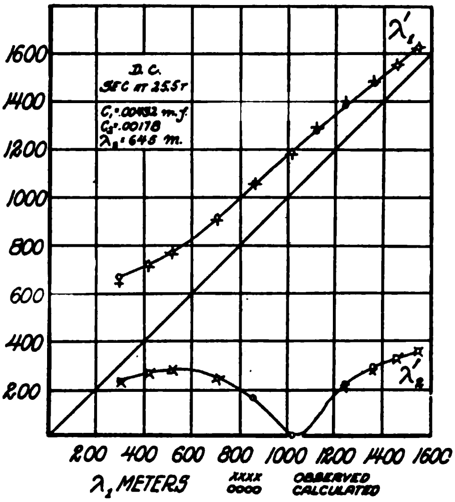

Fig. 61.

Experiment XXXI.: Special Case of the Direct Coupled System, in which the Primary and Secondary are Adjusted Separately to the same period. $\lambda_2 = \lambda_1$. — The observed and calculated values in this

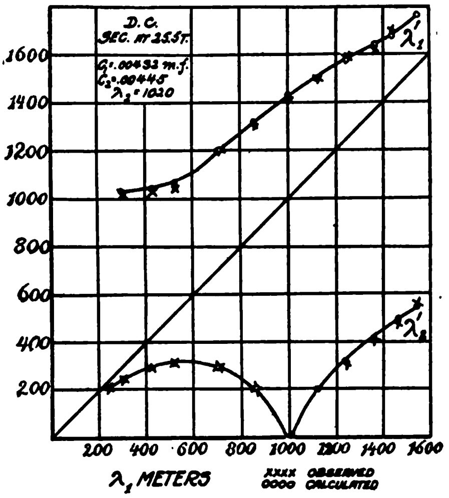

Fig. 62.

case are given in Fig. 64. The figure resembles the corresponding case with the electromagnetic connected system.

In conclusion it should be remarked that the experimental values of the wave-lengths for the entire set of experiments were taken before the calculations were made, and in no case were any of the experimental values redetermined. Each measurement of wave-length was made hastily and with a single setting of the wave-meter No effort was made to obtain high accuracy in the results, as might have been done by averaging several settings of the wave-meter. The results show that the theoretical formulas given above are sufficiently accurate to be used for computing the wave-lengths of the oscillation occurring in connected systems of condenser circuits. Also, on the other hand, the agreement between observed and calculated values show that the wave-meter used is correctly calibrated and is serviceable for the range of wave-lengths between 200 meters and 1,800 meters. It is proposed to utilize the apparatus in direct measurements with wireless telegraph circuits instead of the simpler condenser circuits of the present research.

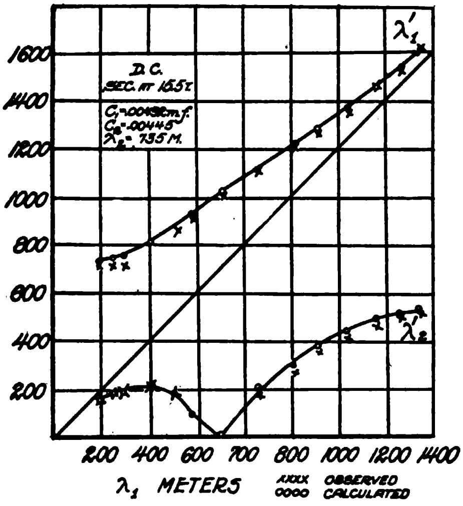

Fig. 63.

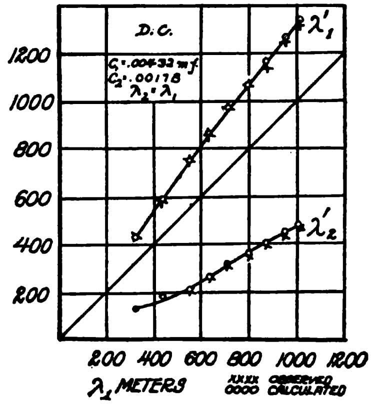

Fig. 64.

Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.
November 6, 1906.

THE TRANSFORMATION INTO AN ELECTRIC CURRENT OF RADIATION INCIDENT ON A MOVING SURFACE.

By Bergen Davis.

A PLANE polarized beam of light is generally considered to consist of a train of waves in which the electrical component alternates in sign, but remains in the same plane perpendicular to the wave front. As the wave passes through a surface, a point on the surface would be subject to an alternating field of force.

It would be a matter of great importance to rectify the small currents that are set up in a conducting surface by such alternating fields of force. The detection of these currents would not only confirm the electro-magnetic theory of radiation, but would furnish a direct method for distinguishing the electric and magnetic components of the wave.

An account of some experiments by the late Professor A. Bartoli on the direct transformation of radiation into electric currents has recently been published.[1] The writer has lately repeated the experiments of Professor Bartoli, making such changes in the apparatus as our present knowledge of the electronic nature of electric conduction would warrant. Because of the importance of Bartoli's experiment, I think it advisable to give an outline of his reasons for expecting the effect and a translation of that part of his paper which describes his apparatus and the results obtained.

1. Professor Bartoli proceeds from his well-known thermo-dynamic proof of radiation pressure. He performed some experiments seeking to observe the expected radiation pressure, but failing to detect it, concludes that it does not exist. It occurs to him that perhaps his failure is due to the conversion of the incident energy into currents in the conducting surface. The experiment

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[1] Su la transformazione in correnti ellectriche delle radiazioni incidenti Sopra una superficie riflettente in movimento. Accademia dei Lincei, 1903, 5th Series, 12.

described below was undertaken for the purpose of detecting these transformed currents.

It is, perhaps, worth while to give a translation of that part of his paper which describes his apparatus and the results obtained.

"Upon a disc *H*, of copper 4 mm. thick and 80 cm. in diameter, a copper band *LPM*, is firmly fixed and perfectly silvered, so that it will reflect at least 92 per cent. of the incident radiation. This band is insulated by means of well-dried pieces of wood, after they have been boiled in linseed oil, to which it is firmly attached by a great number of brass screws.

"The two extremities *L* and *M* of the band are joined by strips of copper, *l* and *m*, to two rings of copper *BB′*, which are 3 mm. thick and insulated by two small ebonite wheels 4 cm. in diameter. The disc *H*, as well as the two small wheels *BB′*, are firmly fixed normally to the steel axis *PS*, which is 3 cm. in diameter but thicker in the middle. Above the two copper rings *BB′*, there are two insulated copper springs *AA′*, which by copper adjusting screws and thick copper wires, communicate with a mirror galvanometer of small resistance and of that type which is completely astatic.[1]

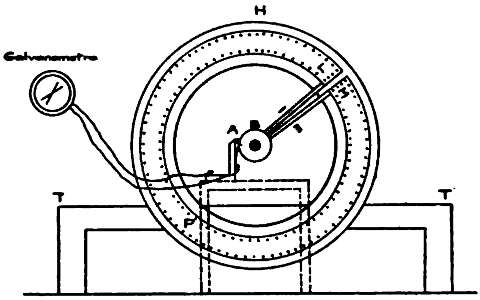

Fig. 1.

"The axle *PS*, was resting on bronze bearings which were supported by a strong bed frame of a turning lathe of more than 1,000 kilograms and which was firmly fixed to the ground.

"It was possible to impress a very rapid rotation to the disc *H* by means of a belt passing over the iron pulley *P* 4.5 cm. in diameter. The motion was generated by two large wheels at each of which from six to eight men were working. A counting device similar to Savart's toothed wheel made it possible to count the number of revolutions of the disc in a stated time. After a half-hour's work it was possible to impart to the disc *H* its maximum velocity which was about 100 to 150 revolutions per second. In that case the velocity of a point on the band *LPM* was about 240 to 410 meters per second.

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[1] A two-needle mirror galvanometer of Magnus, constructed by Sauerwald, Berlin.

"Before commencing the action, a pencil of solar rays concentrated upon the band *LPM* by means of plane and concave mirrors did not produce any thermo-electric current that made the galvanometer deviate more than two to three scale divisions. During the motion of the disc no thermo-electric current whatsoever (owing to the warming of the rings pressing against the springs, or owing to any other cause) made the galvanometer deviate while the moving disc was in the dark. While the galvanometer needle was motionless and the disc had attained its maximum velocity I let the solar pencil (by pulling up the diaphragm) fall upon the band *LPM* (at about its middle). I could then observe a deviation which at times attained 42 scale divisions. This deviation was always observed. It depended upon the direction of rotation. In fact revolving the disc in the opposite sense at the same speed produced a deviation of about 38 scale divisions in the opposite direction. This deviation was entirely dependent on the velocity of rotation; thus, with a rotatory velocity of about one half that of the maximum a deviation of 20 scale divisions was obtained, *i. e.*, about one half the preceding deviation. The deviation persisted as long as the solar pencil was striking the silvered band; when the rays were cut off by the diaphragm the galvanometer returned to zero; when the diaphragm was pulled up the galvanometer deviated anew.

"These experiments were made in the months of August and September, 1880, in the school of design which is situated on the ground floor of the Technical Institute of Florence. I was assisted in these experiments by Dr. Guido Alessandri, then my assistant, now professor in the Royal Lyceum of Potenza.

"For lack of sufficient motor and local conveniences, I could not continue the experiments which I have only indicated for the sake of recording them. I am satisfied however with the discovery of the fact. I reserve for some other time the repetition of these experiments with improved means in order to determine the laws governing this phenomenon."

It is to be regretted that Professor Bartoli does not give the sensitiveness of his galvanometer. It is impossible to estimate the magnitude of the effect which he observed. It is not probable however that as early as 1880 he possessed an instrument so sensitive as those now at our disposal.

There are some conditions of the experiment just described that make it somewhat doubtful if the effect sought for was really observed. The silver surface was deposited on a copper band of considerable thickness. The light was reflected for the most part and certainly not at all transmitted through the strip. The disturbances produced in the strip would be confined entirely to the surface. The electro-motive force that may have been developed did not extend through the strip, consequently if a current were produced in the surface it would flow around through the back of the strip. The current would be completely closed locally and there could be no effective electro-motive force tending to send a current through the external circuit to the galvanometer.

There are other and more fundamental reasons for the improbability of the real existence of the effect described by Bartoli. Some of these objections will be considered in the second part of this paper.

II. It can be shown however that the alternating currents produced in a moving conductor by incident radiation will not be symmetrical, but during one half period will be of greater or less amplitude than the next half period provided the force acting on an electron is modified by its velocity in an electric field.

Suppose a plane polarized beam of light to pass through a thin conducting film in motion. The plane of the electric component is in the direction of motion. The electro-motive force of the light wave will act through the whole thickness of the film, but its intensity will decrease with the depth owing to absorption. The current produced will be proportional to the electro-motive force at any depth.

Let the sinusoidal form in Fig. 2 represent such a wave propagated downward through a conducting film *CD*. The electric component is in the plane of the paper. Consider that the carriers of the current are free negative electrons. If the film is at rest as the train of waves passes through, the electrons will be subject to a symmetrically alternating force which will produce a symmetrical alternating current. If the film is in motion the alternating currents will be unsymmetrical or symmetrical according as the force acting on an electron is a function of its velocity in the field or not.

Assume the film to be in motion and the light wave at a given instant to occupy the position represented by case *A*, Fig. 2. The electrons will have a motion relative to the field from left to right due to the motion of the film. This direction is opposite to that which the field of the wave tends to give them at this instant. The second case *B* represents the position of the wave a half period later than case *A*. The motion of the electrons relative to the field is here in the same direction as that which the field tends to produce. The relative motion of the electrons with respect to the field will thus be a variable one. If the force acting on an electron moving parallel to and in an electric field depends on its velocity in the field, the currents in the film will no longer be symmetrical but will have a greater amplitude in one direction. The current will be alternating but unsymmetrical. Since the resistance is constant the effective electro-motive force will also be unsymmetrical.

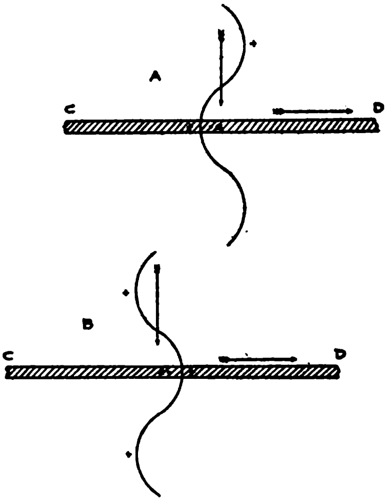

Fig. 2.

I shall make the most simple assumption possible for the dependence of the force on the velocity, namely that it will be modified by a quantity proportional to $(u/v)^2$, where u is the velocity of the film and v is the velocity of light.

Then during the first half period (case A) the effective E.M.F. will be

$$E_1 = X\left[1 + \left(\frac{u}{v}\right)^2\right] \sin pt.$$

The mean value of this is

$$E_1 = \frac{2}{\pi}\left[1 + \left(\frac{u}{v}\right)^2\right]X.$$

During the next half period (case B) the effective E.M.F. will be

$$E_2 = X\left[1 - \left(\frac{u}{v}\right)^2\right]\sin pt,$$

the mean value of which is

$$E_2 = \frac{2}{\pi}\left[1 - \left(\frac{u}{v}\right)^2\right]X.$$

The effective E.M.F. tending to produce a current in one direction in the film is

$$E = (E_1 - E_2) = \frac{1}{2}\,\frac{4}{\pi}\left(\frac{u}{v}\right)^2 X = \frac{2}{\pi}\left(\frac{u}{v}\right)^2 X,$$

where X is the maximum electrical intensity of the sinusoidal light wave. This would be the mean electro-motive force for slow frequencies of vibration. The light wave, however, is of very high frequency, and it is well known that alternating currents of such frequencies cannot be propagated along conductors because of the impedance. Neglecting this impedance the theoretical value of E may be readily found.

Taking the solar constant as 3 gram-calories per minute the energy per cubic-centimeter of sunlight is 2.1×10^6 ergs and the value of X is found to be about 6.2 volts per centimeter.

If the linear velocity of the moving film is 10,000 cm. per second, the ratio $(u/v)^2$ becomes — $\frac{1}{9} \times 10^{-12}$.

Introducing the value of X and $(u/v)^2$ in (3) we have

$$X = 3.6 \times 10^{-11} \text{ volts per centimeter.}$$

If the sunlight is concentrated by a lens to 100 times its natural intensity, the electrical intensity becomes

$$X = 3.6 \times 10^{-10} \text{ volts.}$$

This is the electrical intensity at the upper surface of the film. At the lower surface the electrical intensity will be much less owing to reflection and absorption in the film. Much of the current developed at the upper surface would complete its circuit at the under surface. The effective electrical intensity would thus be due to the intensity of the radiation that passed through the lower surface.

The electrical intensity (3.6×10^{-10}) is too small to be easily detected on account of the unavoidable disturbances produced by the high velocity of the film.

Also although the electrical intensity $2/\pi(u/v)^2X$ is always in one direction in the film, yet it is not continuous but impulsive, and, as I have pointed out, impulses of the order of frequency of light vibrations can not be propagated along conductors.

In spite of these considerations which indicate an improbability of an effect in view of Bartoli's experiments I have though it worth while to seek to test his observations by a modified method.

Apparatus and Experiment.

The method used was to move a semi-transparent film in a plane perpendicular to a concentrated beam of sunlight falling upon it.

The rotating device for giving a great linear velocity to the film *f*) is shown in Fig. 3. An aluminium disc *D* is mounted on a hollow steel axle *A*. The disc is recessed to firmly hold the glass plate *g*. The disc and plate are rotated by an air motor *mm'*. Air compresed to about 25 pounds pressure per square inch was directed by suitably constructed nozzles upon the toothed wheels *mm'*. By this device the glass plate could be given a speed of 200 revolutions per second and the direction of rotation could be quickly reversed.

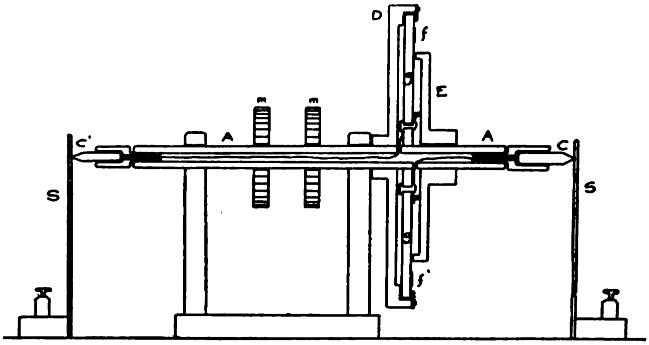

Fig. 3.

A thin circular silver film (*f*) about 8 mm. broad was deposited by the chemical method upon the glass plate. The strip of silver film formed nearly a closed ring. A radial silver strip extended from each end of the film toward the center and attached to aluminium wires by the small screw bolts *pp′*. These wires (insulated) were carried through the hollow axle to the ends of the shaft where they were attached to aluminium cones *cc′*. These cones were also insulated from the shaft. Aluminium springs *ss′* pressed against these conical points. The points of the cones were made to coincide as nearly as possible with the axis of rotation of the shaft. This reduced the motion between the points of the cones and the contact springs to a minimum.

The greater part of the glass disc was covered by an ebonite cover *E*. This was just large enough to cover nearly all the plate to the inner edge of the film. This cover prevented disturbances arising from the friction of the air against the radial parts of the film and the small bolts *pp′*. Great disturbances were always observed in the galvanometer when the apparatus was used without this cover.

The galvanometer was a D'Arsonval type recently constructed for our laboratory by the Weston Electric Instrument Company. Its resistance is 31.3 ohms. The volt sensitiveness is 81.6×10^{-10} volts per scale division, the scale being one meter from the instrument.

The sunlight was concentrated on the film (f) by means of a large lens. The films were of various thicknesses transmitting from .05 to .005 of the incident light. The sunlight could be admitted to or shut off from the film by a movable diaphragm. The greatest rate of rotation attained was about 180 revolutions per second, which gave a linear speed to the film of 6,000 cm. per second.

Very little disturbance was produced in the galvanometer by this rapid rotation. There was occasionally a slow drift, sometimes in one direction sometimes in the other.

A number of films of various thicknesses were experimented with but in no case was there a deflection of the galvanometer that could be traced to the act of admitting the light to the film or to suddenly shutting it off.

Three silver films and one platinum were tried. Their resistances were as follows :

A. Resistances 150 ohms.
B. " 500 ohms.
C. " 200 ohms.
D. This film was formed as follows:

The glass plate was silvered. This silver was coated with a thin film of gelatine. A platinum strip 8 mm. wide was then deposited on this gelatine by the cathode discharge. The platinum film was insulated from the silver by the gelatine. The resistance of the platinum film was 410 ohms.

The order of the effect that might have been expected had there been no impedance already referred to may be approximately calculated. The sunlight was concentrated to about 100 times its natural intensity. The maximum electrical intensity of the electrical component was about 600 volts, of which the resultant components parallel to the direction of motion was 300 volts. The thinnest film transmitted not more than five per cent. of the incident light. The electrical component at the back of the film was thus about 15 volts per centimeter. The velocity of the film was about 6,000 cm. per second. The mean impulsive electrical intensity was

$$x=\frac{2}{\pi}\left(\frac{u}{v}\right)^2 X=4\times 10^{-13} \text{ volts per cm.}$$

This was beyond detection by the galvanometer.

With a resistance of 531 ohms in the circuit, one scale division corresponded to an electro-motive force of 1.39×10^{-7} volts.

The effect if it exists was thus about three million times too small to be detected by this instrument.

CONCLUSION.

Professor Bartoli does not give the sensitiveness of his galvanometer. A direct comparison of the two experiments is consequently impossible. It may however be doubted if at that time (1880) he possessed an instrument of much greater sensitiveness than the one used by the writer. It is true that he attained a greater linear velocity of his conducting strip (40,000 cm. per second). But as I could certainly detect a throw of 4 scale division I should easily have observed an effect that gave him a throw of 42 scale divisions. If the galvanometers were equally sensitive this would allow for a peripheral speed one tenth that obtained by Professor Bartoli.

The fact that with my apparatus, which was on the whole probably as sensitive as that of Professor Bartoli, no effect was observed, and also that, if there was any partial rectification, the electrical intensity must have been impulsive and so destroyed by the impedance, leads at least to the conclusion that the existence of the effect described by Professor Bartoli is not established and must remain in doubt until again observed by other experimenters.

Phœnix Physical Laboratory,
Columbia University,
October 27, 1906.

THE ELASTIC MODULUS FOR SMALL LOADS AT THE ELASTIC LIMIT.

By Henry W. Bearce.

IN June, 1899, there was published in the Physical Review a paper by Mr. C. P. Weston, in which was described a series of investigations relating to the application of Young's Modulus when small loads are used. Even with loads of the order of magnitude of 0.5 gram, it was found that the ratio between the stress and the strain holds true.

The present paper describes a series of experiments where the work of Mr. Weston was repeated with the exception that the rods had been previously weighted so as to bring them to their elastic limits.

This was done by applying a load at the center of the rod and increasing this load by small amounts until a permanent "set" could be detected in the rod. A telescope with a micrometer head was used for this purpose and a very slight deviation from the original position of the rod could be detected as the successive loads were removed. The load which caused the first noticeable permanent "set" was taken as the elastic limit and in that part of the experiment in which Young's Modulus was determined, this was used as the initial load to which the 0.5 gram loads were added.

The deflections in the rods were measured by the method of interferences of light waves. The movable mirror of an interferometer was removed and fastened by a pellet of wax to the center of the rod to be tested, the interferometer being placed on its side and illuminated by a sodium flame placed at the right of the apparatus.

The proper height of the mirror was found by raising and lowering the knife edges, and the proper angle by slightly moving the mirror on the pellet of wax and by the adjusting screws of the stationary mirror. By careful adjustment of the distance and angles of the mirrors, straight and clear-cut fringes were found. Then when the 0.5 gram weight was added the fringes were seen to pass across the field of view. By a count of the number of fringes passing a point of reference the amount of deflection was calculated, assuming as the wave length of sodium light, 0.00006 cm. In order that the fringes could be accurately counted, it was necessary to add the 0.5 gram very slowly and this was done by suspending the weight by a fine silk thread from a horizontal arm moved up and down by a slow motion screw.

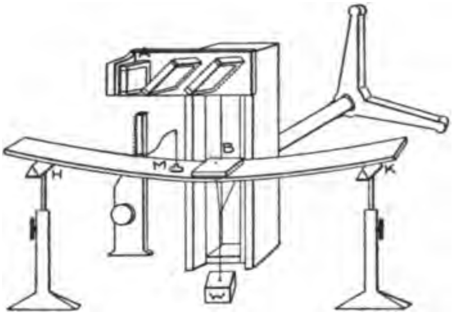

Fig. 1.

The accompanying figure will show the details of the experimental work.

The results of the deflection experiments appear in the table.

TABLE I.

Iron Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflection in cm.	Deflection for 0.5 Gram.
1	0.560	1.68	0.000101	0.0000902
2	1.025	3.59	0.000215	0.0001048
3	1.585	5.21	0.000312	0.0000985
4	1.995	7.05	0.000423	0.0001083

Elastic load — 6.5 *k*.

Using the common formula for the modulus

$$M=\frac{FL^3}{4bh^3d}$$

$F=$ Wt. in grams × 980.6

$L=$ Length $=60$ cm.

$b=$ Breadth $=.970$ cm.

$d =$ Deflection in cm. for 0.5 gram $= 0.000101$

$$M = \frac{0.4985 \times 980.6 \times 216000}{4 \times 0.970 \times .125 \times 0.000101} = \frac{105481008}{.000050304} = 20.96 \times 10^{11}.$$

Table II.

Steel Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflection in cm.	Deflection for 0.5 Gram.
1	0.560	1.34	0.0000804	0.0000718
2	1.025	2.44	0.0001464	0.0000714
3	1.585	3.67	0.0002202	0.0000695
4	1.995	5.01	0.0003006	0.0000753
		Elastic load — 24.25 *k*.		

$L = 60$ cm. $h^3 = .154854$

$b = 1.008$ cm. $d = 0.000720$

$h = .537$ cm. $M = 21.04 \times 10^{11}.$

Table III.

Copper Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflection in cm.	Deflection for 0.5 Gram.
1	0.560	1.35	0.0000810	0.0000723
2	1.025	2.74	0.0001644	0.0000802
3	1.585	4.54	0.0002724	0.0000859
4	1.995	5.92	0.0003552	0.0000891
		Elastic load — 13.25 *k*.		

$L = 61$ cm. $h = 0.639$ cm.

$b = 1.087$ cm. $d = 0.0000819$

$M = 12.26 \times 10^{11}.$

Table IV.

Brass Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflections in cm.	Deflection for 0.5 Gram.
1	0.56	1.35	0.0000810	0.0000723
2	1.025	2.68	0.0001608	0.0000784
3	1.585	4.02	0.0002412	0.0000768
4	1.995	5.51	0.0003306	0.0000829
		Elastic load — 19.5 *k*.		

$L = 60$ cm $h = 0.638$ cm.

$b = 1.094$ m. $d = .0000776$

$M = 11.59 \times 10^{11}.$

A comparison of the above results with those obtained by Prof. Weston is shown in the next table.

TABLE V.

Comparison of Results under Ordinary Conditions with those at Elastic Limit.

Bar.	At Elastic Limit (interferometer).	Ordinary Conditions (interferometer).	Ordinary Conditions (large loads used).
Steel.	21.04×10^{11}	19.19×10^{11}	19.56×10^{11}
Copper.	12.26×10	11.87×10	12.00×10
Brass.	11.59×10	10.64×10	10.62×10

It will be seen that the results for the modulus when the bars are at the elastic limit are slightly in excess of those under ordinary conditions.

The accompanying curves show the relation between loads differing by 0.5 gram, and the resulting deflections measured in wave-lengths of sodium light. In the plot each vertical space equals 0.5 gram, and each horizontal space one wave-length.

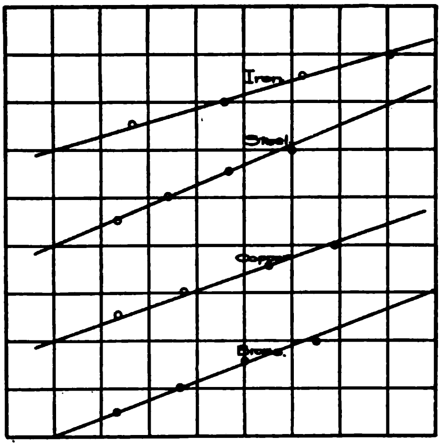

Fig. 2.

In conclusion it may be said:

1. These experiments seem to show a slight increase in the elastic modulus as bars reach the elastic limit.

2. Within the limits of errors of observation Hooke's law holds at the elastic limit as well as under normal conditions.

This work was suggested by Prof. James S. Stevens and carried on under his direction.

PHYSICAL LABORATORY,
UNIVERSITY OF MAINE.

INDEX OF REFRACTION AND DISPERSION WITH THE INTERFEROMETER.

By C. A. Proctor.

To determine the index of refraction of a dispersion medium by means of an interferometer it is necessary to introduce into one arm of the instrument a known thickness of the substance to be tested, and to count the number of fringes which cross the field while this change in the light path is actually occurring, or else to employ two or more light sources of different wave-length. For if we merely determine, by moving one of the mirrors, the shift of the white light fringes due to the substance, the results obtained will be incorrect owing to the shift due to dispersion.[1]

The usual laboratory method of determining the true value of this fringe shift may be briefly outlined as follows:[2] A plate of uniform thickness of the substance, *e. g.*, glass, whose index is to be determined, is cut in two, and one half is mounted in each arm of the interferometer so as to cover half the field. For this purpose the instrument is provided with two extra frames capable of rotation about vertical axes, one of them being equipped with a slow-motion attachment for rotating. The two pieces of glass are arranged to cover the same portion of the field of view and are set normal to the light paths, and the interferometer is adjusted for white light fringes. When these adjustments are made the fringes in the portion of the field covered by the glass will be continuous with those in the remainder of the field. One glass is now turned through a convenient angle. The other glass is then turned by means of the slow-motion attachment, the fringes of sodium light being counted as they cross the field until the white-light fringes again occupy their former position. The angle through which either glass turns is measured with the aid of a telescope and scale. If this angle is i, the fringe count $2N$, the thickness of the glass t, the index of the glass μ, and the wave-length of sodium light λ, it may readily be shown that

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[1] C. R. Mann, Manual of Adv. Optics, p. 37.

[2] Loc. cit., p. 61.

$$\mu = \frac{(t - N\lambda)(1 - \cos i) + \dfrac{N^2\lambda^2}{2t}}{t(1 - \cos i) - N\lambda}. \tag{1}$$

The term $N^2\lambda^2/2t$ is negligible.

If we now restore one glass to a position normal to the light path and move one of the mirrors until the white light fringes in the part of the field covered by the glasses once more occupy their former position, the number of sodium fringes crossing the field will be greater than $2N$, the difference, which we will call $2N'$, depending upon the dispersion. If we assume the relation

$$\mu = A + \frac{B}{\lambda^2},$$

we have

$$N' = \frac{2Bt'}{\lambda^3}. \tag{2}$$

In this expression t' is the thickness of glass introduced into the path by the rotation. In this way we can obtain both the index and the dispersion of our substance, using light of a single wave-length.

The objection to this method is that it demands a sample of glass for test in the form of two plane parallel plates of the same thickness, and a specially equipped interferometer. This makes it in many cases unavailable, either as a practical method of determining indices, or as a laboratory exercise.

It occurred to the writer that the above procedure might be considerably simplified, and that resort to the laboratory scrap pile should furnish materials to satisfactorily take the place of the slow motion attachment on the interferometer for rotating the glass. Accordingly a very rough micrometer screw with its nut was taken from a disused coefficient of expansion apparatus. The graduated disc was removed and in its place was put an arm of brass about 10 cm. long and 0.5 cm. thick. The nut was then secured in a clamp mounted on a heavy laboratory support in such a way that, as nearly as could be judged by eye, the screw was vertical. A spherometer from which the legs and disk had been removed was held by another clamp on the rod of the same support in such a way that the screw was in a horizontal position with its point bearing against the arm attached to the vertical screw. This contact was maintained with the aid of a spiral spring. The glass to be tested was waxed to the lower end of the vertical screw, and a galvanometer mirror to the upper end. This device was placed so that the glass projected into one path of an interferometer far enough to cover the upper half of the field. It furnished a slow motion for turning the glass which gave complete satisfaction.

The method of experiment employed was as follows. From a fragment of plate glass two pieces were cut approximately 3 × 3 cm. each, one of which was mounted on the slow-motion apparatus as described above. As is generally the case the surfaces of this glass were not parallel, so that when it was introduced into one arm of the interferometer the fringes disappeared from the field owing to the deviation of the light passing through the glass. This effect could of course be corrected by the readjustment of one of the mirrors but such a procedure would entail the loss of the fringes in the lower part of the field, and even if the surfaces were nearly enough parallel so that the fringes were visible in both portions of the field, they would be at a very low visibility in one if white light fringes were present in the other. As it is convenient, though by no means essential, to have the fringes in the lower part of the field for purposes of reference, the second piece of glass was mounted in the other arm so that it covered the same part of the the field as did the first. This glass was then turned in its own plane until the fringes were of the same width in both portions of the field. This adjustment is possible only when the lack of parallelism of the surfaces of the two squares of glass is nearly the same, a condition which will generally be fulfilled if they are adjoining portions of the same plate. The glass attached to the slow-motion device was then set normal to the light, *i. e.*, in the position of minimum retardation, and the central fringe of the white-light system brought to a fiducial line in the field. The glass was then turned and the sodium fringes counted until any desired number had crossed the field. Following the notation used above we will call this number $2N$. The angle i through which the glass turned was measured with a telescope and scale. The movable mirror was then shifted and the fringes counted until the central fringe in white light was again on the fiducial line. This count was larger than $2N$ by a number, $2N'$ given by equation (2). That such must be the case appears from the following considerations: If we call the central white light fringe at the start that of order zero, the fringe on the fiducial line after the glass has been turned will be that of order $2N$. If now we move the mirror till $2N$ fringes have crossed the field in the opposite direction the fringe of order zero will have returned to its former position. It will, however, be no longer the center of the white light system because of the shift of that system due to dispersion. As this shift is always in such a direction as to increase the apparent retardation we shall accordingly have to move the mirror further until $2N'$ more fringes have crossed the field before the new central fringe, which is that of order $-2N'$, coincides with the fiducial line.

From N, N', i, and the thickness of the glass t, we can, with the help of equations (1) and (2), compute the index and the dispersion constants of the glass.

Two of the sources of possible error in the above method are worthy of special consideration. These are the lack of parallelism of the surfaces of the glass, and the determination of the angle through which the glass is rotated.

If the faces of the glass are not parallel an error may be introduced owing to the fact that as it is rotated the thickness introduced into the path will not be given by the simple relation on which equation (1) is based, and will be different in different parts of the field. Such a condition will be indicated by a continual change in the width of the fringes as the glass is rotated. This difficulty may be entirely eliminated by so mounting the glass that the line of intersection of its two faces shall be perpendicular to the axis of rotation. In the case of glass No. 2 measurements on which are given below (a fragment of a microscope slide) the general irregularity of the thickness was such that the fringes were very much distorted. However, as the pattern and its position in the field did not alter appreciably during a shift of 150 fringes it may be assumed that no error in the fringe count is introduced. Of course the thickness must be measured at the point where the fringe shift is observed.

The angle to be measured is the rotation of the glass about an axis in its own plane. If the actual axis is not vertical (we assume the light path to be horizontal) the angle determined with the telescope and scale may not be the one desired. This possibility may be guarded against by setting the scale parallel to the glass at the start, and tilting it until its image in the telescope shows no vertical displacement as the glass is turned. The determination of the scale distance is however a matter of some difficulty, and it is in this measurement that the greatest source of error seems to lie. In the apparatus used by the writer for example the absolute error may well have amounted to a half millimeter or even somewhat more. This might be bettered by having the axis of rotation of the glass more definitely marked.

To illustrate the foregoing some determinations of the index and dispersion constants of two samples of glass were made. No 1 was a piece of plate glass and No. 2 a fragment of a microscope slide. In each case the constants *A* and *B* were determined from observations on sodium fringes. As a check the index for the mercury and for the hydrogen line was computed from *A* and *B*, and also directly determined using vacuum tubes as the light source. The thickness of the glass was measured with micrometer calipers. In the case of the readings on No. 1 with different scale distances the glass was re-set each time the scale was moved. The results are given in the appended Table.

The method above described may be applied with satisfactory results to samples of glass varying in thickness from 1.5 mm. up. By using polarized light and properly mounting the plate under test it may also be adapted to the test of crystals of a like range of thickness. Another method, however, suggests itself which might be more conveniently used with crystals, and which is adapted to use with much thinner samples of the material. This may be outlined as follows:

The interferometer is adjusted for white-light fringes, and the lamina whose index is to be determined is introduced into one arm

Glass No. 1. *Thickness* = 8.741 *mm.* $A = 1.5102$, $B = 50.5 \times 10^{-10}$.

Source.	First Count $2N$.	Second Count $2(N+N')$.	Scale Distance Cm.	Index Computed From A and B.	Index Determined From N.
Na. $\lambda = 5896$	300.0		55.39		1.5254
“	400.0		55.39		1.5247
“	500.0		55.39		1.5243
“	150.0		75.42		1.5254
“	250.0		75.42		1.5258
“	350.0		75.42		1.5256
“	200.0		68.79		1.5240
“	350.0		68.79		1.5249
“	400.0		69.95		1.5244
“	399.9	414.1	59.90		1.5247
Hg. $\lambda = 5461$	400.6		59.90	1.5271	1.5270
H. $\lambda = 6563$	400.7		59.90	1.5219	1.5221

Glass No. 2. *Thickness* = 1.702 *mm.* $A = 1.5033$, $B = 51.0 \times 10^{-10}$.

Source.	First Count $2N$.	Second Count $2(N+N')$.	Scale Distance Cm.	Index Computed From A and B.	Index Determined From N.
Na. $\lambda = 5896$	110.1	114.1	60.03		1.5180
Hg. $\lambda = 5461$	118.0		60.03	1.5205	1.5212
H. $\lambda = 6563$	100.0		60.03	1.5152	1.5156

so that it covers a portion of the field and is normal to the light. One mirror is then moved till the white-light fringes return to their former position in the part of the field covered by the lamina, the fringes of sodium light which meantime cross the field being counted.

If $2N_1$ = number of fringes counted,
μ_1 = index of substance for sodium light,
λ_1 = wave-length of sodium light,
B = second constant of Cauchy's formula,
t = thickness of lamina,
we have the relation

$$(\mu_1 - 1)t = N_1\lambda_1 - \frac{2Bt}{\lambda_1^2}.$$

If we repeat this using light of wave-length λ_2 (*e. g.*, green mercury light) we shall have

$$(\mu_2 - 1)t = N_2\lambda_2 - \frac{2Bt}{\lambda_2^2}.$$

If now we attach the lamina to one of the mirror frames, we can obtain white-light fringes by reflection from either of its surfaces.

If we count the number of sodium fringes $2N_3$ which cross the field as we pass from one of these white-light systems to the other we shall have

$$\mu_1 t = N_3\lambda_1 - \frac{2Bt}{\lambda_1^2}.$$

N_3 must be corrected for the half wave-length change of phase occurring on reflection at the front surface.

From these three equations and Cauchy's formula the dispersion constants may be readily obtained.

University of Missouri,
September 20, 1906.

THE LUMINOUS EQUIVALENT OF RADIATION.

By P. G. Nutting.

THE visual response to radiation depends upon the intensity and quality of the radiation and the time during which it is operative, as well as upon various subjective conditions. Some function of energy and wave-length is therefore necessary in order to translate radiation into light or visual brightness, that is, to make possible a purely physical definition of light. The construction of this and subsidiary functions is the subject of this discussion.

The problem is essentially one of determining the properties of the eye as a physical instrument, as a species of radiometer. It is necessary to relate the indications of this instrument to the quality and intensity of the incident radiation, in order to translate exciting stimulus (radiation) into scale reading (light).

The same amounts of radiation in watts per unit wave-length in different parts of the spectrum, affect the eye in different degrees. Hence, other things being equal, the sensibility of the eye is a function of the wave-length, say $V(\lambda)$, call this *visibility* of the radiation. For any arbitrary spectral distribution of radiation $E(\lambda)$ then, the product EV will give a third function of wave-length, say $L(\lambda)$, which we may call the *luminosity* or optical intensity of the radiation. The determination of the visibility function $V(\lambda)$ is the first step toward the solution of the general problem.

Again, the sensibility of the eye varies with the intensity of the incident light. In range, the eye is comparable with an ammeter capable of registering millions of amperes and milliamperes as well. With increase of intensity the sensibility decreases according to some function $S(L)$ of the luminosity. This sensibility function $S(L)$ may be derived from data on the photometric constant and includes Fechner's law as a special case.

Now the sensibility of any instrument at any particular value of the exciting stimulus is the derivative of scale reading with respect to exciting stimulus. Hence, once $S(L)$ is determined, the *visual* *brightness* function $B(L)$ is determined by the general integral of SdL. This brightness function $B(L)$ or $B(E, \lambda)$ is the one ultimately desired.

One might proceed further with the sensibility of the eye for differences in wave-length, thus dissecting $V(\lambda)$ into three or more components $V_1(\lambda)$, $V_2(\lambda)$, $V_3(\lambda)$, $\cdots$, but this has little physical significance except in determining the whiteness of light.

The surface integral of the spectral integral of $E(\lambda)$ is evidently the total energy emitted by a source. Similarly the same integrals of $L(\lambda)$ is the total light emitted. This light integral, tempered or not, according to whiteness, is what possesses commercial value in illumination. The ratio of the light integral to the energy integral is the *luminous efficiency* of a source.

THE VISIBILITY FUNCTION.

The visibility function gives the relative sensibility of the eye to radiation of different wave-lengths. It is by definition ($L \equiv EV$) the ratio of visual intensity L to energy intensity E. It may be determined (*a*) by taking the reciprocal of the threshold value E_0, or (*b*) from the luminosity, determined visually, of a source whose energy curve is known. The identity of visibility with reciprocal threshold value will appear later in the discussion of sensibility as a function of intensity, and is abundantly confirmed by experiment.

Langley[1] measured the least energy by which it was necessary to illuminate a logarithm table to make it legible. A König[2] determined threshold value and luminosity at both high and low intensity. Pflüger[3] measured threshold value with a thermopile. In all, about fifty sets of experimental data on visibility are available. All show a pronounced maximum in the green, a rapid falling off toward blue and red to a very small but finite value in the extreme red and violet.

The position of the maximum of visibility at low intensity varies somewhat with the individual, but the mean of the forty used by the writer[4] is at 510 $\mu\mu$. The maximum lies between 500 and 520 for nine out of every ten individuals. Eleven curves from the best data of Langley and König reduced to the same maximum ordinate are reproduced in Fig. 1. All but two of the remaining curves would have fallen within this same galaxy but could not be reproduced without crowding the figure.

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[1] S. P. Langley, Energy and Vision, Am. Journ. Sci., 36, 359–380, 1888.

[2] A. König and C. Dieterici, Z. Psy. Phys. d. Sinnesorgane, 4, 241–348, 1893.

[3] A. Pflüger, Ann. d. Physik, 9, 185–208, 1902.

[4] A more extended treatment of this subject with full quotations of all data used is to appear in the Bulletin of the Bureau of Standards.

The physical problem does not require a special function that will closely represent each individual curve. What is required is a single general function of the simplest possible form and with fixed mean parameters that will represent the properties of the average human eye as closely as any other function that might be constructed. Special work will always require a special investigation of the properties of the observers eye at the time he is working. General problems of illumination, however, require the specification of the properties of a mean average standard eye.

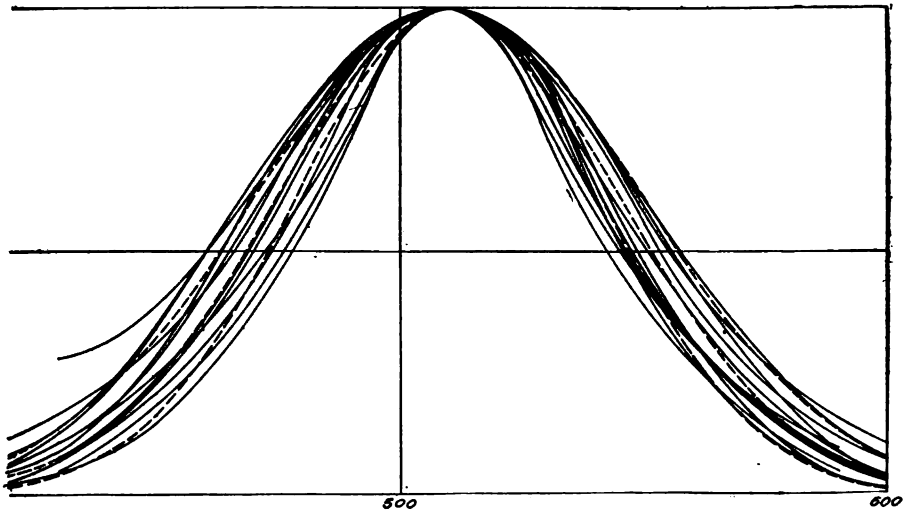

Fig. 1.

From the observed data it appears that the mean visibility curve $V(\lambda)$ is very nearly if not quite symmetrical and similar to the probability curve in form. The dotted curves in the figure are the plotted values of the probability function.

$$V = V_1 e^{-a(\lambda-\lambda_m)^2}$$

with $\lambda_m = 5.10$ s.-m. ($\equiv 510\mu\mu$) and $a = \pi$, 4 and 5. The parameter four represents the mean value very closely. The parameter π would have the advantage of giving the curve unit area per unit maximum ordinate but is obviously too small.

The sole condition thus far imposed upon the parameters V_1, a and λ_m is that they are not functions of wave-length. V_1 depends upon the unit of luminosity chosen; a decreases and λ_m increases somewhat with increase of intensity. The values given, $a = 4$ and $\lambda_m = 5.1$ s.-m., refer to very low intensities approaching the threshold value. The mean visibility curve for low intensities is practically the same for all individuals, being sensibly independent in both form and position of color blindness, partial or complete, and of the absolute sensibility of the individual eye.

The visibility function at high intensities however, varies considerably in cases of partial color blindness. For normal eyes, the only data is that of König, on his own and one other eye. The curves are broader than those at low intensity, and displaced toward the red. The visibility function given fits them fairly well for $a = 2$ and $\lambda_m = 5.6$. With such scant data however, these values cannot be regarded as established. The shift of the maximum with intensity may easily be demonstrated by means of a small grating spectograph and sunlight. As the slit is opened the maximum of brightness is seen to move over from deep green toward the yellow.

Another method of obtaining the variation of a and λ_m with intensity suggests itself. Suppose the threshold value of the luminosity L_0 is determined, as was done by König and Brodhun,[1] in terms of a rather large light unit, say one meter-candle. If now the visibility function $V(\lambda)$ for the higher intensity were identified with that $V_0(\lambda)$ at low intensity, $L_0(\lambda)$ would be constant. In any case $L_0 = V : V_0$.

In Fig. 2 are shown König's values of L_0, the visibility curve V_0 for $a = 4$ and $\lambda_m = 5.1$ together with the curve L given by the product $V_0 L_0$ at each wave length. The curve V' is that obtained by König from luminosity and energy at high intensity, but is subject to a large and uncertain correction for the spectral energy variations. It has the same maximum but is much broader than the calculated visibility curve.

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[1] König and Brodhun, Sitz. Ak. Berlin, July 26, 1888.

Calculating L_0 as the ratio $V : V_0$

$$L_0 = e^{-2(\lambda-5.6)^2} : e^{-4(\lambda-5.1)^2} \mp e^{2(\lambda-4.6)^2},$$

a curve of the same general form as that obtained directly from König's data. The values $a = 2$ and $\lambda_m = 5.6$ for the parameters of the visibility function at high intensities cannot be regarded as established on the scant data at present available, much less can the general functions $a(E)$ and $\lambda_m(E)$ be determined as would be required for a complete solution of the general problem.

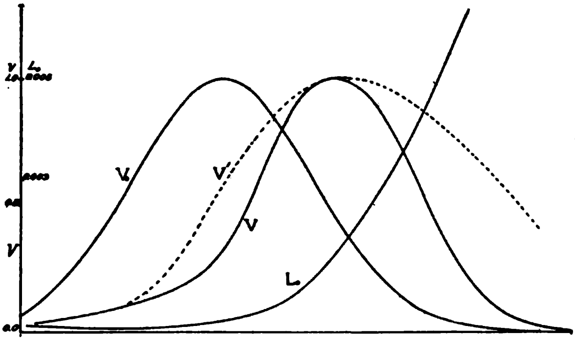

Fig. 2.

VISUAL BRIGHTNESS.

The eye will always underestimate actual differences in intensity, other things being equal, on account of the decrease in sensibility with increase in intensity. But since the magnitude of a sensation can be estimated only with the roughest approximation, indirect methods must be resorted to in order to determine the brightness function of luminosity $B(L)$: The most accurate data in this field is that relating to photometric sensibility and to persistence of vision.

Photometric sensibility is generally given as the ratio of the least perceptible increment to the total luminosity, $\delta L : L$. Call this $P(L)$. Measured in terms of a fixed unit of luminosity this increment would be LP. But sensibility is, properly speaking, the inverse of this, since the least perceptible increment is greater as sensibility is less. Sensibility to luminosity is then proportional to $1/PL$. Now the sensibility of an instrument at any part of the scale is the derivative of the scale reading with respect to the stimulus measured, so that $S = dB : dL$. If then P can be determined as a function of L, B may be established.

König and Brodhun's data[1] on photometric sensibility is very complete for six different wave-lengths and intensities ranging from 0.01 to 100,000 m.-c. This data is shown graphically in Fig. 3, where $\delta L : L$ is plotted against log nat. L. The dotted ordinates represent threshold values (L_0) of the luminosity. Since at the threshold value, the least perceptible light is the whole, $\delta L : L$ must there approach unity in value. The observed curves have been extrapolated to that value. At high intensities all the curves approach the same minimum constant value

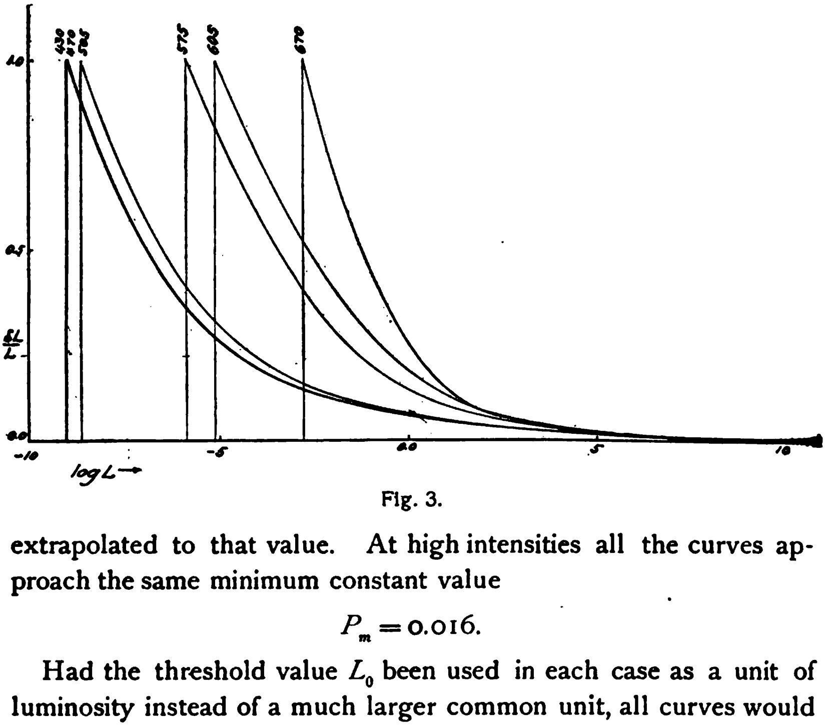

Fig. 3.

$$P_m = 0.016.$$

Had the threshold value L_0 been used in each case as a unit of luminosity instead of a much larger common unit, all curves would be shifted to a common dotted ordinate and, execept the extreme red curve, would become congruent to far within the error of observation.

This form of curve may be represented by the simple function

$$P = a + be^{-cx},$$

where $x \equiv \log\,(L : L_0)$, $a \equiv P_m \mp 0.0016$, $b \equiv 1 - P_m \mp 0.984$ and

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[1] König and Brodhun, l. c.

$bc \mp 0.26 \pm 0.01$ (read from a large plot) when x is in natural logarithms. This would give for $P(L)$

$$P = P_m + (1 - P_m)(L_0 : L)^c.$$

Taking $S \equiv 1/PL$

$$S = \frac{1}{L(P_m + (1 - P_m)(L_0 : L)^c)}$$

$$\text{hence } B = \int S dL = \frac{1}{P_m c} \log (1 + P_m(L^c - L_0^c)),$$

the integration constant being zero since visual brightness must be zero for $L = L_0$ at the threshold value.

This gives then for the final form of the brightness function

$$B = B_0 \log (1 + P_m(L^c - L_0^c)),$$

or at moderate and large intensities, the simple form

$$B = a \log L + b.$$

Had we chosen a different variable T by which to measure the visual sensation, such that $B : B_0 \equiv \log T$, we should have for moderate and large intensities

$$L \mp kT^4,$$

a very simple and suggestive relation.

The Purkinje Phenomenon is implied in several of the relations given above. Mathematically considered, this phenomenon involves the rate of change of luminosity or of visual brightness with energy. This should be greater for red and yellow than for blue and green. However it is demonstrated, the phenomenon rests ultimately upon the variation, $V(E)$, of visibility with intensity. If this variation did not exist there could be no Purkinje or other allied phenomenon.

The simplest demonstration is given by the ratio of two luminosities $L = EV$ and $L_0 = E_0 V_0$, the latter of which is very small. Then

$$\frac{L}{L_0} = \frac{E}{E_0} \frac{V}{V_0}.$$

But $V : V_0$ is always greater for red than for blue and green light (see Fig. 2) since the parameter a decreases and λ_m increases continuously with increase in E. Hence a given variation in E will produce a much larger variation in L in the red than in the blue or green, judging by the data of Fig. 2, it might in some cases be 50 or 100 times as great. Taking instead of $L : L_0$, $dL : dE$ or $dB : dE$ leads after substitution to the same result.

Fechner's Law. — Fechner's law states that to produce a constant increment to a sensation requires a constant percentage increment to the stimulus. In this case the law will give $\delta L : L =$ const., where δL is the least perceptible luminous increment. For moderate and high intensities, this constant is about 0.016. But at the threshold of vision, evidently the least perceptible increment is the whole, hence $\delta L : L$ approaches the value unity and Fechner's law $\delta L : L =$ const. cannot hold at these low intensities.

To extend Fechner's law to cover these low intensities we must consider $\delta L : L = P$, the general photometric function discussed above. Hence, if the function $P(L)$ there constructed be correct, the complete form of Fechner's law is

$$\frac{\delta L}{L} = P_m + (1 - P_m)\left(\frac{L_0}{L}\right)^c$$

which is the simple general form

$$\frac{\delta S}{S} = a + bL^{-c}.$$

THE TIME FUNCTION.

Upon visual brightness as a function, $B(t)$, of the time depend persistence of vision and other allied phenomena. Since data in this field is of considerable accuracy and obtained with the simplest apparatus, a function relating brightness to a time is of great interest.

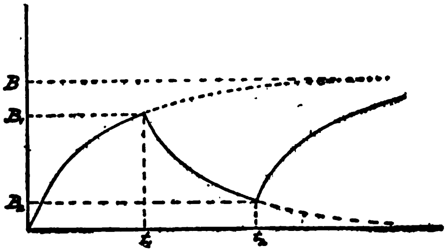

Fig. 4.

$B(t)$ we know to be of the general form shown in Fig. 4, starting at zero, increasing rapidly at first and then less rapidly, and reaching a constant value after a short time.

$$B(t) = B(1 - e^{-at})$$

satisfies these conditions.

If now the eye be illuminated by light passing through a rotating sectored disk, $B(t)$ will be cut off after a time p_1, and again renewed after a further interval p_2 and so on. Let B_1 be the value attained by $B(t)$ after a period p_1, and B_2 the value if screened for a time p_2 after having attained the value B_1. Then

$$B_1 = B(1 - e^{-a_1p_1})$$
$$B_2 = B_1e^{-a_2p_2}.$$

If the speed of the sector be varied until the sensation of flicker just vanishes, obviously

$$B_1 - B_2 = PL$$

the photometric constant measured as an absolute increment of luminosity. But

$$B_1 - B_2 = B(1 - e^{-a_1p_1})(1 - e^{-a_2p_2})$$
$$= B(1 - e^{-ap})^2$$

if $a_1p_1 = a_2p_2 \equiv ap$, that is if the segments cut from the disk equal those remaining and if initial rate of growth equals rate of decay of visual impressions. Hence, under these conditions

$$PL = B(1 - e^{a-p})^2$$

In the preceding work both P and B have been set up as functions of L and certain constants. Hence if L is a constant in any variation of conditions (say wave length) the critical frequency, p, must be constant as observed by Ferry.[1]

Substituting the values of P and B found above

$$(1 - e^{a-p})^2 = \frac{PL}{B} = \frac{P_mL + L_0^c(1 - P_m)L^{1-c}}{B_0 \log(1 + P_m(L^c - L_0^c))}$$

a form rather too complex to be useful as it stands. But the quantity in parenthesis is small so that its expansion in power series converges very rapidly. Further, the right hand member at moderate and high intensities is approximately proportional to $L/(\log L + a)$, hence

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[1] E. S. Ferry, Am. J. Sc., 44, 192–207, 1892.

$$a^2p^2 = k\frac{L}{\log L + a}$$

is an approximate form of the above.

LUMINOUS EFFICIENCY.

It is the spectral integral of luminosity L to which the illuminating power and hence the commercial value of radiation is proportional. It may be well here to summarize the relations subsisting between these quantities and luminous efficiency.

Suppose the spectral *energy* $E(\lambda)$ of the radiation be given in watts per s.-m. per unit solid angle ω. From this suppose the spectal *luminosity* $L(\lambda)$ derived in arbitrary units (say *lumens*) per s.-m. per unit solid angle by means of the subsidiary visibility function $V(\lambda)$. Then

Specific Emission $= \int_0^\infty E d\lambda$ in watts per unit solid angle.

Total Emission $= \int_s d\omega \int_0^\infty E d\lambda$ in watts.

Specific Illumination $= \int_0^\infty L d\lambda$ in lumens per unit solid angle.

Total Illumination $= \int_s d\omega \int_0^\infty L d\lambda$ in lumens.

Luminous Efficiency $= \dfrac{\text{total illumination}}{\text{total emission}} =$ etc., lumens per watt.

Since the space distribution of radiation and illumination must be identical, the space integrals may be dropped in calculating efficiency. And since $L = EV$.

$$\text{Luminous efficiency} = \frac{\int_0^\infty EV d\lambda}{\int_0^\infty E d\lambda}$$

the numerical value of which therefore depends upon the unit V_1 of visibility adopted in $V = V_1 e^{-a(\lambda - \lambda_m)^2}$ for $\lambda = \lambda_m$. This is the value assigned to the luminosity when the radiation is one watt per unit wave-length at $\lambda \equiv \lambda_m$.

Luminosity and Temperature. — If the spectral energy $E(\lambda)$ is known in functional form, say the Wien function, taking $V(\lambda)$ as derived gives the luminosity $L(\lambda)$ in terms of wave-length and certain parameters including temperature. The total luminosity then, which is the definite integral of $Ld\lambda$, is in the form of a function of these parameters. Unfortunately the Wien function itself leads to a form

$$L = EV = C_1 V_1 \lambda^{-5} e^{-c_2/\lambda T - a(\lambda - \lambda_m)^2}$$

which is very difficult or impossible to integrate. Once the integral of $Ld\lambda$ is obtained, visual brightness follows as a linear function of the logarithm of this integral.

Quality of Illumination. — Light is "white" when (*a*) $E(\lambda) =$ constant and (*b*) light of three or more colors is combined in certain proportions. According to the most widely accepted theories of color vision, the visibility curve $V(\lambda)$ is a composite of three elementary visibility curves such that

$$V = V_1 + V_2 + V_3.$$

These elementary curves appear to be indeterminate at present chiefly because there are more variables to be determined than equations to determine them. By assuming them of equal area (König and Dieterici) or equal maximum ordinate or of exactly the same form they may be deduced by means of the observed sensibility of the eye to slight color differences. Assuming them to be known, and taking $L_1 = EV_1$, etc., evidently the criterion for white light is that

$$\int L_1 d\lambda : \int L_2 d\lambda : \int L_3 d\lambda = \int V_1 d\lambda : \int V_2 d\lambda : \int V_3 d\lambda,$$

whatever the form of the spectral energy curve $E(\lambda)$.

SUMMARY.

1. Radiation $E(\lambda)$ may be translated into light $L(\lambda)$ by means of the relation $L = EV$ where $V(\lambda)$ is the visibility function of the simple form

$$V = V_1 e^{-a(\lambda - \lambda_m)^2}.$$

The parameter V_1 is arbitrary while for low intensities $a = 4$ and $\lambda_m = 5.1$ s.-m. a decreases and λ_m increases steadily with increasing intensity, reaching the values $a = 2$ and $\lambda_m = 5.6$ at an intensity of about 100 m. c.

2. The least perceptible percentage differences in luminosity to which the eye is susceptible is closely represented by the function

$$P(L) = P_m + (1 - P_m)(L_0 : L)^c,$$

where $P_m \underset{\cdot}{=} 0.016$ and $c \underset{\cdot}{=} 0.26$ and L_0 is the threshold value of the luminosity. P_m and c are nearly if not quite independent of the individual and of the color.

3. Fechner's law $\delta L : L =$ constant, holding for moderate and high intensities, may be extended to cover low intensities as well if written in the form

$$\frac{\delta L}{L} = P_m + \frac{1 - P_m}{(L : L_0)^c}.$$

4. Visual brightness or apparent luminosity appears to be a function of the form

$$B = B_0 \log.(1 + P_m(L^c - L_0^c)).$$

No attempt has been made to derive these functions analytically from physiological data, such attempts would but obscure the points at issue. And as the methods of synthetic function theory are as yet largely those of cut-and-try, the functions here obtained are little more than tentative at best. Still they are vastly more than empirical formulæ and of more value than any equations derived from largely hypothetical physiological data.

My object has been not so much to arrive at the final form of these functions as to show the essential relations between them. When better data, covering a wider range and more individuals has been obtained, the fundamental functions and their parameters may be more firmly established.

Bureau of Standards,
Washington, D. C.,
November, 1906.

STANDARD CELLS.

By K. E. Guthe and C. L. von Ende.

1. According to recent investigations electrolytically prepared mercurous sulfate seems to be superior to chemically prepared sulfate as the depolarizer in standard cells[1] and the first results of a series of observations on cadmium cells[2] containing this substance showed for a period covering several months a remarkable agreement of their electromotive forces. For this reason one of the authors made use of Clark and cadmium cells with electrolytic mercurous sulfate for the determination of the electromotive forces of standard cells by absolute measurement.[3]

These cells have been transported from Washington to Iowa City, being carefully protected against mechanical disturbances. During the past winter they stood, however, for some weeks in diffused light and were exposed for a few days to large temperature variations. A comparison of the cells after this time showed that the electromotive force of the cadmium cells had decreased relatively with respect to that of the Clark cells, but accurate measurements could not be made at that time on account of lack of sufficiently sensitive instruments. The comparisons reported on in this paper were made with a calibrated Wolff potentiometer and a galvanometer allowing with certainty a reading to the fifth decimal place.

The cells were placed in a thermostat whose temperature could be kept constant at 25° C. within 0.02 degrees. The temperature was measured by means of a thermometer subdivided into fiftieths of a degree and calibrated at the Bureau of Standards. Table I. shows the relative change of the electromotive forces of the cells in the interval of a year. The R series are Clark cells with electrolytic mercurous sulfate, C_3 and K_{10} cadmium cells with chemically prepared sulfate and the rest cadmium cells with the electrolytic material. More detailed information as to the construction of these cells may be found in the Bulletin of the Bureau of Standards, vol. 2, p. 60. Some of the cells have been set up by Professor Carhart, some by Professor Hulett. As reference value in the following table the electromotive force of the Clark cell as determined by absolute measurement was chosen, namely, 1.42040 volts at 25° C.

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[1] Wolff, Trans. Amer. Electroch. Soc., 5, 49, 1904. Carhart and Hulett, ibid., 5, 59, 1904. Hulett, Zeitschr. f. physik, Chem., 49, 483, 1904; PHYS. REV., 22, 321, 1906. Compare also: Reichsanstaldt, Zeitschr. f. Instr. kunde, 26, 120, 1906.

[2] Carhart, Trans. Internat. Electr. Congr. St. Louis, vol. 2, 125, 1904.

[3] Guthe, Bull. Bur. Standards, vol. 2, 33, 1906.

Table I.

Old Cells.

	Date of Construction.	September 17, 1905.	October 15, 1906.	November 28, 1906.	January 12, 1907.
R_2	May 1904.	1.42040	1.42040	1.42040	1.42040
R_4		40	40	40	41
C_3	Nov. 1903.	1.01857	1.01842	1.01844	1.01842
E_2	Jan. 1904.	31	04	01	1789
F_8	Feb. 1904.	27	19	22	1822
F_9		28	08	07	1799
K_6	July 1904	33	24	25	1826
K_{10}		33	11	12	02
O_1	Feb. 1905.	33	23	23	24
O_2		33	19	22	21

The change in most of the cadmium cells amounts to about 0.0001 volt. Since the potentiometer was carefully calibrated and the thermometer correction known, this change is larger than the possible error in the apparatus. The relative values of F_8, K_6 O_1 and O_2 have remained about the same as last year, but for the others the decrease has been larger and quite irregular.

2. To decide the question if a change had taken place in the Clark cells chosen as reference standards it became necessary to construct new standard cells. At the same time we wished to ascertain how closely such cells would agree in electromotive force if constructed by different observers and with material from different sources. No such comparative investigation has yet been published so far as we know.

Professor Hulett kindly sent us some of his electrolytic mercurous sulphate and we prepared two different lots following closely the method published by him. We shall designate these two as lot 1 and 2 respectively. The sulfate was made from pure mercury in an acid prepared by mixing one part of concentrated sulfuric acid of specific gravity 1.84 with six times its volume of distilled water. The current density was between two and three amperes per 100 square centimeters. The first lot consisted of very fine crystals; the second was stirred longer after the breaking of the current and was therefore coarser.

The sulfate was filtered in a Gooch crucible and after the acid had been thoroughly drained off, was washed repeatedly with alcohol and saturated zinc or cadmium sulfate solution. We found it necessary to be very careful in washing, especially with the finest crystals. After the last washing with the salt solution the top layer was removed and the paste prepared in the usual manner.

The mercury was either distilled by Hulett's method[1] or distilled twice in vacuo in an ordinary mercury still.

The zinc amalgam was prepared in the usual way by dissolving zinc in mercury. In some of the cadmium cells we used an amalgam prepared electrolytically.

CLARK CELLS.

3. Two series were set up, the *a* and *b* series. In the *a* series the zink amalgam contained 9 per cent. of zinc. Our mercurous sulfate No. 1 was used in the *a* series. In the *b* series the amalgam contained 10 per cent. of zinc, and Hulett's mercurous sulfate was used. Mercury distilled by Hulett's method was used in cells a_1, a_2, b_1 and b_2, the other mercury in a_3 and b_3. The zinc sulfate crystals were recrystallized from a neutralized solution of Schuchardt's c. p. sulfate and the solution in the cells made from these crystals.

In table II, the electromotive forces as determined on different days are tabulated. The values for the cells with the very fine mercurous sulfate were at first somewhat low, but increased rather suddenly between the ninth and fourteenth day, then slower until they reach a value a little higher, but still agreeing well with that of the old Clark cells. The *b* cells, the mercurous sulfate of which had not been as difficult to wash, had immediately after construction a value agreeing within 0.00003 volt with that of the old cells and remained constant ever since.

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[1] Hulett and Minchin, Phys. Rev., 21, 388, 1905.

The great difference in the size of the mercurous sulfate crystals had apparently no great influence upon the final value of the electromotive force ; the two different samples of mercury behaved electrolytically exactly alike.

TABLE II.

Clark Cells.

	Date of Construction.	Oct. 14, 1906.	Oct. 20, 1906.	Oct. 26, 1906.	Nov. 6, 1906.	Nov. 20, 1906.
a_1		1.42025	1.42029	1.42037	1.42041	1.42043
a_2	Oct. 13, 1906	—	—	—	40	43
a_3		24	30	38	42	43
b_1			38	39	38	38
b_2	Oct. 19, 1906		37	40	39	38
b_3			37	39	38	38

	Date of Construction.	Dec. 5, 1906.	Dec. 24, 1906.	Jan. 19, 1907.
a_1		1.42044	1.42045	1.42045
a_2	Oct. 13, 1906	44	45	45
a_3		45	44	45
b_1		38	38	—
b_2	Oct. 19, 1906	38	38	38
b_3		37	38	38

We also prepared some cells in which the directions were not followed very closely. In a_4, a_5, and a_6 the mercurous sulfate (Hulett's) was not thoroughly freed from alcohol which produced a lowering of the electromotive force by over 0.00040 volt without an appreciable change in course of time. Cells b_4 and b_5 were prepared with our mercurous sulfate No. 1, thoroughly washed but it was mixed with about the same bulk of zinc sulfate crystals and the paste made very thin. These cells had from the start an electromotive force 0.00015 volt higher than the normal value and remained perfectly constant.

It is apparent that the treatment of the mercurous sulfate plays a much more important rôle than the sulfate in itself. a_1, a_2 and a_3 as well as b_4 and b_5 were made with our sulfate, but the former cells show quite a different behavior from that of the latter; the same is true of the cells in which Hulett's sulfate was used.

After a few weeks a_3 a_4 and a_5 as well as b_4 and b_5 were placed for three hours on a circuit of 1,000 ohms external resistance Nov. 4 and Nov. 23 respectively. It was hoped that, if free acid had

TABLE III.

Clark Cells.

	Date of Construction.	Oct. 14, 1906.	Oct. 18, 1906.	Oct. 20, 1906.	Oct. 26, 1906.	Nov. 6, 1906.
a_4	Oct. 13, 1906	1.41968	1.41992	1.41992	1.41994	1.41982
a_5		80	1.42004	1.42003	99	80
a_6		87	06	05	1.42005	93
b_4	Oct. 19, 1906			1.42056	1.42057	1.42056
b_5				56	57	56

	Date of Construction.	Nov. 23, 1906.	Dec. 5, 1906.	Dec. 24, 1906.	Jan. 19, 1907.
a_4	Oct. 13, 1906	1.41980	1.41981	1.41980	1.41980
a_5		80	81	81	80
a_6		91	91	91	91
b_4	Oct. 19, 1906	1.42053	1.42054	1.42055	1.42054
b_5		53	54	54	54

been present, this treatment would change the electromotive force. After opening the circuit the electromotive force had dropped about 0.00040 volt, but it returned in about two hours to about its original value and remained constant after that.

CADMIUM CELLS.

4. The cadmium sulfate crystals were obtained from three different solutions; of these *A* was made from Merck's pure cadmium sulfate, *B* from Baker's c. p. material, and *C* from mixed turbid crystals picked out from either *A* or *B*. In the *c* series lot 2 of our electrolytic mercurous sulfate washed with alcohol and saturated cadmium sulfate solution (made from selected clear crystals) was used. The amalgam contained 12.5 per cent. cadmium and was made by dissolving Kahlbaum's cadmium (käufflich) in warm mercury.

Since it is rather troublesome to obtain a sufficient number of perfectly clear crystals from a small bulk of cadmium sulfate solution, we used also somewhat turbid crystals. The crystals in c_1 were selected large clear crystals from solution *C*, in c_2 small selected, somewhat turbid crystals from solution *B*, and in c_3 large selected crystals somewhat cloudy, from *A* and *C*.

In the *d* series Hulett's mercurous sulfate was used, treated the same way as ours in the *c* series. In d_1 and d_2 selected clear crystals were used from solution *C*, in d_3 and d_4 small crystals which had been formed at the bottom of the crystallizing dish from solution *A* and which were washed a few times with distilled water, after the mother liquor had been filtered off. No attempt was made to select the crystals for these cells. The solution in the cells was always made from the crystals used in filling. d_1 and d_3 contain the same amalgam as the *c* cells, d_2 and d_4 electrolytically prepared amalgam.

Table IV. shows that the cadmium cells *c* gave at first a value for the electromotive force of nearly 1.01840 volt. which dropped, however, within a few days by about 0.0001 volt. and remained fairly constant after that. The drop in the *d* cells is, however, considerably larger than in the *c* cells, and especially pronounced in d_3 and d_4. The different methods of preparing the amalgam does not seem to have an effect upon the electromotive force.[1]

Table IV.

Cadmium Cells.

	Date of Construction.	Nov. 3, 1906.	Nov. 6, 1906.	Nov. 10, 1906.	Nov. 12, 1906.	Nov. 17, 1906.
c_1	Nov. 2, 1906	1.01840	1.01838	1.01836	1.01835	1.01835
c_2		38	36	35	34	35
c_3		39	37	36	33	33
d_1	Nov. 9, 1906			1.01842	1.01840	1.01834
d_2				42	42	37
d_3				46	40	28
d_4				47	45	35

	Date of Construction.	Nov. 23, 1906.	Nov. 28, 1906.	Dec. 5, 1906.	Dec. 13, 1906.	Jan. 19, 1907.
c_1	Nov. 2, 1906	1.01834	1.01833	1.01831	1.01829	
c_2		35	34	32	30	1.01829
c_3		32	32	31	28	28
d_1	Nov. 9. 1906	1.01827	1.01823	1.01815	1.01811	1.01805
d_2		33	31	24	19	13
d_3		19	14	10	06	03
d_4		24	19	11	07	02

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[1] Towards the end of December, 1906, we exchanged Clark cell b_1 and cadmium cell c_1 for a Clark and cadmium cell constructed by Dr. Wolff at the Bureau of Standards. Wolff's Clark cell has an electromotive force 0.00006 volt. lower, and his cadmium cell an electromotive force 0.00003 volt. lower than ours of the same type. Dr. Wolff writes that he has found practically the same difference in his comparisons at the Bureau which shows that the cells have not been affected by transportation.

5. In a general way this investigation corroborates Hulett's recently published results,[1] that Clark cells containing electrolytic mercurous sulfate when set up with proper precautions will have an electromotive force of 1.42040 volts at 25° C. within a few one-hundred thousandths of a volt and remain constant in course of time. The cadmium cells show immediately after the setting up of an electromotive force of 1.0184 volts, closely agreeing with the value given by Hulett.

There is, however, one notable difference. Hulett's *A* cells, set up October 21, 1905, have an electromotive force of 1.01843 volts and have kept this value for at least six months, later results not having been published. His old cells (*F* series) are now about 0.00011 volt lower. Hulett, therefore, drew the conclusion that in the course of time the electromotive force slowly decreases and described a number of very interesting experiments which seem to show that the system in the cathode leg of the cells is not in equilibrium. But the *F* cells, though originally of the same high value, decreased to their present low value within a week, and have remained, with one exception, constant since then. A similar rapid decrease is noticeable in the *J* and *K* cells on which Professor Carhart[2] has reported. Hulett explains this difference in behavior by a difference in the depth of the paste. We have made the paste in the cells over two centimeters deep, but nevertheless the initial decrease is rapid. This difference in the behavior of Hulett's and our cells can hardly be due to the mercurous sulfate and we are at a loss to explain it unless it is to be found in the unstable equilibrium between the cadmium sulfate solution and the rest of the material at the cathode or in a difference in the method of construction of which we are not aware. Our results do not extend over a sufficiently long period to decide the question whether or not there will be a continuous slow decrease of the electromotive force in course of time.

After the satisfactory results obtained with the Clark cells the apparent difficulty for different observers to set up cadmium cells, which remains constant from the start and are practically identical in their electrical behavior, seems to indicate that there is still some factor of uncertainty in the method of construction for the latter and this demands further investigation.

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[1] Hulett, Phys. Rev., 23, 166, 1906.

[2] Carhart, l. c.

The large changes in the electromotive forces of the old cadmium cells mentioned above, are, possibly, not due to a continuous gradual decrease in course of time but to sudden changes brought about by the severe treatment to which they were subjected. It is however very gratifying to know that the causes for such changes have had no influence whatever upon the electromotive force of the Clark cells, since the latter agree so closely with the newly prepared cells.

CONCLUSIONS.

1. The electromotive forces of standard cells set up with electrolytic mercurous sulfate by different observers and from different sources agree under otherwise perfectly equal conditions within a few one hundred-thousands of a volt.

2. Mercury twice distilled in an ordinary vacuum still gives the same results as mercury distilled by Hulett's method.

3. Amalgam prepared in the usual way gives the same results as electrolytic amalgam.

4. Clear and cloudy cadmium sulfate crystals produce no difference in the electromotive force of the cadmium cells.

5. Clark cells with electrolytic, coarsely grained mercurous sulfate have immediately after construction an electromotive force of 1.4204 volts at 25° C. within a few one hundred-thousandths of a volt and remain constant in course of time.

6. We have been unable to construct cadmium cells which do not show an identical decrease in electromotive force.

THE STATE UNIVERSITY OF IOWA,
IOWA CITY, January, 1907.

AN OPTICAL DEVICE FOR DEFLECTION INSTRUMENTS.

BY EDWIN F. NORTHRUP.

THE system of a suspended instrument, as a galvanometer, when subjected to vibrations will have a motion that is compounded of a rotation about a vertical axis and a rotation about a horizontal axis. If the system is held between an upper and a lower filament that are under tension, the horizontal component of the vibratory motion may be practically eliminated by accurately balancing the system about the line of the suspension. Sudden horizontal movements will then have little or no tendency to produce rotations.

Furthermore, in d'Arsonval galvanometers the electromagnetic damping forces quickly reduce to rest a system started into oscillation about a vertical axis. Oscillations of the system about a horizontal axis are not damped out by electromagnetic forces and are more difficult to eliminate by balancing or weighing the system. The vertical movements of the image of the scale seen in a telescope blurr the image, and often are so bad as to make it impossible to read the scale. To use suspended instruments in places subject to considerable vibration, the julius suspension is successfully employed, but this apparatus is both costly and cumbersome.

I give here an optical device which I have devised and employed that *entirely* eliminates the vertical component of the movement of the image of the scale. This device used in conjunction with a well-balanced system enables the scale to be easily read when the system itself is in violent vibration about a horizontal axis.

The device is clearly explained by reference to the diagram. Here *B* is a side elevation of a D'Arsonval coil fitted with two mirrors, their faces making an angle of 90 degrees with each other. *A* is a front elevation of the same.

The course of the ray from scale to mirrors and thence to telescope is shown in dotted line. It is easily seen that rotation of the ninety-degree-mirror system about a horizontal axis does not change the direction of the light ray from scale to telescope. Furthermore, if the scale and telescope are vertically raised or lowered together in an arc of which the mirror is the center the image of the scale continues visible in the telescope. It is therefore very easy "to find the scale" as the necessity for the up and down adjustment of telescope and scale when made to move together is dispensed with.

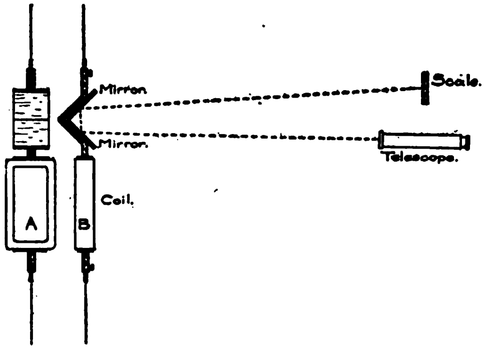

Fig. 1.

The loss of light by the double reflection is not noticeable and the double mirror system operates for deflections about a vertical axis in all respects like a single plane mirror.

If it is required to cast a spot of light on a scale this may be effected by placing a plano-convex lens of suitable focal length, in front of the mirror system.

THEORY OF THE ELECTRODELESS RING-DISCHARGE.

A Correction.

By Bergen Davis.

PROFESSOR R. WACHSMUTH, of Berlin, has kindly written me a letter pointing out a lack of clearness and an error in a paper with the above title published in the Physical Review, March, 1905.

The method there developed of finding the electrical intensity at a point within a flat spiral coil carrying a varying current is not free from criticism, and I take this occasion to give a more complete and simpler solution of the problem.

The final value of the electrical intensity found by the corrected method does not differ materially from the value previously found, and hence does not affect the constants of the discharge as calculated by means of the theory.

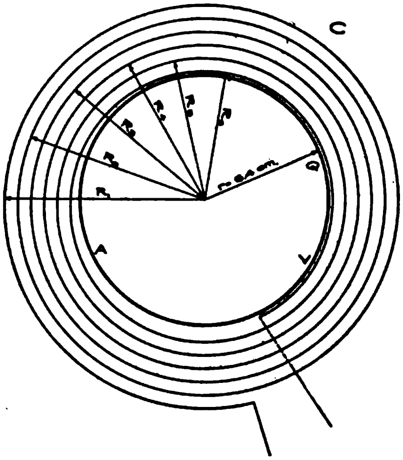

Fig. 1.

The problem is to find the electrical intensity X acting at the inner wall of the vessel AQ when an oscillating current γ flows through the coil C.

The coil consisted of six turns of wire wound in a flat spiral. It was connected to the outer coatings of two Leyden jars, the inner coatings of which were connected to a static machine and a spark gap.

On the passage of a spark a current flowed through the coil, which may be represented by

$$\gamma = \frac{CP}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}}$$

if the damping is neglected.

C is the capacity of the Leyden jars, L the self-induction of the system and P is the potential to which the jars were charged at the instant of the passage of the spark.

The mean radii of the separate turns of the coil were:

$R_1 = 9.3$ cm.

$R_2 = 9$

$R_3 = 8.82$

$R_4 = 8.58$ cm.

$R_5 = 8.34$

$R_6 = 8.1$

The inner radius of the vessel was $r = 6.4$ cm.

It is desired to find the electrical intensity X acting along the inner wall of the vessel AQ.

At any point within the coil

$$\text{Curl } E = -\frac{\partial H}{\partial t}. \qquad (1)$$

Where E refers to the electrical intensity arising from the varying magnetic field due to the current γ in *one* of the turns of the coil.

Then along the wall of the vessel AQ whose length is $l = 2\pi r$, by Stokes' theorem we have

$$\int_0^{2\pi r} E dl = \int\int \text{curl } E ds$$

or

$$E = -\frac{1}{2\pi r}\frac{\partial}{\partial t}\int\int H ds. \qquad (2)$$

$\int\int H ds$ is the total magnetic flux through the circle AQ due to the current in *one* of the turns of the coil. It is also equal to the coefficient of mutual induction M between the circle AQ and one of the turns of the coil multiplied by the current γ. That is

$$\int\int H ds = M\gamma.$$

Introducing this in equation (2) together with the expression for γ and differentiating

$$E = -\frac{1}{2\pi r}\frac{P}{L}\cos\frac{t}{\sqrt{LC}} M.$$

The mean value of which is

$$E = -\frac{1}{\pi^2 r}\frac{P}{L} M. \qquad (3)$$

For the six turns of the coil (3) becomes

$$X = E_1 + E_2 + \cdots E_6 = -\frac{1}{\pi^2 r}\frac{P}{L}\sum_1^6 M. \tag{4}$$

The coefficient of mutual induction between a turn of wire and a circle AQ is given by Maxwell (Elect. & Mag., Vol. II., p. 345).

It may be expressed in the terms of this coil and summed as follows:

$$\sum_{R=1}^{R=6} M = \sum_{R=1}^{R=6} 4\pi r \log\frac{8r}{R-r}\left(1+\frac{1}{2}\frac{R-r}{r}+\frac{1}{16}\left(\frac{R-r}{r}\right)^2-\frac{1}{32}\left(\frac{R-r}{r}\right)^3+\cdots\right)$$
$$+\sum_{R=1}^{R=6} 4\pi r\left(-2-\frac{1}{2}\frac{R-r}{r}+\frac{3}{16}\left(\frac{R-r}{r}\right)^2-\frac{1}{48}\left(\frac{R-r}{r}\right)^3+\cdots\right).$$

Where R has the values R_1, $R_2 \cdots R_6$ and r is the radius of the circle AQ.

Solving the above expression for the various values of R, equation (4) reduces to

$$\frac{X}{P} = .0017.$$

This is the ratio of the electrical force acting on the gas at the inner wall of the vessel to the potential at which the spark passes.

This value of X/P differs but little from the value given in my previous paper which was

$$\frac{X}{P} = .00165.$$

This correction is less than the experimental errors, consequently it will make no material change in the constants of the discharge calculated by means of the theory from the experimental data.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
October 6, 1906.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE THIRTY-FIFTH MEETING.

THE annual meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York City, December 27–29, 1906.

The business meeting of the Society was held Friday morning December 28, President Barus being in the chair. Tellers having been duly appointed the Society proceeded to the election of officers for the year 1907. The following officers were elected:

President. — Edward L. Nichols.

Vice-President. — Henry Crew.

Secretary. — Ernest Merritt.

Treasurer. — William Hallock.

Members of the Council. — H. A. Bumstead and C. E. Mendenhall.

All the other sessions of the Society were joint sessions with Section B of the American Association for the Advancement of Science. President Barus occupied the chair during the morning session of December 28 and both sessions of December 29, while Vice-President Sabine of Section B presided for the session of December 27 and the afternoon session of December 28.

The following papers were presented.

1. A Colorimetric Determination of the Relative Humidity of the Air. HENRY E. WETHERILL.
2. The Kinetic Theory of Gases. LUIGI D'AURIA.
3. The Effect of a Magnetic Field upon the so-called Spontaneous Ionization of Air. W. W. STRONG.
4. The Standard Cell. F. A. WOLFF and C. E. WATERS.
5. The Equilibrium of Mercurous Sulphate and Mercury and Cadmium Sulphate. F. A. WOLFF and C. E. WATERS.
6. Projections of the Sphere Appropriate for Studying the Dynamical Problems of the Atmosphere. (Read by title.) CLEVELAND ABBE.
7. Final Report on Ether Drift Experiments. E. W. MORLEY and D. C. MILLER.

8. The Compensated Two-circuit Electrodynamometer for Alternating Current Measurements of Precision. Edward B. Rosa.

9. The Power Factor and Temperature Coefficient of Mica Condensers. E. R. Rosa and F. W. Grover.

10. The Transmission of Roentgen Rays through Metallic Sheets. J. M. Adams.

11. The Transformation into an Electric Current of Radiation Incident on a Moving Surface. Bergen Davis.

12. Energy necessary to Ionize a Molecule by Impact of Negative Electrons. Bergen Davis.

13. The Influence of Electrical Fields upon Spectral Lines. G. F. Hull.

14. Production of Radium by Actinium. Bertram B. Boltwood.

15. Production of Radium from Actinium. E. Rutherford.

16. On the Conductivity of the Air caused by Certain Compounds during Temperature Change. Fanny C. Gates.

17. Fluorescence Absorption. E. L. Nichols and Ernest Merritt.

18. The Absorption of Some Solids for Light of Short Wave-length. Theodore Lyman.

19. The Distribution of the Energy emitted by a Righi Vibrator. C. R. Fountain and F. C. Blake.

20. Diffraction of Electric Waves of Short Wave-length. A. D. Cole.

21. The Constants in Gas-Viscosity. Willard J. Fisher.

22. On the Variation of the Heat of Mixture with Concentration and Temperature. Bertha M. Clark.

23. A Study of the Reversible Pendulum. John C. Shedd and James A. Birchby.

24. On the Nature of Optical Images. Albert B. Porter.

25. The Mutual Inductance of two Coaxial Coils. E. B. Rosa.

26. The Mutual Inductance of a Circle and a Coaxial Helix. The Lorenz Experiment and the Ayrton-Jones Absolute Electro-dynamometer. E. B. Rosa.

27. Mutual Inductances for Laboratory use. E. B. Rosa.

28. A Color Mixer. Albert B. Porter.

29. Wave-Metrical Measurements with Wireless Telegraph Circuits. (Read by title.) G. W. Pierce.

30. Geometrical Theory of Radiating Surfaces with Discussion of Light Tubes. Edward P. Hyde.

31. The Hissing Point of the Metallic Arc. W. G. Cady.

32. On the Determination of Specific Inductive Capacities by the Gold Leaf Electroscope. (Read by title.) J. C. McLennan.

33. Some Notes on the Magnetic Properties of Heusler's Alloys. J. C. McLennan.

34. On the Magnetic Susceptibility of Mixtures of Salt Solutions. J. C. McLENNAN.

35. Helion, a new Incandescent Lamp Filament. H. C. PARKER and W. G. CLARK.

36. The Magnetic Rotation of Na Vapor at the D Lines. R. W. WOOD.

37. Fluorescence Spectra of Mercury Vapor. R. W. WOOD.

38. Hydraulic Analogy of the Welsbach Mantle and other Radiators. R. W. WOOD.

39. The Shielding of a Highly Sensitive Galvanometer. E. F. NICHOLS and S. R. WILLIAMS.

40. On Magnetic Shielding. A. P. WILLS.

41. On the Temperature of the Mercury Arc. CHAS. T. KNIPP.

42. Remarkable Optical Properties of Carborundum. LEWIS E. JEWELL.

43. Models illustrating the Motion of a Violin String. HARVEY N. DAVIS.

44. The Motion of a Violin String under light Bowing. HARVEY N. DAVIS.

45. Distributions of Ions and of Nuclei in Dust Free Air. (Read by Title.) CARL BARUS.

Adjourned Saturday afternoon, December 29, 1906.

ERNEST MERRITT,
Secretary.

ON EFFECTS OF THE ELECTRICAL DISCHARGE ON THE ACETYLENE FLAME.[1]

BY C. F. LORENZ.

THE observation that the brightness and whiteness of the light emitted by a flame of ordinary illuminating gas was increased slightly by the passage through it of the discharge from an induction-coil, the primary of which was fed with alternating current, led to experiments with various carbon containing flames in order to bring out this effect in a more pronounced manner. It was found that in the case of a cylindrical jet of acetylene issuing under considerable pressure from a small hole the light could be increased about five-fold. The formation of soot, at first troublesome, was avoided by placing the pointed brass electrodes in the invisible mantle of vapor surrounding the flame out of contact with the luminous part, one being placed near the base and the other near the top or middle of the flame on the opposite side. Simultaneous measurements of current, candle-power and voltage were made on a flame having an initial candle-power of 7.5, flame height of 59 mm. and electrode distance of 24 mm., the rate of flow of gas being .34 cubic feet per hour. There was a linear increase of candle-power with current up to a limiting value of 38.0, the current being then 45 milliamperes and the difference of potential between the electrodes 2,700 volts. The difference of potential first increased with increase of current, as in the case of an ordinary resistance and then decreased as in the case of an arc, the maximum value, amounting to 3,000 volts occurring when the current was about 10 milliamperes, this being long before the beginning of appreciable arcing at the electrodes.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

Observation of the flame in a rotating mirror shows that it shortens and brightens each time the discharge passes. The flame, of course, emits a humming sound. On employing direct current, obtained from a number of small generators in series, an almost noiseless steady light resulted. Several more or less useful applications of such a current carrying acetylene flame, which gives a white light strong in the violet end of the spectrum, suggest themselves. For example, it may be applied in cases in which it is desired to produce rapid variations in the intensity of a light by means of small currents, such as microphone currents; the secondary of an induction coil, which may serve at the same time as as a steadying resistance, is put in series with the flame and the variations or interruptions made in a circuit containing the primary.

THE RADIATION CONSTANTS AND TEMPERATURE OF THE NERNST GLOWER.[1]

BY C. E. MENDENHALL AND L. R. INGERSOLL.

FROM a spectrophotometric study of a Nernst glower at several known temperatures (the melting-points of gold, palladium and platinum) the constants in the isochromatic radiation formula have been determined, on the assumption that the equation is of Wien's form: $\log E = K_1 + K_2 1/T$. This establishes a radiation-temperature scale for the glower, and by exterpolation the temperature of the glower with normal or any desired power consumption can be estimated. Experience has shown that the isochromatic radiation function gives about the most reliable method of estimating extremely high temperatures. The normal temperature is found to be about 2,300 abs., disagreeing with the recent determination of Hartman, 1,800° abs., which can readily be shown to be too low. The use of a Nernst filament as a meldometer for determining very high melting points is also discussed, as also the relative advantages of radiation and thermoelectric methods for determining flame and other high temperatures.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

CHANGE OF PHASE DUE TO THE PASSAGE OF ELECTRIC WAVES THROUGH THIN FILMS, AND THE INDEX OF REFRACTION OF WATER FOR SUCH WAVES.[1]

BY WM. R. BLAIR.

I. The apparatus for the production of the waves consists of a high frequency device and a finely adjustable gap in oil. The interferometer is of the Michelson type, paraboloidal reflectors being used about the gap and the receiver. Two receivers are used, one receiving the direct radiation from the gap, and the other the energy of the interfering waves. They are connected with the galvanometer so as to oppose each other. The time during which the current passes through the galvanometer is the same for all readings, being controlled by a pendulum.

II. It is found that the change of phase is not a linear function of the thickness of the film used.

III. These results seem to make it necessary to modify the conclusions drawn from previous experimental work upon thin wedges of absorbing media.

ON THE MAGNETIC BEHAVIOR OF CERTAIN ALLOYS OF NICKEL.[1]

BY BRUCE V. HILL.

MANY nickel alloys are of great importance. Such are the nickel steels and a number of alloys used as resistances. The thermal, mechanical and electric properties seem to be closely connected with the magnetic. These relations may be accounted for by the allotropic theory.

Since the depression of the transformation point of nickel in an alloy is very like that of the freezing point of a solvent by a solute it was sought to find whether the formula of van't Hoff

$$\Delta t = .02\, T^2 a / ML$$

does not apply to the case of alloys. Alloys of nickel with copper, tin and silver were studied and the results are tabulated together with those of other observers upon other alloys of nickel. It is seen that the depression of the point of transformation of the nickel is not inversely proportional to the atomic weight and no simple relation has been found.

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

THE VISCOSITY OF WATER AT LOW RATES OF SHEAR.[1]

BY LAWRENCE E. GURNEY.

BECAUSE of its bearing on the theory of fluids an attempt was made to measure the coefficient of viscosity of water at much lower rates of shear than have been used hitherto. A ring of water 1 cm. thick, 5 cm. in radius and about 5 cm. in effective length was enclosed between concentric brass cylinders; the outer one rotating at uniform speed — the inner one hung upon a small steel wire. End effects and the constants of the suspension were eliminated experimentally. Three rates of shear were used of about 5, 1.3, and .6 radians per second. The results of these three series were self-consistent within .3 per cent. and within experimental error appeared to agree with the values obtained at ordinary rates of shear.

Prof. A. W. Duff (Phil. Mag., May, 1905) describes experiments from which he concludes that the soluble constituents of glass may enormously increase the viscosity coefficient of water at low rates of shear. This conclusion was tested by the method given above and was not confirmed.

Incidental observations were made showing that under certain conditions the surface film of water may behave like a rigid body.

ON DISTRIBUTIONS OF NUCLEI AND IONS IN DUST-FREE AIR.[2]

BY CARL BARUS.

I HAVE recently found it desirable to gather my data together for comparison. There is in fact a serious discrepancy between Mr. C. T. R. Wilson's results and mine when reduced to the same scale. Mr. Wilson's supersaturations for negative ions and cloud are distinctly higher, which certainly can not mean that my fog chamber is in these regions inferior to his own. Thus in moderately ionized dust-free air my condensations begin at a drop of about 18.5 cm. from 76 cm. as compared with 20.5 in Wilson's apparatus; similarly my fogs begin at the drop 20.3, Wilson's at 27.7. Furthermore at low ionization even the vapor nuclei of dust-free wet air become efficient in the presence of ions. It seems impossible therefore that any positive ions should fail of capture.

Notation. — The whole case may best be represented graphically. In my apparatus, however, the adiabatic volume expansion v_1/v is a troublesome datum to compute; it appears as

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[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

[2] Read by title at the New York meeting of the Physical Society, December.

$$\frac{v_1}{v}=\frac{(p-\pi)\left(\frac{p'-\pi'}{p-\pi}\right)^{\frac{k-c}{k}}\left(1-\frac{\pi_1'-\pi_1}{p_1-\pi_1}\right)^{c/k}+\frac{v''}{V}}{(p_2-\pi)\left(1+\frac{v''}{V}\right)},$$

where p and p' are the pressures in the fog and vacuum chambers before exhaustion, p_2 their common pressure when in communication after exhaustion, always at the same temperature. p_2 is very nearly the adiabatic pressure. The volume ratios of the chambers is $v''/V = .064$; the π's denote the different vapor pressures and k and c the specific heats. With a large vacuum chamber the approximation

$$v_1/v=[(p-\pi)/(p_2-\pi_1)]^{c/k},$$

suffices, so that if $\delta p = p - p_2$, the convenient variable for the comparison of exhaustions is the relative drop $[\delta p-(\pi-\pi_1)]/(p-\pi)$. This is used in the diagram with the approximate equivalent of v_1/v.

Graphs. Dust-free Air. —The chart contains a number of curves, showing the nucleation in thousands per cubic centimeter (computed from the angular diameter of coronas) in terms of the exhaustion. Typical cases are given, in their lower parts only. Thus the curve for the vapor nuclei of dust-free air begins appreciably below .25 ($v_1/v = 1.22$; adiabatic drop from 76 cm., 20.3 cm.), but it hugs the axis until about .31, after which it sweeps upward far beyond the chart into the hundred thousands and millions. The position of Wilson's negative ions and positive ions are indicated at .27 and above .31. Wilson's fog point would lie at .365 in the chart and there would be an air curve to the right beyond.

Weak Ionization. — If a weak ionizer (radium 10,000 X, 10 mg., sealed in an aluminum tube) is placed at $D = 40$ cm. from the glass fog chamber, the curve rises slightly above .24, becomes nearly constant slightly below .26, until above .29, after which it also begins to sweep with great rapidity into the hundred thousands of nuclei. In other words at weak ionization, the vapor nuclei of dust-free wet air become efficient in the presence of the ions. There are but two steps in the curve the initial one scarcely leaving the axis, the other at a height of about $n = 15,000$.

Moderate Ionization. — Let the radium tube be attached to the outer surface of the fog chamber. The curve obtained begins appreciably slightly above .22 ($v_1/v = 1.19$, adiabatic drop from 76 cm. about 18.4 cm.) but it scarcely rises until above .24. From this point it also sweeps upward, but can not get much above 80,000 nuclei per cubic centimeter which condition is reached at about .27. To make this curve rise into the hundred thousands, *i. e.*, to make the vapor nuclei of dust-free wet air efficient in the presence of the ions, the exhaustion must be carried to about .50, much beyond the lateral limits of the diagram; but the fog is then intense and without coronas. No other apparatus has heretofore succeeded in catching the nuclei of dust-free air in the presence of such ionization. Again there are but two steps, one very near the axis not appreciably influenced by the greater ionization and the other above $n = 80,000$. Persistent nuclei are not produced, however long the exposure.

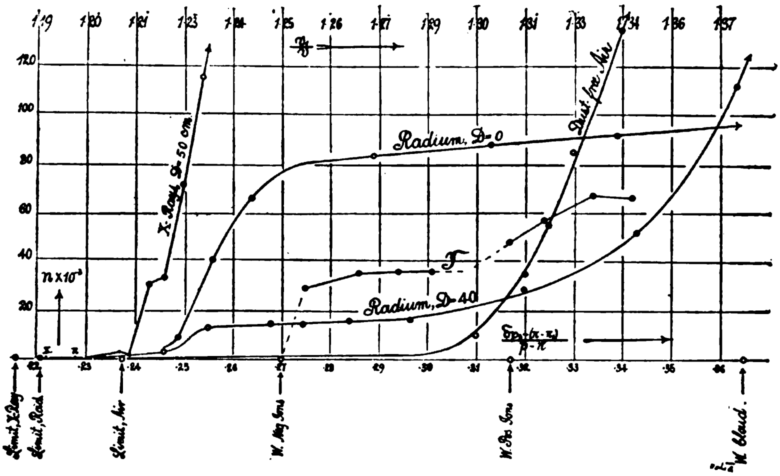

Fig. 1.

Strong Radiation. — If an ordinary X-ray bulb (4 inch spark) is placed at a distance of about 50 cm. from the fog chamber, the condensation produced begins appreciably somewhat below. 22 ($v_1/v = 1.18$; adiabatic drop from 76 cm. about 18 cm.); but the graph scarcely rises until nearly .23, when the upward sweep into the hundred thousands begins. Exposure of a few seconds produces fleeting nuclei only; exposure of one or more minutes produces persistent nuclei. In spite of intense ionization, the first step (.22 to .24) near the axis has scarcely risen, the other is indefinitely high beyond the reach of coronas.

Other Nucleations. — I have ventured to place J. J. Thomson's data (Phil. Mag., vol. V, 1903, p. 349) in the same chart at *T*. They must be interpreted, however, relatively to Wilson's points (negative ions $v_1/v = 1.25$, positive ions 1.31, cloud 1.38). In relation to the other curves of the chart Thomson's graph must be shifted bodily toward the left until the lower and upper steps of the curve correspond with the other cases. In none of the experiments made with my apparatus does the initial step (which should correspond to Thomson's branch for negative ions), rise much above the horizontal axis, no matter how intense the ionization. This rise begins at about .24 in the chart and continues thereafter in a way to correspond with the ionization.

The diagram is too small vertically to include Thomson's second group of experiments in which the initial steps ($v_1/v < 1.33$) lies at an average height of $n = 85,000$.

Conclusion.—The ratio of one half in the height of the steps occurring in the region of transition from negative to positive ions, the accelerated motion of the piston producing exhaustion, and the agreement in size of predicted and observed ions, is the strong evidence of Thomson's interpretation. Nevertheless I am not prepared to abandon my own apparatus. H. A. Wilson with his method obtains a division of ions into three or more groups. What I get is another division of the totality of ions into two groups, one a numerically small group with large nuclei, and a numerically large group with relatively small nuclei and containing all the ions. This occurs even in such cases where I catch the vapor nuclei of dust free air in presence of the ions (radium at $D = 40$ cm.), and hence all ions, positive and negative, must have been caught in an earlier stage of the exhaustion.

The slopes of the air graph and the strong X-ray graph represent the initial branches of a general law of distribution of molecular aggregates, such as is given by the theory of dissociation. They may therefore be expected to be similar in their slopes as they actually are.

The question is finally to be asked why I catch the negative ions, etc., at an apparently much lower supersaturation than C. T. R. Wilson. I have entertained doubts whether the inertia of the piston in his apparatus is initially quite negligible; whether in any apparatus the computed adiabatic temperatures were actually reached. Nobody has proved it, and the case is worst for tubes. Moreover in every apparatus there must be a limit at which the smaller nuclei of a graded system can no longer be caught in the presence of the larger nuclei. But I do not believe that the real discrepancy will be found in any of these misgivings. It seems to me to be inherent in this; in Wilson's apparatus the results are given from the observed volume ratio v_1/v of adiabatic expansion; in my method the results follow from the observed pressure ratio p/p_1. It seems questionable whether the customary constants by which one passes from one group of data to the other are really applicable, to wet air at very low temperatures. Moreover when the exhaust cock in my apparatus is opened for

$$t = 0 \qquad .25 \qquad .5 \qquad 1 \qquad 2.5 \qquad 5 \qquad \infty \qquad \text{sec.},$$

the isothermal pressures of the fog chamber (caet. par.) read

$$\bar{p}_2 = (57.8) \qquad 53.2 \qquad 52.7 \qquad 52.0 \qquad 51.5 \qquad 50.9 \qquad 50.4 \qquad \text{cm.},$$

so that in the first quarter second of opening the final isothermal pressure p_3 (chambers communicating) is already reached to more than 60 per cent., and my smallest fog chamber holds over 6 litres. One naturally asks whether the importance of this in its bearing on the measurements of the ratio of specific heats k/c, has ever been adequately appreciated.

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Providence, R. I.

On the Variation of the Heat of Mixture with Concentration and Temperature.[1]

By B. M. Clark.

When two liquids are mixed, there is in general either an absorption or a generation of heat, the amount of heat depending upon the relative amounts of the liquids present, and upon the temperature at which mixture occurs.

The variation of the heat of mixture with temperature — for any definite concentration — bears theoretically a simple relation to the specific heats of the components of the mixture and the specific heat of the resultant mixture, and the present experiments were begun with a view to determining experimentally the numerical value of this relation in the case of some of the simpler solutions.

If c and c' represent the specific heats of the components of the solution, then the specific heat K of the solution will not in general be given by the simple law

$$K = c(1 - x) + c'x,$$

but will be greater or less than this value. Calling Q the heat of solution due to the mixture of any two liquids, we have from thermodynamic considerations the following relation

$$-\frac{d(Q_{xt})}{dt} = K - K'.$$

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

When K is the observed specific heat of the resultant solution, K' the specific heat calculated from the above formula. This shows that only where $K = K'$ is the heat of solution independent of the temperature at which mixture occurs.

Observations were made with infinitely soluble as well as partially soluble liquids.

The values obtained for the heat of solution at various temperatures show that for the liquids tested the thermodynamic relation

$$-\frac{d(Q_{xt})}{dt} = K - K',$$

holds within the limits of experimental error. The value $\partial Q/\partial t$, known as the temperature coefficient of the heat of mixtures, varies in some mixtures with the temperature.

Anilin xylol, anilin-toluol gave evidence of a temperature coefficient equal to zero, *i. e.*,

$$\frac{\partial Q}{\partial t} = 0.$$

Numerical values and curves will be given in a paper to appear shortly.

The Constants in Gas-Viscosity.[1]

By Willard J. Fisher.

In 1893 W. Sutherland[2] proposed an explanation, based on the assumptions of the Kinetic Theory, with the additional one of attractive forces between molecules, which has been shown by numerous investigators to agree better with the observed facts of gas-viscosity than any of the empirical formulas. His formula is

$$\eta \propto \frac{\theta^{\frac{1}{2}}}{1 + \frac{C}{\theta}},$$

in which η is the viscosity-coefficient, θ the absolute temperature, C a constant depending on the gas. The formula holds only for temperatures above the critical and pressures such that Boyle's Law is true. Taking

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] W. Sutherland, Phil. Mag. (5), 36, p. 507, 1893.

η_0, the viscosity-coefficient at 0° C., as a second constant of the equation, all writers hitherto have thrown it into the form

$$\eta = \eta_0 \left(\frac{\theta}{273}\right)^{\frac{1}{2}} \frac{1 + \frac{C}{273}}{1 + \frac{C}{\theta}},$$

determining η_0 and C by the method of Least Squares.

Observing that in Sutherland's original article, in terms of molecular quantities, C has the value

$$\frac{1}{c} m^2 f\left(\frac{1}{2a}\right),$$

where m is the mass of a molecule, a its radius, $c/2$ the factor converting absolute temperatures into molecular kinetic energy, and f symbolizes the unknown law of potential energy between molecules, the writer plotted a curve between C and m^2, using the relative molecular weights of Chemistry; and finds that the elements which have been studied through sufficiently wide ranges of temperature give an approximately straight line, with equation $C = 0.058M^2 + 74$; in which M is the relative molecular weight.

Combining this with Sutherland's expression for C, for these gases,

$$f\left(\frac{1}{2a}\right) = \frac{c}{h^2}\left(0.058 + \frac{74}{M^2}\right),$$

in which h is the mass in grams of a Hydrogen atom. It is to be noted that the values of C are in considerable disagreement, as given by the different authors.

Also, instead of writing the equation with the variation sign, it may be put in the form

$$\eta = K \frac{\theta^{\frac{1}{2}}}{1 + \frac{C}{\vartheta}}$$

The proportionality constant K, hitherto unnoticed, has the value by Sutherland's article

$$\frac{.064 c^{\frac{1}{2}} m^{\frac{1}{2}}}{(2a)^2}$$

which suggests that K should be linear in $m^{\frac{1}{2}}$. Determining K from published data for various elements and compounds by a graphical method, it is found that no obvious law holds for the compounds, but that the four elements, H, N, O, Ar, give a straight line of formula $K \cdot 10^7 = 0.593M^{\frac{1}{2}} + 66$, in which M is the relative molecular weight.

Helium, in this respect, as in several others, does not form part of the series, and Argon deviates about 4 per cent.

No other elementary gases have been studied under conditions such as to justify farther conclusions.

REMARKABLE OPTICAL PROPERTIES OF CARBORUNDUM.[1]

By LEWIS E. JEWELL.

SOME transparent blue plates of carborundum, found in 1904, gave by preliminary measures with a microscope a very high value for the refractive index.

Later, in material received from the Carborundum Company of Niagara Falls, were found some transparent and colorless plates, some of which were fairly thick. The polarization of the plates was studied with the polariscope and one specimen of twinned crystals produced a well-defined, transparent, colorless prism where the plates were joined.

Measurements with the spectrometer were made and the refraction proved to be greater than that of the diamond, and the dispersion more than twice as great.

Spectrograms were obtained of the light transmitted through the plates and a thickness of one thirtieth of a millimeter absorbed practically all light having a wave length less than 4,000 Angström units.

Interference bands in the spectrum of light transmitted through thin parallel plates gave greater values of the dispersion than the measurements with the spectrometer. Spectrograms have also been obtained of the light reflected from the surfaces of the crystals and the reflective power, which is extremely high, differs very little in the visible and ultra-violet portions of the spectrum until a wave-length of about 2,400 is reached, beyond which there seems to be an absorption band.

There may be a slight increase in the reflective power in the violet and ultra-violet, but it is not very pronounced. After the measurements were made some better material was obtained and larger and much more perfect prisms have been made, with which much more accurate values of refractive indices are to be made, also the interference fringes in the spectra produced by carefully selected plates are to be used for this purpose also.

Very remarkable markings have been found upon some plates, the most remarkable of which take the form of spirals, some of which are hexagonal and others triangular, changing to hexagonal farther out. Most of the spirals are however circular, some of them are so close together and so regular as to give brilliant spectra, even to the sixth or eighth order; the lines in some cases are as close together as thirty or fifty thousand to the inch. Many of these markings are extremely beautiful.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

Sometimes there are two outer coatings to the plates, the outer an oxidation product giving the brilliant surface colors which are due to a thin film. Sometimes there is another layer with an exceptionally brilliant silver-like surface. Either or both of these coatings may be present and in some cases neither one.

The first mentioned coating is comparatively soft and easily removed, the other coating is nearly or quite as hard as the corborundum itself.

THE MUTUAL INDUCTANCE OF A CIRCLE AND A COAXIAL HELIX. THE LORENZ EXPERIMENT AND THE AYRTON-JONES ABSOLUTE ELECTRODYNAMOMETER.[1]

BY E. B. ROSA.

THE constant of the Lorenz apparatus for the absolute measurement of resistance and of the Ayrton-Jones electrodynamometer for the absolute measurement of current require the calculation of the mutual inductance of a circle and a coaxial helix, the circle in the first case being the edge of the rotating disc and in the second case one end of the suspended coil, the helix being the fixed coil carrying the primary current.

Jones obtained an expression for the mutual inductance in question by integrating spirally along the entire length of the helix, his result being an exact expression but appearing in terms of elliptic integrals of all three kinds and hence being a very tedious and difficult expression to use in numerical calculations. Lorenz's expression is an algebraic series less difficult but also less accurate than Jones's. Using an expansion in zonal harmonics, I have obtained an expression in the form of an algebraic series, similar to Lorenz's but more accurate, which is far more convenient to use than Jones's and agreeing with the latter to less than one part in a million (in a particular numerical test) thus being amply accurate for the most refined experimental work. This expression is obtained on the assumption that the current is distributed over the solenoid in a uniform current sheet, whereas Jones's expression assumed the current flowing in a helix. Their agreement is a confirmation of the theorem that a spiral distribution of current is equivalent to a current sheet. For a short coil, however, a question arises as to how to introduce the current into the helix so as to meet the mathematical conditions of equivalence to a current sheet. It is shown in the paper that for such sizes of wire and coils as are likely to be employed in measurements of precision the variation in the constant of the coil due to the lead wires not conforming to the mathematical conditions may be made wholly inappreciable, so that the formula of Jones or the more convenient series formula may be used without any correction for leads.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

Mutual Inductances for Laboratory Use.[1]

By E. B. Rosa.

THE most accurate as well as the most convenient method of calibrating a ballistic galvanometer is by means of a mutual inductance, through the primary of which a measured currrent flows. Some laboratories use a standard solenoid with a secondary wound within or without, calculating the mutual inductance of the primary and secondary coils from their dimensions. Such a solenoid made large enough and carefully enough to give the mutual inductance with fair precision is both bulky and expensive, and not very portable. Being of considerable length, its magnetic field often extends to a considerable distance and may disturb other work. On the other hand suitable primary and secondary coils may be wound on a thoroughly seasoned wood spool (or a marble spool) only 10 or 12 cm. in diameter and afford, when calibrated, an accurate and convenient mutual inductance for the calibration of galvanometers that is cheap, portable and reliable. Such inductances are so inexpensive and occupy so little space that one can be left with its secondary in circuit with the galvanometer, ready to be used for calibrating the latter at any time. They can be made in the laboratory at trifling cost and can be sent by mail or express to the Bureau of Standards and calibrated at very slight expense. Specifications for such coils will shortly be published by the bureau.

The Mutual Inductance of Two Coaxial Coils.[1]

By E. B. Rosa.

ABSOLUTE measurements of resistance have been made by Rowland, Glazebrook and others using two coaxial coils the mutual inductance of which was computed from their dimensions. By the methods employed by Kohlrausch and Rayleigh the mean radius of such coils can be obtained with great precision by comparing them with larger coils wound with a single layer of wire, the mean radius of which may be determined with sufficient accuracy by direct measurement. The formulæ employed by Rowland and Glazebrook in calculating the mutual inductances of their coils are both approximate, and do not agree with one another closely unless the coils have very small sections or are rather far apart. Weinstein and Stefan both gave formulæ for use in calculating the mutual inductance of such coils, but they do not agree with one another as accurately as is desirable for precision work. I have revised and corrected Weinstein's formula and derived a new formula starting where Stefan began, these two formulæ agreeing very closely and proving more accurate than any of the others. These formulæ are sufficiently accurate for the most refined work in the absolute measurement of resistance.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

MODELS ILLUSTRATING THE MOTION OF A VIOLIN STRING.[1]

BY HARVEY N. DAVIS.

THE function $u(x, t)$ which gives the displacement, at the time t, of a point x units from one end of a violin string, can be represented graphically by a surface with the x, t plane as a base-plane. If the units were properly chosen, this graph would be the surface which the string would generate if, while it vibrated, it were also carried along in a direction perpendicular to the plane of its vibration; and in any case, a section of such a surface parallel to the x axis represents, usually on a magnified scale, the configuration of the string at some corresponding time, $t = t_0$, while one parallel to the t axis represents the displacement of a corresponding point, $x = x_0$, as a function of the time.

Five surfaces of this kind, modeled in three dimensions, were shown, representing, one the general Helmholtzian solution and the others the motion of a string bowed at points $\frac{1}{5}$, $\frac{2}{5}$, $\frac{2}{7}$ and $\frac{3}{8}$ of its length from one end. These particular cases were chosen because, besides being typical, they have some interest in connection with Young's and Krigar-Menzel's observations on the absence or dominance of certain overtones when a string is bowed at or near one of their nodes.

THE MOTION OF A VIOLIN STRING UNDER LIGHT BOWING.[1]

BY HARVEY N. DAVIS.

THIS paper discusses the influence which the pressure of the bow upon the string has on the resulting vibration form.[2] For each bowing speed there is, for comparatively great pressures, a considerable range within which the only effect of a change in the pressure is a slight corresponding change in the position of equilibrium about which the string vibrates, the vibration form being always that described by Helmholtz and others, and the amplitude remaining the same. For smaller pressures both the amplitude and the vibration form change with the pressure. In particular, if the pressure is below a certain critical value, determined partly by the materials and condition of the apparatus and partly by the bowing speed, no vibrations can be maintained. For pressures slightly greater than this critical pressure there are no overtones, the time graph of the displacement of the point under the bow being a sine curve tangent at its point of greatest slope to the line representing the speed of the bow. As the pressure is increased beyond this value, the bowing speed remaining constant, the mode of vibration goes over continuously into the Helmholtzian form.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] The vibrations actually observed were longitudinal rather than transverse, the apparatus used being a modification of that described to the Society in December, 1905.

On Magnetic Shielding.[1]

By A. P. Wills.

ASSUMING the results of a previous paper, giving the "shielding ratio" for a set of three concentric spherical iron shells and for a similar set of cylindrical shells, the following problem was discussed: Given the innermost and outermost radii of the system in each case, what values should the remaining four radii have in order that the shielding shall be a maximum? Starting with the expression giving the "shielding ratio" (the ratio of field impressed to field within innermost shell), derived in the paper referred to above, the conditions for a maximum of the expression, under the conditions imposed, are examined; and it is found that approximately the best conditions are obtained when the radii of the shells are in geometrical progression. This holds for both spherical and cylindrical systems.

A Color Mixer.[1]

By Albert B. Porter.

THE simplest means of mixing and matching colors is undoubtedly Maxwell's color-top. It is, however, defective in that one cannot continuously vary the positions of the slotted discs while seeking a color-match, but must usually stop the top many times, readjust the discs, and again set up the rotation, before a satisfactory match is secured.

In the present instrument the slotted discs remain stationary while their image is rapidly rotated. This is effected by viewing the discs through a right-angled prism which rotated with its hypothenuse parallel to the axis of rotation and to the line of sight. As the slotted discs remain stationary, their angular exposure can be easily and continuously varied until the best possible color-match is secured.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

ON THE NATURE OF OPTICAL IMAGES.[1]

BY ALBERT B. PORTER.

A CONSIDERATION of the method by which light is propagated is sufficient to show that an ordinary optical image is merely a particular case of an interference pattern. This may be easily shown experimentally by using as an object a coarse, black-line grating illuminated by a parallel beam of monochromatic light passing through the grating and then through a convex lens. On the far side of the lens a system of sharply defined interference fringes is formed which can be seen with an eyepiece, or intercepted on a screen, at any point over a considerable range along the axis. Somewhere in this system of fringes is the geometrical image of the grating, but it is visually quite indistinguishable from other sections of the fringe system. If the angle of incidence of the light falling on the grating is changed, the whole fringe system shifts to one side or the other except in the focal plane, where it remains stationary. This shows that the focal plane of the lens is merely the plane in which the interference fringes formed by light of all incidences coincide, and that the so-called geometrical image is really a superposition of coincident interference patterns; while the usual absence of a sharp image outside the focal plane is due to the more or less uniform illumination caused by the overlapping of fringe systems formed by light coming from various points in the source. When the grating is illuminated by a parallel beam of white light the effects are similar, except that outside the focal plane the interference fringes are colored. This shows that the focal plane is also the plane of achromatic interference, *i. e.*, the plane in which the fringes due to light of various wave-lengths coincide.

These experiments clearly show why it is in general essential to use a condenser to illuminate the field of a microscope in order to obtain a critical image, *i. e.*, an image which comes sharply into and out of focus and which is hence as free as possible from confusion with details of structure lying above and below the focal plane.

In the case of self-luminous objects, although the geometrical image is still to be considered as an interference pattern, the effects outside the focal plane are greatly modified by the lack of phase relations between the waves given off from various points of the object.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, held December 27–29, 1906.

THE ABSORPTION OF SOME SOLIDS FOR LIGHT OF SHORT WAVE-LENGTH.[1]

BY THEODORE LYMAN.

THE paper deals with light of extremely high frequency whose existence was first discovered by Schumann. This region of the spectrum presents problems of considerable interest, for to those questions of pure spectroscopy — the radiation and absorption of gases and solids and the reflection of metals — must be added problems connected with the photo-electric effect and with the physiological action of light.

The grating spectroscope which the author has used in his measurement of the hydrogen spectrum between 1650 and 1250 Angström units, though well adapted for the determination of wave-lengths is not especially fitted for rapid work in other branches of the subject. A prism spectroscope would be preferable for many purposes on account of its short light path and the superior intensity of its spectrum.

The object, therefore, of the present research was a purely practical one, namely, to find some substance from which the prism and lenses of such an instrument might be constructed. White fluorite is the only known substance fitted for the purpose, and it was hoped that a less costly substitute might be discovered.

In order to test rapidly the transparency of a great number of substances an instrument of special type was constructed. In it the light from a discharge tube fell upon a concave mirror and was thrown by it through a fluorite prism onto a screen coated with willimite. The whole apparatus was enclosed in an air-tight case which could be exhausted and filled with hydrogen. The specimens to be examined were introduced into this case in such a manner that eight of them could be interposed in succession between the mirror and source of light without opening the apparatus. The extent and intensity of the spectrum on the fluorescent screen was observed through a glass window.

To check the results obtained with this instrument the absorption spectra of some of these substances were photographed by a method already described.[2]

The substances examined were: Colored fluorites, quartz, topaz, gypsum, celestite, rock-salt, barite, alum, colemanite, sugar (rock candy), borax, adularia, calcite, chrysoberyl, sanidin, anagonite, apophyllite, diamond and kunzite. The results are:

I. No substance shows so great transparency as white fluorite.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] Astrophysical Journal, Vol. XXIII., No. 3.

II. The transparency of colored fluorites varies through a considerable range, but some light green specimens are nearly as good as the white variety.

III. Colored fluorites may be deprived of their color by heating. The process does not materially alter their transparency.

IV. Quartz in thicknesses of about 1 mm. is transparent to λ 1500. The absorption increases with such rapidity with the thickness that thick prisms and lenses would cut the spectrum off at about λ 1750 for all practical purposes.

V. Of the remaining substances Ceylon topaz is the best, being transparent to λ 1560.

VI. Rock salt is transparent to λ 1750 only.

On the Conductivity of the Air Caused by Certain Compounds During Temperture-Change.[1]

By Fanny Cook Gates.

INVESTIGATIONS on the conductivity of the air caused by the presence of the sulphate of quinine under certain conditions, indicate that it accompanies hydration and dehydration, although phosphorescent effects to which it may be directly due, appear at the same time. The phenomenon is most easily observed during the cooling of the quinine from something over 100° C. to room temperature.

Heretofore, the only other substance known to produce a similar effect is cinchonine, which like quinine, hydrates and phosphoresces upon cooling. Both substances are so complex in structure, that the exact chemical changes to which they are subject are studied with difficulty.

During the investigations described in the present paper, a large number of substances were heated to about 150° C., and the conductivity of the surrounding gas was tested while they cooled. Of all the substances tested, none was found to give so large an effect as that which results from the sulphate of quinine, but a very definite and marked conductivity was found to accompany the cooling of anthracene and to a less degree that of grape-sugar and æsculin.

The Transmission of Röntgen Rays Through Metallic Sheets.[1]

By J. M. Adams.

FOR some years after Röntgen's discovery of the X-rays, it was a matter of doubt whether the absorption of the rays in metallic sheets was accompanied by the development of heat. To investigate this question by means of an instrument different from those already used for this purpose, a radiomicrometer was constructed. The metals forming the thermal couple were constantan and copper. At one of the junctions was placed a small disk of thin platinum to receive the rays, the other junction being shielded from them. The sensitiveness of this radiomicrometer was such that a radiation of 5.6×10^{-8} gm. cal. per sec. per sq. cm. of its sensitive surface produced a deflection of one scale division.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

Evidence of heat developed in the platinum by the absorption of Röntgen rays in it was readily obtained, and a quantitative measurement of the whole energy radiated per second in the form of Röntgen rays by a tube was made and was found to be of the same order of magnitude as the results of those investigators who had been able to observe the heating effect at all.[1] The necessity of making correction for the incomplete absorption of the rays in the platinum of the instrument, together with the well known fact that the character of Röntgen rays is changed by passage through substances, made it seem desirable to investigate the phenomena of the transmission of the rays through metallic sheets more fully than has been done heretofore.

The general law of the absorption of the rays in a metal, viz., that each successive equal increment of thickness is less effective as an absorbing medium than the one preceding it, was confirmed by experiments with the radiomicrometer; and curves showing the relation between the thickness of a metallic sheet and its absorbing power were plotted and were found to have the same general characteristics as similar curves which other investigators had obtained by means of the fluoroscope, the photographic plate, or the ionization electroscope.

The dependence of the absorbing power of a given metallic sheet upon the intensity of the rays incident upon it was examined for sheets of silver, platinum, copper, tin, and aluminium, and in every case it was found that the effectiveness of a sheet as an absorbing medium is independent of the intensity of the incident rays; in other words, the deflection of the radiomicrometer suffered an equal percentage reduction upon the interposition of a metallic sheet in the path of the rays, whether the latter were strong or weak. The rays were weakened by moving the tube away from the rest of the apparatus.

It was found that the effect of the *surfaces* of metallic sheets upon transmission is small, in the case of copper and of aluminium. To show this, a laminated plate of the metal in question was prepared, equal in total thickness to another solid plate of the metal. These two plates were interposed in turn in the path of the rays, and produced equal reductions of the deflection of the radiomicrometer.

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[1] Several recent measurements by other investigators afford additional confirmation of the earlier results, so far as the order of magnitude is concerned.

The transmission of a beam of Röntgen rays through a metallic sheet has generally been supposed to render the beam more penetrating toward a second sheet of the same or of any other metal than the original beam was. Experiments with the radiomicrometer have confirmed this view in most cases; but the effect of transmission through silver on penetrating power for aluminium was found to be very small, while transmission through aluminium appeared to decrease the penetrating power of the rays for silver, so far as the deflections of the instrument are an indication.

Evidence of transformation of one sort of ray into another, in transmission through a metallic sheet, was sought, with a negative result. The experiment was performed by interposing in the path of the rays a plate made of two sheets of different metals placed face to face. If transformation occurs, the deflection of the instrument should depend upon the order in which the rays traverse the two sheets, for the effect of the second metal upon rays produced by the first would not be likely to be quantitatively the same as the effect of the first metal upon rays produced by the second. In fact, the order of the sheets in the plate made no difference in any case.

ON THE MAGNETIC PROPERTIES OF HEUSLER'S ALLOYS.[1]

BY J. C. MCLENNAN.

IN this paper the author describes some experiments made during the past year by Messrs. Dawes, McTaggart, and Robertson, and Miss L. B. Johnston on the magnetostriction and permeability of Heusler's alloys of varying composition.

From measurements on both phenomena the alloys are shown to be in an exceedingly unstable condition when freshly made and from observed changes in their magnetic behavior the conclusion is drawn that profound modifications are made in their structure through the lapse of time as well as by changes in temperature and by repeated magnetizations and demagnetizations.

In connection with the phenomenon of magnetostriction the gradual shortening observed by Austin[2] with rods of the alloys subjected to long continued high magnetic fields was shown to disappear when the rods reached a stable condition after being repeatedly magnetized.

In a series of rods containing the same amount of manganese with varying amounts of aluminium the greatest elongation was observed with rods in which the manganese and aluminium were present in the ratio of their atomic weights.

The paper concluded by emphasizing the suggestion made by Hill[3] that possibly the magnetic properties of these alloys at room temperatures are largely determined by the temperatures from which they have been cooled, and that by suitably heating samples of the alloys to different temperatures and then chilling them their magnetic properties at these temperatures may be ascertained just as the structures of other alloys at different temperatures have been investigated in this way by Neville[1] and Heycock.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] PHYS. REV., 23, p. 498, 1906.

[3] Phil. Trans. A., CXCIV., 201, 1900.

GEOMETRICAL THEORY OF RADIATING SURFACES WITH DISCUSSION OF LIGHT TUBES.[2]

BY EDWARD P. HYDE.

THEORETICAL photometry assumes two general laws of radiation. (1) The law of variation of the intensity of illumination of a surface in inverse proportion to the square of the distance of the surface from the luminous source is merely a statement of a geometrical property, if the rectilinear propagation of light is assumed. (2) Lambert's law of variation of the intensity of a luminous surface in direct proportion to the cosine of the angle of emission is an empirical law based primarily on the observation that a uniformly bright sphere, when viewed at a distance appears as a uniformly bright disk. It would seem to follow from Kirchhoff's law that Lambert's cosine law can be true only for a black body, but no satisfactory experiments have been made so far as the writer knows, to test the law empirically in its application to glowing surfaces.

These two laws, the inverse square law and the cosine law, are applicable to the infinitesimal elements of a radiating surface, and large errors may result if they are assumed to apply to an extended source as a whole. Particularly is this so in the case of the inverse square law, which underlies the great majority of photometric measurements. In a previous paper[3] the writer solved the case for a finite cylinder of uniform brightness, in connection with a study of Talbot's law as applied to the rotating sectored disk. The curve of deviation from the inverse square law for a cylinder 20 mm. long and 1 mm. radius (the distances used in applying the inverse square law being measured to the center of the cylinder) is plotted, and is resolved into two simple curves corresponding to the two extreme cases of (1) a cylinder 20 mm. long but with an infinitesimal radius, and (2) a cylinder with a radius of 1 mm. but having an infinitesimal length.

Another simple illustration of the treatment of a radiating surface, as distinguished from a point source, is had in the case of a uniformly bright circular disk of finite radius, for which the curve of deviation from the inverse square law is plotted.

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[1] Proc. Roy. Soc., LXIX., 320, 1902.

[2] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906. To be published in full in the Bureau of Standards Bulletin.

[3] Bureau of Standards Bulletin, Vol. 2, p. 1, 1906.

In each of the above two cases the expression is deduced for the illumination at different distances in a single plane or along a single line symmetrical with respect to the radiating surface. In the case of a uniformly bright strip of infinite length but of finite width it is not difficult to derive the expression which will give the illumination at any point in space. Instead of illumination, however, the more general term *specific luminous flux* is substituted. At every point in space there is some definite direction in which the flux of luminous energy is a maximum. The quantity of luminous energy which in one second flows normally across a surface of unit area placed perpendicular to the direction of maximum flux is defined as the *specific luminous flux* at the point. It is a vector quantity, and the component in any direction equals numerically the difference in illumination on the two sides of an infinitely thin material screen placed perpendicular to the direction.

Since the problem of the infinite strip is one of two dimensions the complete solution is obtained from a consideration of a single plane normal to the strip and intersecting it in the line MN. Denoting the specific luminous flux by Φ_0 it is found that Φ_0 at any point is in the direction of the tangent at the point to the hyperbola having M and N as foci. If we draw the system of confocal hyperbolas and ellipses having M and N as foci, the hyperbolas intercepting equal distances on the strip MN, and if we remember that this system of hyperbolas and ellipses is a plane section of hyperbolic and elliptic cylinders, the hyperbolic cylinders of unit height may be considered as tubes of light, the specific luminous flux being along the tube at every point.

By deducing the flux of light across successive elliptic cylinders between two consecutive hyperbolic cylinders, it is found that the specific luminous flux is solenoidal, and therefore its divergence is zero. The elliptic cylinders may be considered as level surfaces, and a special form of potential function Ψ, may be derived such that along any *one* hyperbola Φ_0 is proportional to the rate of decrease of Ψ in the direction of the tangent at the point, At the intersection of different hyperbolas with any one ellipse, however, Φ_0 is proportional to the rate of decrease of Ψ in the direction of the normal to the ellipse, multiplied by $\sin y$, where $\sin y$ is the semi-minor axis of successive hyperbolas corresponding to different values of y. It readily follows that the second differential parameter $\Delta\,\Psi = 0$.

Since two examples have been given of the errors incident to assuming for a finite surface the inverse square law, which only applies to the elements of the surface, the above case of an infinite strip is used to show the errors incident to assuming for the strip as a whole the cosine law which is true only for the elements of surface of the strip. The values of Φ_0 at different distances in a direction normal to the strip at its middle point, and in a direction making an angle of 45° with the normal at the middle point are calculated. For any definite distance the former multiplied by cos 45° would equal the latter if the cosine law, which has been assumed for the elements of surface, applied to the surface as a whole. The difference between the two values gives the errors for the distance used, and by plotting the errors for different distances a curve of deviation from the cosine law is obtained.

THE ELECTRODE EQUILIBRIUM OF THE STANDARD CELL.[1]

BY F. A. WOLFF AND C. E. WATERS.

SOME experiments recently described by Hulett[2] indicating the existence of a state of unstable equilibrium in the Weston cell have, on account of the importance of the subject, led to a further study of the question at the Bureau of Standards.

The equilibrium between cadmium amalgam and cadmium sulphate was first studied. Saturated solutions of the latter were shaken up in air, nitrogen, hydrogen and in vacuo with cadmium amalgam, but no difference as great as 10 microvolts was obtained when the treated solutions were employed in setting up Weston cells, even though the samples shaken in the presence of air were cloudy from the basic cadmium sulphate formed.

The equilibrium of mercury, mercurous sulphate and cadmium sulphate, and the corresponding system of the Clark cell was then studied in special cells, so constructed that the above materials could be rotated and the effect determined without opening the cells.

The cell consisted of a tube about 2 cm. in diameter and 12 cm. long, provided at the lower end with a small bulb into which a platinum wire was sealed. The bulb was connected to the main tube by a narrow neck, so that, with sufficient mercury in the cell, the platinum terminal was never in contact with the solution. A shorter internal tube about 1 cm. in diameter with several lateral openings, sealed in at the other end of the main tube, was charged with amalgam and crystals of cadmium or zinc sulphate which were held in place by a plug of asbestos wool. A platinum wire, sealed into a glass tube, to protect it from the solution extended from the amalgam through the upper seal, made after the introduction of the above materials.

The mercurous sulphate or cadmium sulphate or zinc sulphate crystals were introduced over the mercury through a side tube by which the cell was next exhausted and through which a saturated solution of cadmium or zinc sulphate was subsequently introduced. After filling, the side tube was sealed off, leaving only a small air bubble in the cell, thus practically eliminating the influence of the air.

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[1] Abstract of paper read at the New York meeting of the American Physical Society, December, 1906.

[2] PHYS. REV., 23, 166, August, 1906.

The cells were then placed in holders so arranged that one dozen could be rotated simultaneously with their axes parallel to the axis of rotation and inclined at a small angle to the horizontal.

Altogether 12 Weston and 5 Clark cells of this type were set up with samples of mercurous sulphate, made not only by the electrolytic method but also by several chemical methods.

The Weston cells after rotating from one to two months in a constant temperature oil-bath show changes which do not exceed 0.01 per cent. for all the gray samples (the grayness probably due to very finely divided mercury) except in one case in which the platinum terminal was purposely exposed to the solution.

The white samples generally showed effects several times as great as the gray ones, but not as large as found by Dr. Hulett. There also seems to be a tendency to reach a maximum, and then to approach normal values.

One Weston cell set up with basic cadmium sulphate has remained as constant as any of the others, its value being a few parts in 100,000 below normal.

The Clark cells showed effects of the same nature as the Weston, but in general the differences were even less, particularly for the white samples of mercurous sulphate.

No initial low values, as described by Hulett, were ever obtained.

The above results are given to show that *not all* samples of mercurous sulphate exhibit the behavior observed by Hulett and that the gray samples were, after a rotation of several months, still in agreement with cells of the usual construction to within 0.01 per cent.

This investigation is still under way and when completed the results, with full details, will be published in the Bulletin of the Bureau of Standards.

THE STANDARD CELL.[1]

BY FRANK A. WOLFF AND C. E. WATERS.

THE paper is a resumé of the work done by the authors, at the Bureau of Standards, on the reproducibility of the Clark and Weston standard cells.

Mercurous sulphate, the component to which most of the differences, in the past, may be attributed, was prepared by the following methods.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

1. The electrolytic method, devised independently by Carhart and Hulett and one of the authors, in which the mercurous sulphate is formed by the passage of the current from a mercury anode, the electrolyte being sulphuric acid. Current densities from less than 0.1 to more than 9 amperes per square decimeter were employed while the strength of acid varied from 20 to 400 grams per liter. Two different forms of apparatus were used and the stirring was varied between wide limits.

2. By the recrystallization of commercial samples from strong sulphuric acid under various conditions (5 samples).

3. By the action of fuming sulphuric acid on metallic mercury (1 sample).

4. By treatment of mercurous nitrate with sulphuric acid (2 samples).

5. By reduction of mercuric sulphate by metallic mercury (2 samples).

6. By reduction of mercuric sulphate by SO_2 (2 samples).

7. By the direct action of rather strong sulphuric acid, containing a small percentage of nitric acid, on mercury (6 samples).

In addition six commercial samples were treated as follows:

8. By digestion with hot 1:4 sulphuric acid and mercury for several hours (6 samples).

9 and 10. By digestion in the cold with 1:4 sulphuric acid since April, 1904 (two lots of 5 and 4 samples respectively).

11. By digestion in the cold with 1:6 sulphuric acid and mercury since October, 1905 (6 samples).

In addition two samples of mercurous sulphate made by Professor Hulett were available.

Weston and Clark cells were set up for each of these samples, the remaining ingredients being:

Mercury redistilled at reduced pressure.

Cadmium — electrolytic, obtained from Kahlbaum and the same redistilled several times in vacuo.

Cadmium sulphate — two samples obtained from Kahlbaum. Recrystallized — cloudy crystals.

Zinc from Kahlbaum grade I. and the same redistilled in vacuo.

Zinc sulphate — samples from Kahlbaum and Baker recrystallized.

THE RESULTS.

Of 55 Weston cells set up with 38 samples of electrolytic mercurous sulphate, the deviation of 31 from the mean of all was within ± 10 microvolts, of 14 from 10 to 15 microvolts and of 5 from 15 to 20 microvolts. Of the remaining five (maximum deviation less than 50 microvolts) there were 4 duplicates of which the deviation did not exceed 11 microvolts.

Resumé of Results.

Mercurous Sulphate.	Number of Samples.	Weston Cells.		Clark Cells.	
		Number of Cells.	Average Deviation From Mean of Electrolytic Cells.	Number of Cells.	Average Deviation From Mean.
Method 1	38	55	± 11 microvolts.	57	± 10 microvolts.
2	5	5	−12 "	5	−10 "
3	1	1	−04 "	1	−02 "
4	2	4	−18 "	2	00 "
5	2	2	−12 "	2	−04 "
6	2	2	+18 "	2	+05 "
7	6	9	−14 "	10	−11 "
8	6	6	+ 4 "	8	+18 "
9	5	5	+27 "	5	37
10	4	4	+50 "	—	+— "
11	6	6	+70 "	8	+90 "
Hulet's I.	1	2	+30 "	3	+32 "
" II.	1	3	−01 "	3	+07 "

Of 57 Clark cells set up with the same samples of mercurous sulphate, 35 differed from the mean of all by less than 10 microvolts, 15 by 10 to 15 microvolts, 6 by 15 to 25 microvolts, while the remaining cell differed by 36 microvolts from the mean.

The results for all the samples of mercurous sulphate are given in the table above.

In addition 22 Weston cells were set up to determine what variations might be introduced from the cadmium sulphate, and whether there were any electromotive differences between clear and cloudy crystals as suggested by Prof. Hulett. Eight samples in all, recrystallized from acid, basic and neutral solutions, were tried and, excluding one sample precipitated at high temperature and no doubt inverted, the mean deviation from the electrolytic cells was found to be — 6 microvolts with a range of + 9 to — 29 microvolts.

The values obtained from day to day generally agree to within a few parts per million and although most of the Weston cells of the Bureau are seven months old no tendency toward a decrease in electromotive force as observed by Hulett has been found. A small number of cells which showed high initial values have decreased, but not below the mean of all the cells made with electrolytic mercurous sulphate. The same remark applies to samples made by the various chemical methods employed.

In addition, the ratio of Clark to Weston cells was determined from time to time, but the small differences observed, amounting to two or three parts in 100,000, and including errors of temperature measurement and those due to defective insulation of the cells in the bath and between the individual coils of the high resistance potentiometer employed, indicate an apparent *increase* of the Weston cells, or a *decrease* of the Clark cells.

A number of Weston cells recently set up have somewhat *smaller* values than the older cells, prepared from the same materials, but the difference does not exceed 10 microvolts.

Clark and Weston cells have been exchanged with Professor Hulett and very recently with Professor Carhart and Professor Guthe. In addition four electrolytic Weston cells made at Cornell University by Mr. Wold have been compared at the Bureau. The results of these comparisons are given below in terms of the means of the Bureau cells.

	Weston Cells.		**Clark Cells.**	
	Cell.	**Mean Difference in Microvolts.**	**Cell.**	**Mean Difference in Microvolts.**
Prof. Hulett	A_8	+45	H_4	−15
" "	A_{52}	+27	H_{18}	+5
" "	O_4	40		
Prof. Carhart..........	No. 5	+1	C_2	00
" "	No. 8	+4		
Prof. Guthe..........	C_1	+25	A_1	+65
Mr. Wold	No. 24	−9		
" "	No. 27	−10		
" "	No. 20	−24		
" "	No. 21	−15		

In addition two other cells made by Professor Hulett have been under observation since July, 1906. F_7, one of the first electrolytic cells set up by him several years ago, has steadily decreased form — 10 to — 150 microvolts, while O_5 has shown great fluctuations, the value ranging from — 100 to + 32 to — 60.

While the direct comparison of cells made by Professor Hulett with those made at the Bureau shows a discrepancy of 25–45 microvolts, cells made by the anthors with sample II. of mercurous sulphate show that the differences are probably not due to the mercurous sulphate used but to some difference in the other materials or in the specifications employed.

Samples of the various materials will therefore be exchanged with other investigators in this field, to determine the cause of difference, and it is confidently expected that as excellent agreement as is here shown can be reached by all when the same specifications are followed.

Further work is in progress at the Bureau of Standards on the remaining cell materials and on other factors which might influence the electromotive force, *e. g.*, size of grain, depth of paste, influence of diffusion, influence of added impurities. A careful determination of the temperature coefficients will also be made.

The main conclusions to be drawn are that since samples of mercurous sulphate of uniform electromotive properties can be made by a number of different methods, and, since the remaining materials used in the construction of standard cells can be obtained in an exceedingly pure state, that Clark and Weston standard cells are reproducible to within two or three parts in 100,000, that they reach a permanent value within a few days, and that when set up with proper materials the Weston cell can be depended on as constant well within the above limits for a considerable period.

Another conclusion that might be drawn is that the standard cell has a marked superiority, both in ease and accuracy of reproduction over the silver coulometer as a *fundamental* electrical standard, particularly as the results obtainable with the latter can only be preserved by the aid of a standard current balance, or the combination of a standard resistance and a standard cell, all of which must be assumed constant between silver coulometer measurements.

With materials at hand, and if necessary, these could be procured by investigators from the various National Physical Laboratories, a number of cells can very readily be set up in far less time than that required for a coulometer measurement, and with a greater certainty of reproducing the volt than is possible by the coulometer and standard resistance, both of which are subject to error.

The choice of the standard cell and the standard resistance as fundamental standards is besides the most logical as practically all voltage and current measurements of precision are made by the potentiometer method and thus directly in terms of the standard cell or the combination of a standard cell and a standard resistance.

Considerable work has already been done along other lines and will, with the full details of this paper, be published an early number of the Bulletin of the Bureau of Standards.

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THE

PHYSICAL REVIEW.

MODIFICATIONS OF THE MAXWELL-RAYLEIGH AND THE ANDERSON METHODS FOR THE MEASUREMENT OF THE COEFFICIENT OF SELF-INDUCTION.

By Anthony Zeleny.

I. The Maxwell-Rayleigh Method.

THE Maxwell-Rayleigh method[1] for the measurement of the coefficient of self-induction is theoretically more free from doubtful corrections than any other known method. It has been discarded, however, on account of an experimental difficulty. The form in which the method was left by Lord Rayleigh necessitates, in addition to balancing the Wheatstone bridge and a ballistic galvanometer throw, a deflection of the galvanometer produced by the continuous current when one arm of the bridge has been overbalanced by a very small and known amount. After the introduction of this small resistance and during the interval required for the deflecting needle or coil to come to rest the resistance of the copper inductance coil is changed by heating due to the current and by variations in the temperature of the room, so that when the galvanometer reading is observed the deflecting current is produced by an overbalancing resistance which differs from the small known resistance introduced. The amount of change in the resistance of the copper coil is unknown and is usually sufficient to render results uncertain within one per cent.

The modification of the method here described renders the objectionable measurement unnecessary and makes of it a convenient and an accurate method. In the Wheatstone bridge, Fig. 1, let

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[1] Phil. Trans., R. S., 1882, p. 677; Henderson, Practical Electricity and Magnetism, 1898, p. 301.

R_1 = the inductive resistance,
R_2, R_3, R_4 = the non-inductive resistances,
G = resistance of the galvanometer branch,
L = coefficient of self-induction of R_1,
I = current in the main circuit,
I_1 = current in the inductive coil R_1.

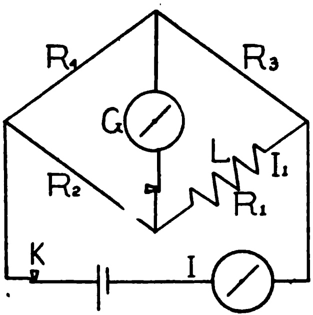

Fig. 1.

Also let i = induced current flowing through R_1,
q = the quantity of induced electricity passing through the galvanometer,
Q = whole quantity of induced electricity passing through the bridge on the opening of the main circuit,
R = the resistance of the Wheatstone net around the circuit with R_1, the sources of the induced electromotive force, as the origin.

When the circuit is opened by means of the key K, after a balance has been obtained, the induced electromotive force

$$e = -\frac{LdI_1}{dt}$$

and

$$L = -\frac{e}{\frac{dI_1}{dt}} = \frac{\bar{e}}{\frac{I_1}{t}} = \frac{iR}{\frac{I_1}{t}} = \frac{QR}{I_1} \qquad \text{(I)}$$

Q is calculated from the measured quantity q and the known resistances. R is obtained from the known resistances, and I_1 from the measured current I and the known resistances. All of these quantities can be determined with accuracy.

For convenience of calculation the terms from which the quantities Q, R and I_1 are calculated will be substituted for these quantities in the equation (I).

$$q = \frac{(R_2 + R_4)Q}{R_2 + R_4 + G}, \therefore Q = \frac{R_2 + R_4 + G}{R_2 + R_4} q = \left(1 + \frac{G}{R_2 + R_4}\right) q. \qquad \text{(II)}$$

$$R = R_1 + R_3 + \frac{(R_2 + R_4)G}{R_2 + R_4 + G} = R_1 + R_3 + \frac{1}{\frac{1}{G}\left(1 + \frac{G}{R_2 + R_4}\right)}. \quad \text{(III)}$$

$$I_1 = \frac{(R_3 + R_4)I}{R_1 + R_2 + R_3 + R_4} = \frac{I}{1 + \frac{R_1 + R_2}{R_3 + R_4}} = \frac{I}{1 + \frac{R_2}{R_4}}. \quad \text{(IV)}$$

$\left(\frac{R_1 + R_2}{R_3 + R_4} = \frac{R_2}{R_4}\right.$ because when the bridge is balanced $\left.\frac{R_2}{R_4} = \frac{R_1}{R_3}\right)$.

Substituting these values in equation (I),

$$L = \frac{QR}{I_1} = \frac{q}{I}\left(1 + \frac{R_2}{R_4}\right)\left[\left(1 + \frac{G}{R_2 + R_4}\right)(R_1 + R_3) + G\right] \quad \text{(V)}$$

When the quantity q is measured by comparison with the free charge[1] of a standard condenser,

$$q : CE :: d : D$$

and

$$q = \frac{CE}{D} d, \quad \text{(VI)}$$

where d is the throw due to q and D the throw due to the quantity CE.

Substituting this value of q in equation (V),

$$L = \frac{CE}{D} \cdot \frac{d}{I}\left(1 + \frac{R_2}{R_4}\right)\left[\left(1 + \frac{G}{R_2 + R_4}\right)(R_1 + R_3) + G\right]. \quad \text{(VII)}$$

When the quantity q is measured by comparison with the quantity induced in the secondary circuit of a standard mutual inductance coil[2] whose coefficient of self-induction is M, the quantity of induced electricity, $Q = MI_P/R_S$

$$\therefore q : \frac{MI_P}{R_S} :: d : D$$

and

$$q = \frac{MI_P}{R_S D} d$$

and $L = \frac{MI_P}{R_S D} \cdot \frac{d}{I}\left(1 + \frac{R_2}{R_4}\right)\left[\left(1 + \frac{G}{R_2 + R_4}\right)(R_1 + R_3) + G\right]$. (VIII)

When $R_1 = R_3$ and $R_2 = R_4$, as is usually the case, equation (VII) becomes

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[1] A. Zeleny, The Capacity of Mica Condensers, PHYS. REV., 1906, Vol. 22, p. 65.

[2] A. Zeleny, Galvanometer Measurements, PHYS. REV., 1906, Vol. 23, p. 399.

$$\mathbf{L} = 2\frac{\mathbf{CE}}{\mathbf{D}} \cdot \frac{\mathbf{d}}{\mathbf{I}}\left[2\mathbf{R}_3\left(1 + \frac{\mathbf{G}}{2\mathbf{R}_3}\right) + \mathbf{G}\right] \tag{IX}$$

and equation (VIII) becomes

$$L = 2\frac{MI_P}{R_S D} \cdot \frac{d}{I}\left[2R_3\left(1 + \frac{G}{2R_2}\right) + G\right]. \tag{X}$$

For simplifying the calculations, G can be made some exact multiple of R_2 by introducing the proper resistance into the galvanometer circuit.

ABSOLUTE MEASUREMENTS.

The coefficient of self-induction may be determined by means of any one of the equations (VII), (VIII), (IX) or (X). The equation (IX), however, is to be preferred ordinarily on account of its simplicity and the ease with which the necessary measurements are made. The limit of attainable accuracy is that of direct deflection methods, *i. e.*, about .01 per cent., but may be somewhat increased by use of the differential galvanometer. The error due to the small self-inductance in the non-inductive arms of the bridge and to the electrostatic capacity of all of the four arms is very small and should become appreciable only in the measurement of very small inductances. The error introduced, however, is much smaller than with any of the alternating current methods.

COMPARISON MEASUREMENTS.

This method may be employed to compare coefficients of self-induction with a standard self-inductance. The three non-inductive resistances and the galvanometer resistance are maintained constant, and, in the fourth arm of the bridge, a variable non-inductive resistance is placed in series first with the standard inductance coil and then with the one of unknown inductance. This non-inductive resistance is varied in obtaining the balance first for the standard and then for the unknown inductance. The current may be varied, if desirable, so that approximately equal throws are obtained in the two cases. In the expressions for the values of the coefficients as obtained from equation (VII) all of the quantities except d and I are common to both, so that

$$L : L_1 :: \frac{d}{I} : \frac{d_1}{I_1}$$

and

$$L_1 = \frac{d_1 I}{d I_1} L. \qquad \text{(XI)}$$

But if the current is left unchanged

$$L_1 = \frac{d_1}{d} L \qquad \text{(XII)}$$

and the only observations necessary are the throws given by the known and unknown inductances. If the electromotive force is constant the current need not be observed, because when a balance is obtained the total resistance in the circuit is the same in the two cases.

Experimental Results.

In a series of measurements by this method the apparatus employed was arranged as shown in diagram in Fig. 2. R_2, R_3 and R_4 represent three arms of the Wheatstone bridge. In the fourth arm are the inductive resistance whose coefficient of self-induction

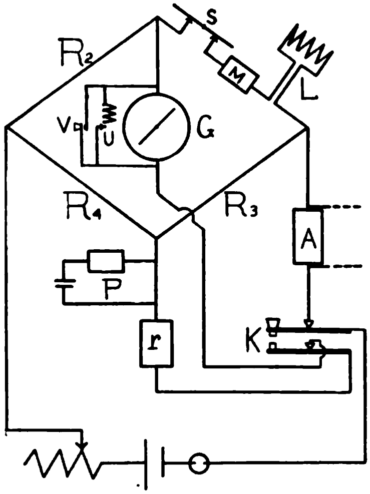

Fig. 2.

L is to be measured, the adjusting resistance M, and the sliding resistance S which is described in detail later. U represents the shunt employed during the adjustment of the balance and V the short-circuiting key used in bringing the coil to rest so as not to pass beyond the null point. *P* is a secondary circuit for compensating any thermo-electric current that may exist in the galvanometer circuit.[1] The resistance *G* in the galvanometer circuit can be varied by means of the resistance *r*. *A* is a standard resistance employed for measuring the current in the main circuit, the difference of potential between its binding posts being determined by a Wolff potentiometer. By means of a single tap, the key *K* opens the main circuit and after all the measurable part of the induced electricity has passed through the Wheatstone net, but before the galvanometer coil has turned an appreciable distance, it opens the galvanometer circuit. The quantity of electricity passing through the galvanometer can then be compared directly with the quantity discharged from a standard condenser. (See former papers of the writer, referred to above.)

A proper construction of the sliding resistance *S* is essential for the convenience and success of this method. The contacts must be good and the slide easily moved so that, if necessary, the operator can give it a continuous slow motion and thereby compensate for the gradual change of resistance due to the alteration of the temperature of the inductance coil. The sliding resistance employed is shown in Fig. 3. It is composed of two wires, each 35 cm. long, one being a No. 16 copper and the other being a No. 16 manganin wire. The resistance of the manganin wire is a little more than .05 ohm, so that with 0.1 ohm as the smallest resistance in the adjusting box, *M*, a perfect balance can be obtained by first adjusting the resistance in the box, then the slide on the manganin wire, and finally, for the most accurate adjustment, the slide on the copper wire. Each slide consists of a block of copper 3.5 cm. long, with a hole through its

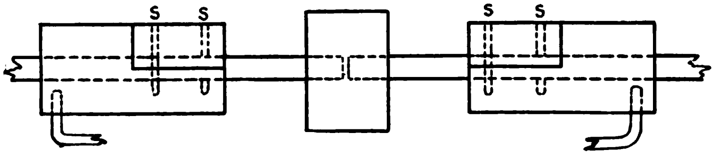

Fig. 3.

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[1] A Leeds and Northrup H type moving coil galvanometer of 127 ohms resistance was employed.

center of the same diameter as that of the wire. A part of each slide is cut into two pieces, as shown in the figure, and the two parts are held together by a strong rubber band (not shown) and kept from sliding sidewise by the pins *SS*. The face of the smaller section was filed, so as to always make a contact at the end of the block. This makes a satisfactory slide, free from contacts having strong thermo-electric power. It is necessary, however, to polish the wires with fine emery cloth before using. This is especially true of the copper wire; a small quantity of oxide making the resistance of the contact variable so that no satisfactory adjustment is possible.

After a balance of the bridge was obtained the thermo-electric current in the galvanometer circuit was compensated and then again the final balance of the bridge made. The quantity of induced electricity passing through the galvanometer at the opening of the main circuit was compared with the quantity discharged from a standard condenser. The resistance of the galvanometer circuit was measured after each set of observations.

The coefficients for two coils wound on the same spool were determined; and to test the accuracy of the values obtained, the coefficients of the two placed in series with the currents first in the same and then in opposite directions were determined as was also the coefficient of mutual induction.

The following observations were common to all the self-induction determinations:

$t = 20^\circ.2$ to $20^\circ.3$ C.

$R_2 = R_4 = 99.99$ ohms.

$A = 9.9966$ ohms.

$R_3 = 40.000$ except in case of coil No. 2, where $R_3 = 10.014$ ohms.

E.M.F. of standard cell on potentiometer = 1.01869 (German) volts.

E.M.F. of standard cell used for charging condenser = 1.01883 (German) volts.

$C = 1.00136 \times 10^{-6}$ farads.

Distance of straight scale = 127.2 cm.

The remaining data are given in Table I.

TABLE I.

Coils.	**Induction Throws (d).**	**Condenser Throws (D).**	**Potentiometer Reading-Volts.**	**R of Galv. Circuit.**	**L in Henrys.**
No. 1 + No. 2	19.421$\pm^{3}$	19.908 $\pm^{2}$	0.50906 $\pm^{3}$	1686.0 $\pm^{1}$	.095391
No. 1 − No. 2	19.471	19.893	0.50895	393.75	.024766
No. 1	19.535	19.824	0.50877	865.82	.051049
No. 2	19.216	19.445	0.93644	363.65	.009042
Mutual inductance of No. 1 and No. 2.	19.730	19.700	0.69205	Resistance of secondary circuit. 1196.12.	M in Henrys .017653

The lower part of Table I contains the observations necessary, in addition to the above data, for the determination of the coefficient of mutual induction of the two coils. The method used was that employing the relation $M = QR_S/I_P$, the main and secondary circuits being opened in succession as in the self-induction determinations. The apparatus was arranged as shown in diagram in Fig. 4, the parts serving a similar purpose as those in Fig. 2 are represented by similar characters, while C represents the two inductance coils.

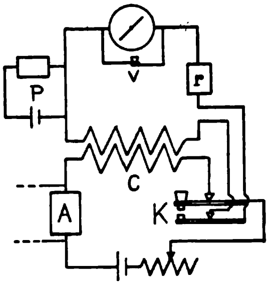

Fig. 4.

The galvanometer throws given are the average of five or more, varying among themselves by an amount rarely exceeding .01 cm., and all have been changed from the observed straight scale to the equivalent circular scale readings.

The coefficients of the two coils when in series with the current in the same direction (L_{1+2}) and with the current in opposite directions (L_{1-2}) are given by the equations:

$$L_{1+2} = L_1 + L_2 + 2M,$$

$$L_{1-2} = L_1 + L_2 - 2M,$$

where M is the coefficient of mutual induction. Hence

$$L_{1+2} + L_{1-2} = 2(L_1 + L_2) \tag{XIII}$$

and

$$\frac{L_{1+2} - L_{1-2}}{4} = M. \tag{XIV}$$

Substituting the determined values as given in Table I:

In equation (XIII)

$$2(L_1 + L_2) \quad = .120182$$

$$L_{1+2} + L_{1-2} = .120157$$

$$\text{Difference} = .000025 = .021 \text{ per cent.}$$

In equation (XIV)

$$M = \frac{L_{1+2} - L_{1-2}}{4} = .017656$$

The independently
determined value of M $= .017653$

$$\text{Difference} = .000003 = .017 \text{ per cent.}$$

This test shows that the results obtained are correct within the limits of observational error as the greater of the two differences, *i. e.*, .021 per cent. would be introduced by a constant error of less than one-half a tenth of a millimeter in the observed throws. It also shows that in the employment of this method the error due to the self-induction in the non-inductive resistances[1] and to their electrostatic capacities must be very small.

II. The Anderson Method.

The Anderson method and its modifications have been accepted by many as the most satisfactory of all the known methods. For this reason, a comparison of the results obtained by the above modified Maxwell-Rayleigh method was made with the results obtained by this one which, however, it was found necessary to modify as described below.

A Modification of the Original[2] *Null Method.* — In the original Anderson method as it is now commonly employed the condenser remains permanently connected in the circuit so that, after a balance is obtained and the circuit is opened, the liberated absorbed charge as well as the free charge of the condenser affects the galvanometer. For this reason the results obtained are dependent upon the period of the galvanometer and upon the resistance in the condenser circuit, and may be in error by more than one per cent. even when a good condenser is employed. No accurate determinations can be made in this way unless the effective capacity of the condenser for the particular apparatus is first obtained, but this is not practicable. The capacity values of standard condensers are usually determined by standardizing bureaus by an intermittent current method giving 100 interruptions per second. This should give the "free charge" capacity values of good condensers with sufficient accuracy for all ordinary purposes. The rated values can therefore be employed when using the condensers as described in a previous paper.[1]

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[1] A Leeds and Northrup box bridge with coils wound on wooden spools was employed.

[2] A. Anderson, Phil. Mag., 1891, XXXI., p. 329.

The determinations by this method were made therefore by using the discharge key shown in Figs. 2 and 4, the main and the galvanometer circuits being opened in succession, as in the cases previously described. The rated capacity value of the condenser was then the effective value. The results obtained were

by the Modified Maxwell-Rayleigh Method $L = .095391 \pm 9$

and by the Modified Anderson Method $L = .095386 \pm 9$,

an agreement well within the limits of observational error. For the same degree of sensitiveness, however, the Anderson method required a current strength three times that employed in the other method.

The Secohmmeter Method. — The value of the coefficient for the same coil was obtained by using the Anderson method modified[2] by alternating the current by means of a secohmmeter. The object was to compare the results with those obtained with the other methods and to observe the variations with the change of speed of the secohmmeter. The electrostatic effects due to the commutator were eliminated in the usual manner.[3] The currents in the various branches did not effect the inductance in the coil, for when a balance was obtained no appreciable change was observed on reversing the coil, which was placed about a meter from the other parts of the apparatus.

The chief difficulty experienced was from the thermo-electric current due to the unequal warming, by friction, of the brushes on the commutator. To correct for this, the sechommeter was brought to rest after a preliminary balance was obtained and the null point taken as affected by the thermo-electric current. The final balance was then made using the new null point. The magnitude of this correction affected the results by .04 per cent. at the lowest and by

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[1] A. Zeleny, loc. cit.

[2] Fleming and Clinton, Phil. Mag., 1903, V., p. 493.

[3] The Electrician (London), 1897, p. 379.

.005 per cent. at the highest speed employed. The thermo-electric current, however, was somewhat irregular so that at the lower velocities an error of .01 per cent. due to this cause is possible. The results obtained are given in Table II.

TABLE II.

No. of Alternations per Second.	L	Increment $\times 10^6$.	No. of Alternations per Second.	L	Increment $\times 10^6$.
0	.095386	—	34.6	.095559	173
10.16	.095442	56	42.6	.095565	179
15.62	.095499	113	50.0	.095581	195
27.6	.095545	159	57.6	.095590	204

For the same current flowing through the main circuit this method for the higher speeds was about 200 times as sensitive as the same method when employed without the secohmmeter, but on account of continuous variations of the balance the actual working sensibility was only about ten times as great. It appears from Table II., that notwithstanding the great sensitiveness of this method it cannot be employed for measurements of the highest accuracy without the application of doubtful corrections of considerable magnitude.

These variations of the values are due principally to the effect of the inductances in the non-inductive branches and to the electrostatic capacity in all the branches, the effect varying with the rate of the alternations. This subject is ably treated by Rosa and Grover.[1] The variations are no doubt due partly also to the varying effect of the absorbed charges in the condenser.

SUMMARY.

The experimental difficulty of the Maxwell-Rayleigh method for measuring the coefficient of self-induction is avoided by omitting the measurement with the small known resistance for disturbing the balance in the Wheatstone net and substituting for it the measurement of the current in the main circuit. The modified method presents for work of precision an advantage over all other known methods on account of a greater freedom from uncertain corrections.

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[1] Bulletin of the Bureau of Standards, 1905, Vol. I., p. 304.

The method as expressed by equation (1) is also valuable from an educational standpoint on account of the simplicity of the mathematical relations involved.

A simple modification enables the comparison of inductances to be made with ease.

The Anderson null method as modified by the writer so as to render only the free charge of the condenser effective is shown, experimentally, to be capable of as high a degree of accuracy as the modified Maxwell-Rayleigh method. It possesses a slight disadvantage from a theoretical standpoint, is less sensitive, and requires a double balancing; but has the advantage of being a null method and of not requiring the measurement of several electrical quantities necessary in the employment of the other method.

The modification of the Anderson method by the employment of a secohmmeter gives results varying appreciably with the rate of the alternations of the current.

PHYSICAL LABORATORY,
THE UNIVERSITY OF MINNESOTA,
December 22, 1906.

THE ABSORPTION OF HYDROGEN BY METAL FILMS.

By Walter Heald.

IT is known that under certain conditions a number of the metals absorb gas. Some, such as silver and gold, are capable when in a molten state of absorbing it from the air, as is the case with other liquids, this gas being given up again when the metal solidifies. Palladium absorbs very large quantities of hydrogen even when in the solid state. For instance, if a piece of palladium is used as the negative pole in the electrolysis of a dilute solution of sulphuric acid, it takes up about nine hundred times its own volume of hydrogen.

Skinner[1] has found that most of the metals give off hydrogen when used as cathode in a discharge tube, which shows that in their natural condition the metals contain a certain amount of absorbed hydrogen.

In the present investigation, while trying to determine the quantity of gas that a given amount of metal evolved when vaporized by a large electric current in an atmosphere of hydrogen, it was noticed that after the vaporization took place, the gas pressure gradually decreased. This could not be accounted for by a decrease in temperature, which could last only a short time, because the pressure continued to decrease in some cases for an hour or more. Before the electric current had passed the gas pressure was noted as constant. After it had been broken, apparently the only change that had taken place in the tube was that a metal film had been deposited on the walls of the tube. Therefore, the conclusion is drawn that the newly deposited metal gradually absorbed hydrogen from the molecular state.

In the case of its absorption by a palladium cathode during electrolysis of a dilute solution of sulphuric acid, the hydrogen is in a nascent state. Such may also be the case when it is absorbed by hot metals, the temperature at least aiding dissociation. If, then, hydrogen, which is in the molecular state is absorbed by a cold metal film, it seems most likely that the metal in this condition dissociates the hydrogen, though possibly it has a strong affinity for and absorbs the hydrogen molecule. At any rate it is interesting to learn something concerning the absorption of hydrogen by films of different metals produced in the above manner.

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[1] Phys. Rev., vol. 21, no. 1, July, 1905.

The purpose of this investigation was therefore to obtain deposits of a number of different metals in a tube containing hydrogen, and to measure the amount and rate with which each absorbed the gas. The deposits were produced as indicated by vaporizing the metal in a discharge tube by means of a large electric current and allowing it to condense on the walls of the containing tube. It was hoped that the quantity of the active metal might also be determined but this proved too small to measure.

APPARATUS AND METHOD OF EXPERIMENT.

The form of discharge tube used is shown by diagram in Fig. 1. The brass rods *R* are sealed with de Khotinsky's laboratory cement in the glass tubes *P* which fit in the ground glass joints *J*. Electrodes *E* represent the metal to be tested. These were either screwed or clamped in the ends of the brass rods. Glass tubing enclosed the brass rods so as to confine the discharge to the metals used as electrodes.

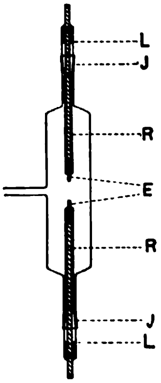

Fig. 1.

The discharge tube was connected to an air pump and a tube containing P_2O_5 for drying purposes. It was also connected to a flask containing dry hydrogen, which was generated by electrolysis of dilute phosphoric acid, and to a McLeod gage used for measuring pressure. This gage magnified the pressure one hundred times.

After exhausting the discharge tube it was allowed to stand in connection with the P_2O_5 tube several hours before admitting hydrogen.

The current used for vaporizing the metal was supplied by a battery of small accumulators. Usually a potential difference of about a thousand volts was employed, the amount of current being regulated by placing a copper fuse wire in the circuit. The current passed until this melted, which took from about a half second to two or three seconds. Most of the time the fuse wire used was No. 32 *B* and *S* gage. In some cases a No. 36 wire was placed in parallel with it.

The discharge tube was filled with hydrogen to a pressure of from one to six millimeters and through this the vaporizing current was passed. The volume of the chamber occupied by the gas was about five hundred cubic centimeters.

With most of the metals there was considerable difficulty in obtaining a sufficient deposit to give appreciable results. Two methods for vaporizing the metal were tried. The first was to use two pieces as electrodes, these being placed about an inch apart. The second was to connect the two brass rods *R* with a wire of the metal to be tested; then to melt it by sending an electric current through it, so that the arc, arising when the connection was broken by fusion, vaporized part of it. The first method proving the more satisfactory was the one used most of the time. Following is given an account of metals tested and results obtained.

CADMIUM.

Cadmium was tested first because of its having a comparatively low boiling point, hence would vaporize easily. With a cadmium disk serving as cathode and a steel disk as anode, the metal film seemed to come wholly from the cathode, since the deposit formed only on the walls of the tube near the cathode. Besides, the surface of the cathode alone should erosion. The same was true with other metals. Therefore it is assumed that the deposit in all cases was of the same material as the cathode.

Three deposits of cadmium were produced each accompanied by some absorption of gas. In the case where it was heaviest the following results were obtained.

Time deposit was made 2:10 P. M.

Time, P. M.	Pressure, mm.
2–10–30	1.365
2–15–30	1.340
2–25–30	1.330

After this the pressure remained constant.

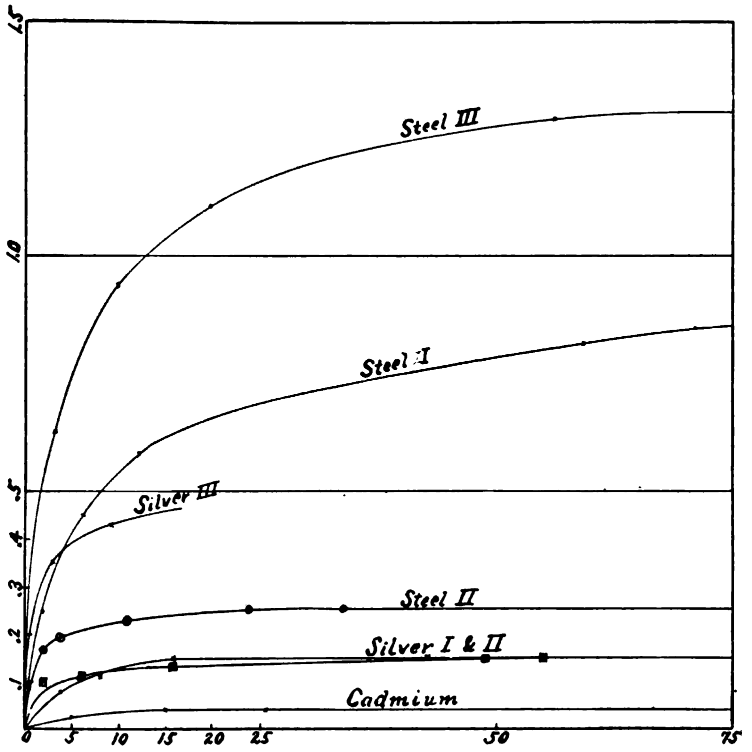

Fig. 2.

The absorption curve plotted from these results is found in Fig. 2 in which the ordinates represent the decrease in gas pressure.

STEEL.

Both methods were tried for a depositing of steel. By the first, with disk electrodes, the following results were obtained:

1. A fairly heavy deposit over a length of three or four centimeters of the tube:

Time deposit was made 10–13–30 A. M.

Time, A. M.	Pressure, mm.
10-14	4.48
10-16	4.23
10-20	4.03
10-26	3.90
11-13	3.67
11-25	3.638
11-58	3.629
P. M.	
2:30	3.628

2. Only a very light deposit:
Time of deposit 8:58 A. M.

Time, A. M.	Pressure, mm.
8-58-10	2.61
9-00	2.44
9-02	2.42
9-09	2.38
9-22	2.355
9-32	2.36

3. The deposit was heavier than either of the other two:
Time of deposit 9:35 A. M.

Time, A. M.	Pressure, mm.
9-35-10	5.626
9-38	5.0
9-45	4.687
9-55	4.515
10-37	4.336
11-37	4.285
P. M.	
1-38	4.302

The absorption curves plotted from these results are also in Fig. 2. Obviously the difference in the curves arises from the quantity of metal deposited.

Using the second method, there was only one case in which any appreciable deposit was obtained, and then not only was the steel wire vaporized but also part of the ends of the brass rods and some of the solder used to hold the clamps; hence results are not given.

SILVER.

A silver disk was used as cathode, a steel disk as anode. Only slight deposits were obtained, except in case 3 in which it was fairly heavy. Following are the results:

1. Time of deposit 9–8 A. M.

Time, A. M.	Pressure, mm.
9-8-10	2.162
9-12	2.08
9-16	2.05
9-24	2.01
9-45	2.015
9-51	2.01

2. Time of deposit 10–35–30 A. M.

Time, A. M.	Pressure, mm.
10-35-40	2.09
10-37-30	1.987
10-41-30	1.976
10-51-30	1.954
11-24-30	1.94
11-35	1.942

3. Time of deposit 11–40 A. M.

Time, A. M.	Pressure, mm.
11-40-5	2.74
11-43	2.39
11-49	2.308

See Fig. 2 for absorption curves plotted from these results.

ZINC.

When making experiments with zinc the first method only was used with electrodes of zinc wire. The results seemed to be rather inconsistent. Part of the time there was apparently a slight increase of pressure and part of the time a slight decrease after the zinc film had been deposited. The effect, however, was in each case small enough to be accounted for by error in reading the pressure.

ALUMINIUM.

Several attempts were made to obtain deposits of aluminium, using pieces of wire for both anode and cathode. In two cases a slight one was obtained. There was very little if any decrease in pressure with time. There was, however, quite a decrease arising from the momentary current. In the first case this was .42 mm., and in the second .17 mm. It is probable therefore that all absorption took place within five seconds after the deposit was made.

PLATINUM.

Using small platinum wires as electrodes, slight deposits were obtained. As with aluminium there seemed to be no decrease in pressure after the deposition took place, but quite a large decrease during the passage of the current. In one instance this amounted to .53 mm. Since it is known that platinum absorbs large quantities of hydrogen, and very rapidly under certain conditions, it may be that with the platinum film as with the aluminium, the gas was absorbed at such a rapid rate as to fully charge the metal before the pressure could be read.

Of the metals studied, cadmium, silver and steel gave definite results as to the rate of absorption of hydrogen by the films ; aluminium and platinum merely indicate that marked absorption occurs on depositing ; while zinc is the only metal tested which did not exhibit this quality.

I wish to express here my appreciation and thanks to Dr. Skinner for his help and inspiration to me in this work, as well as for the laboratory facilities placed at my disposal at the University of Nebraska where the work was done.

Bureau of Standards,
Washington, D. C.

ON THE SUSCEPTIBILITY OF MIXTURES OF SALT SOLUTIONS.[1]

By J. C. McLennan and C. S. Wright.

IN the following paper an account is given of some measurements made on the magnetic susceptibility of solutions of manganese, aluminium and copper sulphates in water, and several mixtures of these with the object of obtaining information which might be of service in explaining the behavior of the magnetic alloys recently discovered by Heusler,[2] and more recently studied by Gumlich,[3] Austin,[4] Fleming and Hadfield,[5] Take,[6] Hill[7] and Guthe and Austin.[8]

The method followed in measuring the susceptibility of the solutions was that suggested by Lord Kelvin,[9] and afterward used by Wills[10] in his experiments on the susceptibility of weakly magnetic solids and liquids, and later by Burton and Phillips[11] in their investigation of the magnetic properties of iron in the colloidal state.

The arrangement of the apparatus is indicated in Figs. 1 and 2. A rectangular slab *S*, Fig. 1, of the substance whose susceptibility is to be studied was suspended from one arm of a balance *B* with its long axis vertical, and its lower face in the strongest part of the field midway between the poles of an electromagnet.

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[1] Read before the American Physical Society at the December meeting in New York, 1906.

[2] Fr. Heusler, W. Starck, and E. Haupt, Verh. der Phys. Ges., 5, p. 219, 1903. Heusler, Starck, and Haupt ueber der ferromagnetischen Augenschaften von Legierung unmagnetischer Metalle, Marburg, 1904.

[3] Gumlich, Annalen der Physik, 16, p. 535, 1905.

[4] Austin, Verh. der Deut. Phys. Gesell., 6, p 211, 1904.

[5] Fleming and Hadfield, Proc. Roy. Soc., 76, p. 271, 1905.

[6] R. Take, Verh. der Deut. Phys. Gesell., 7, p. 133, 1905. Take, Ann. der Physik, Vierte, Folge, Band 20, p. 849.

[7] Hill, PHYS. REV., 21, 335, 1905; 23, No. 6, p. 498, 1906.

[8] Guthe and Austin, Bulletin of the Bureau of Standards, Washington, Vol. II., No. 2, August, 1906, p. 297.

[9] Brit. Assoc. Rep., 1890, p. 745.

[10] Wills, PHYS. REV., April, 1898; ib., March, 1905.

[11] Burton and Phillips, Proc. Camb. Phil. Soc., Vol. XIII., Pt. V.

If k = the susceptibility of the material of the slab.

A = the area of the horizontal base of the slab.

H = the strength of the magnetic field at the point midway between the poles.

H_1 = the strength of the magnetic field at the upper surface of the slab.

Then the pull P in grams due to the magnetic field is given by

$$P = \frac{kA(H^2 - H_1^2)}{2g} \text{ grams.}$$

The electromagnet used in the investigation was provided with prismatic pole pieces with plane vertical faces 5 cm. wide, and about 1.5 cm. in height. These were adjusted to approximately 1.5 cm. apart, and kept in one fixed position throughout the different measurements. In determining k for a selected liquid the slab S was replaced by a deep rectangular glass vessel, Fig. 2, which measured internally 8 cm. in height, .78 cm. in width and 2.44 cm. in breadth. Preliminary measurements were first made on the pull exerted on the empty cell when currents of different intensities were passed through the electromagnet. The solution to be examined was then placed in a glass cell, and the pull on the filled cell again ascertained for different strengths of field. By combin-

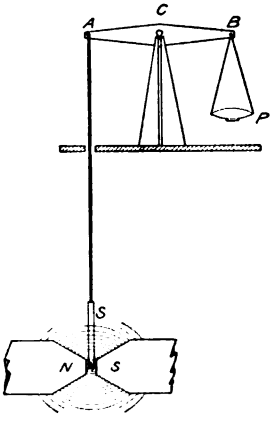

Fig. 1.

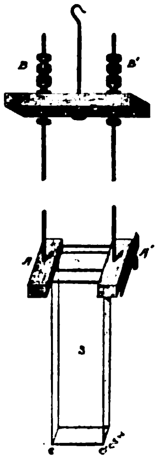

Fig. 2.

ing these two sets of measurements the pull in the selected solution itself was obtained and the susceptibility calculated for the different strengths of field from the formula

$$k = \frac{2pg}{A(H^2 - H_1^2)}.$$

In measuring the intensity of the magnetic fields used in the experiments, a slab of wood whose dimensions were approximately the same as the internal dimensions of the glass cell was inserted in the frame in place of the cell. To the narrow vertical and lower sides of this wooden slab a strip of tinfoil was pasted, and connection was made by contact pieces at A and A', Fig. 2, to the supporting rods. Wires were attached to these at B and B', and through them currents of different strengths sent to the tinfoil circuit.

If the current carried by the tinfoil circuit be i amperes, and the length of the horizontal portion of the foil be l cm., then the pull P in grams exerted by a field H on the lower strip of the foil can readily be shown to be given by the relation

$$P = \frac{Hli}{10g}$$

and consequently the strength of the field by the equation

$$H = \frac{10Pg}{li}.$$

The use of this method permitted the field strengths to be determined with accuracy and also with considerable ease. In all the experiments the current strengths were measured by Weston instruments which previously had been standardized.

I. DETERMINATION OF MAGNETIC FIELD.

As wood itself is slightly diamagnetic the different magnetic fields were found to exert a force on the slab even when no current was passing through the foil. It was therefore necessary to allow for this effect in calculating the field strengths. A set of measurements on the different fields used is given in Table I.

TABLE I.

Magnetic Field.

i in E.M.	i in Foil.	Pull on Slab no i in Foil.	Pull on Slab with i in Foil.	Pull on Foil.	$H = 10Pg/li$. $l = 3.025$ $g = 980.5$.
6.25	.13	−.0094	.1697	.1791	4465
6.4	—	−.0098	—	—	—
6.8	.13	−.0106	.1783	.1889	4710
7.15	.13	−.0113	.1837	.1950	4862
7.4	.13	−.0115	.1899	.2014	5022
7.95	.13	−.0132	.1959	.2091	5214
8.73	.13	−.0149	.2068	.2217	5528
9.05	—	−.0157	—	—	—
9.4	.13	−.0164	.2153	.2317	5777
10.0	.13	−.0177	.2235	.2412	6014
10.25	—	−.0185	—	—	—
10.6	.13	−.0190	.2297	.2487	6201
10.95	.13	−.0198	.2337	.2535	6321
11.41	.13	−.0204	.2363	.2567	6500
11.55	—	−.0209	—	—	—
12.15	.13	−.0224	.2466	.2690	6707
12.95	.13	−.0244	.2553	.2797	6974
14.05	—	−.0268	—	—	—
14.10	.13	−.0269	.2687	.2956	7367

In column I. are given the strengths of the current in amperes in the electromagnet, in column II. the strengths of the current in amperes in the foil circuit, in column III. the pulls on the wooden slab when there was no current in the foil circuit, and in column IV. the pulls on the slab, with the current passing through the foil circuit. Columns V. and VI. contain respectively the deduced values of the pulls on the foil circuit and the corresponding calculated values of the magnetic field strengths.

TABLE II.

Strength of Upper Part of Field.

i in E.M.	i in Foil.	Pull on Slab no i in Foil.	Pull on Slab with i in Foil.	Pull Due to i in Foil.	H (Calc.). $H = 10Pg/li$.
5.85	.1	−.0085	−.0008	.0077	248
9.0	.1	−.0155	−.0062	.0093	302
10.27	.1	−.0183	−.0088	.0095	308
12.6	.1	−.0235	−.0127	.0108	340
14.7	.1	−.0280	−.0164	.0116	376

A second set of measurements was made with the slab reversed, so that the current in the foil circuit passed over the upper surface of the slab instead of along the lower one, as in the previous measurements. A set of these weighings is given in Table II., and from them the value of H_1 deduced. These values, it will be seen, are small compared with the corresponding one for H, and are therefore negligible compared with the latter.

II. Susceptibility of Water.

As a control experiment a preliminary set of measurements was made upon the susceptibility of water, and the mean of a number of results found with fields ranging from 4,000 to 8,000 C.G.S. units, gave -7.33×10^{-7}, a value which is in good agreement with the results of other investigators.[1]

III. Susceptibilities of Salt Solutions.

Manganese Sulphate, $MnSO_4$. — In experimenting with this salt a standard solution was prepared which was found by a gravimetric analysis to contain .0792 gram of manganese per c.c. The susceptibility was determined for fields ranging from 4,700 to 7,315 C.G.S. units, and the results of the different weighings are recorded in Table III.

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[1] Susceptibility of water from a Table given by H. D. Stearns, Phys. Rev., Vol. 16, pp. 1–10, January, 1903.

	T	$k \times 10^7$	Reference.
1885 Quincke.	20°C.	−8.4	Wied. Ann., Vol. 35, p. 137, 1888.
1888 DuBois.	15 C.	−8.6	Wied. Ann., Vol. 35, p. 137, 1888.
1892 Henrichsen.		−7.5	Wied. Ann., Vol. 45, p. 38, 1892.
1895 Curie.	18 C.	−7.9	Jour. de Phys., p. 106, 1895.
1896 Townsend.	10 C.	−7.7	Proc. Roy. Soc., London, 60, p. 186, 1898.
1898-1901 Koenigsberger.	20 C.	−8	Wied. Ann., Vol. 66, p. 698, 1898, and Drude's Ann., Vol. 6, p. 506, 1901.
1898 Fleming & Dewar.		−7.4	Proc. Roy. Soc., 63, p. 328, 1898.
1899 Jaeger & H. Meyer.	18 C.	−6.6	Wied. Ann., Vol. 67, p. 712, 1899. Drude's Ann., Vol. 6, p. 870, 1901.
1903 H. D. Stearns.	22 C.	−7.33	Phys. Rev., Vol. 16, pp. 1–10, 1903.
1904 A. P. Wills.		−7.22	Phys. Rev., Vol. 22, p. 188, 1905.

TABLE III.

Solution $MNSO_4$. .0792 gm. per c.c.

H	Field Pull on Cell Water.	Pull on Cell Solution.	Pull on Salt in Solution.	$10^7 \times K$. $K = \frac{2Pg}{AH^2}$ $A = 1.9$ sq. cm. $g = 980.5$ dynes.
4,700	−.0267	.4448	.4715	213.38
4,955	−.0296	.4953	.5249	213.82
5,250	−.0334	.5545	.5877	213.26
5,500	−.0370	.6173	.6543	216.32
5,550	−.0375	.6245	.6618	214.87
6,000	−.0445	.7383	.7828	217.47
7,315	−.0638	1.0833	1.1471	214.40
				Mean 214.79

If we define molecular susceptibitity, M_s, by the equation

$$M_s = \frac{MK}{d}$$

where M is the molecular weight of the salt, K the susceptibility of the solution, and d the number of grams of the salt per c.c. in solution, we obtain from the above table the value .014915 as the molecular susceptibility of the salt.

In order to test whether the molecular susceptibility depended on the concentration, the standard solution was diluted and the susceptibility found for different concentrations. The results which are given in Table IV., show that the molecular susceptibility did not vary appreciably with the concentration.

Copper Sulphate. — Similar measurements made on a solution of

TABLE IV.

$MnSO_4$. Relation of Concentration to Susceptibility.

Concentration Grams of Mn per c.c.	Susceptibility of Solution $10^7 K$.	Molecular Susceptibility M_s.
.0792	214.7	.014925
.0264	74.51	.01552
.01582	41.28	.01433
.0088	23.78	.01486
		Mean M_s = .01491

copper sulphate containing .0191 gram of copper per c.c., gave a molecular susceptibility of + .00153.

Aluminium Salts. — When the susceptibility of a solution of aluminium sulphate containing .018 gram of metal per c.c. was measured, it was found to give a small negative value showing that this salt in solution is slightly diamagnetic. On account of the fact that aluminium is generally given as being weakly paramagnetic, and that no results seem to have been recorded for the salts of this metal, the susceptibilities of aluminium nitrate and of aluminium chloride were also determined and the results obtained are given in Table V.

TABLE V.

Salt.	Concentration gms. of Metal per c.c.	Susceptibility of Solution $10^7 K$.	Molecular Susceptibility M_s.
Aluminium Sulphate $Al_2(SO_4)_3$	.018	− .64	−.00018
Aluminium Nitrate $Al_2(NO_3)_6$	.097	+ .33	+.00002
Aluminium Chloride Al_2Cl_6	.094	−1.04	−.00005

From this table it will be seen that while the sulphate and the chloride are diamagnetic, the nitrate is paramagnetic. This difference between the susceptibilities of the metal and some of its compounds forms a parallel to the case of copper, where the pure metal is diamagnetic at ordinary temperatures while the salts are largely paramagnetic.

IV. MIXTURES OF SALT SOLUTIONS.

In view of the behavior of Heusler's alloys which exhibit a maximum permeability when the manganese and aluminium are present in the alloy in the ratio of their atomic weights, it seemed of interest to ascertain whether or not the susceptibility of a solution of manganese sulphate was affected by the addition of varying quantities of aluminium sulphate.

With this object in view a solution of manganese sulphate containing .0182 gram of manganese per c.c. was prepared, also one of aluminium sulphate containing .018 gram of aluminium per c.c., and one of copper sulphate containing .019 gram of copper per c.c., and the following mixtures of these solutions were examined.

(1) 8 c.c. $MnSO_4$ sol. + 8 c.c. H_2O.
(2) 8 c.c. $MnSO_4$ sol. + 2 c.c. $Al_2(SO_4)_3$ sol. + 6 c.c. H_2O.
(3) 8 c.c. $MnSO_4$ sol. + 4 c.c. $Al_2(SO_4)_3$ sol. + 4 c.c. H_2O.
(4) 8 c.c. $MnSO_4$ sol. + 8 c.c. $Al_2(SO_4)_3$ sol.

The susceptibilities of all the above solutions were found to be practically the same, showing that the aluminium present in the mixtures produced no modification in the susceptibility of the manganese such as it does in Heusler's alloys.

A second series was prepared in which the water added to make up the mixtures to 16 c.c. in the first series was replaced by the copper sulphate solution, and measurements made with these mixtures are given in Table VI.

TABLE VI.

Mixtures.	Susceptibilities $10^7 K$.	Atomic Ratio of Aluminium to Manganese.
I. $MnSO_4$ sol. 8 c.c. $Al_2(SO_4)_3$ sol. 8 c.c.	24.37	2.02
II. $MnSO_4$ sol. 8 c.c. Al_2SO_4 sol. 4 c.c. $CuSO_4$ sol. 4 c.c.	26.04	1.00
III. $MnSO_4$ sol. 8 c.c. Al_2SO_4 sol. 2 c.c. $CuSO_4$ sol. 6 c.c.	26.65	.50
IV. $MnSO_4$ sol. 8 c.c. $CuSO_4$ sol. 8 c.c.	27.49	——

A curve representing them is shown in Fig. 3, where the abscissæ represent the susceptibilities × 10^7 and the ordinates the volumes of copper sulphate in the mixture.

These results show a regular increase in susceptibility proportional to the amount of copper added, which indicates that the susceptibilities of the mixtures followed a simple additive law, such as was found by Wiedemann[1] to hold for other salts. The susceptibility of aluminium sulphate has been shown to be extremely small, and on the basis of an additive law, its effect in the two sets of mixtures tested should be inappreciable. The susceptibility of any mixture should

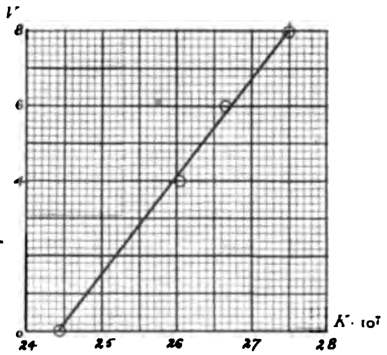

Fig. 3.

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[1] Wied. Elect., III., pp. 955-972.

therefore be determined by the respective susceptibilities of the manganese and copper sulphates, and from the measurements given above it will be seen that this was the result obtained.

With the mixtures tested the atomic ratios of the aluminium to the manganese present in the solutions varied from 2.02 to zero. It is evident, therefore, from the above experiments that the relations the two metals bear to each other under the conditions of the present investigation are quite different from those which exist between them in the alloys. In the latter the aluminium by its presence produces a very pronounced increase in the permeability of the manganese but the measurements made in the experiments described above go to show that no such action takes place between the metals when solutions of their salts are mixed.

PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO,
December 20, 1906.

RESEARCHES ON THE FORMS AND STABILITY OF AËROPLANES.

By W. R. Turnbull.

THE crying need to-day of aëronautics (considered as a science) is research work, and it is simply amazing, when one considers the thousands of scientific workers in all parts of the world devoting their time to refinements and ultra-refinements of well-known physical laws, to note how few persons are willing to take up the laws of aërodynamics and reduce them to an exact science. The field that is open is alluring, even fascinating, and there is more scope for original research, and more chance for reward than in many of the beaten tracks; and yet the student of science still holds aloof, preferring to pass and repass over the footsteps of others.

Just as soon as the mechanical engineer can command at his designing table full data concerning the lift, drift and center of pressure of aëroplanes, the thrust and efficiency of different forms of air propellers, the strength and suitability of materials, and the complete elements entering into the longitudinal and transverse stability of an aërodyne,[1] considered as a whole, just so soon, and not before, can we reasonably expect the problem of aërial navigation to be really solved.

During a careful study of the history of aërodynes, which I have followed to the time of writing (October, 1906), the fact that has most impressed me is that in every one of the proposed machines, both in the tried and untried ones, automatic longitudinal stability is not only a weak feature of the designs but is altogether absent in practically every case.

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[1] I use the word "aërodyne" in preference to "flying-machine," to denote an aëroplane-supported machine, driven by mechanical power through the air. I regret the introduction of a new term in the nomenclature of aërodynamics but there is certainly a term lacking to express the required meaning, for "flying-machine" (Ger. Flugmaschine : Fr. machine volante) is too general and too suggestive of a wing-flapping machine; "aëroplane" (Drachenschweber, Fr. aëroplane) should apply not to the whole machine but to the supporting surface only (moreover, aëroplane is used to denote a soaring machine, etc., which is not power-driven) and "aërodrome" (the term used by Langley) while it is euphonious really denotes an *air course* in the same sense that hippodrome means a course for horse-races. "Aërodrome" in France is also used to mean a balloon-shed (compare Capt. F. Ferber, Revue d'Artillerie, Mars, 1904, also a letter by the writer in The Scientific American, September 22, 1906).

To a fairly satisfactory extent automatic transverse stability (by giving the aëroplanes a dihedral angle or by keels) is a solved problem, but not so automatic longitudinal stability. On this account I took up the laboratory study of aëroplanes with the aim clearly in view of finding if possible a type of aëroplane that should have *automatic* longitudinal stability. In this I have been most fortunate for I have found a type of double curvature aëroplane which not only possesses the elements of automatic longitudinal stability but also gives a very good "lift" and much reduced "drift" when compared with the single curvature surfaces most commonly employed and looked upon as our most efficient lifting aëroplanes.

To make the study systematic let us divide all the possible (*simple*) aëroplanes into five types as follows:

Type I. The plane surface.

Type II. The single curvature surface, convex on the under side.

Type III. The single curvature surface, concave on the under side (the opposite of type II.).

Type IV. The double curvature surface, concave at the forward portion, convex at the after portion, on the under side.

Type V. The double curvature surface, convex at the forward portion, concave at the after portion on the under side.

These are the only possible types (leaving out multiple curvatures of more than two) and we will take up the study of each individually and then compare the results.

In order to further simplify the problem and to make the results with the five different types strictly comparable with one another, I will deal only with surfaces having a rectangular plan-view and in which the width (at right angles to the air current) is twice the length (in the direction of the air current). While this method fails to cover the whole ground, since the number of possible plan-views is practically limitless, I feel assured that, *so far as general laws are concerned* the two-to-one rectangle is fairly typical of all forms.

Throughout the experiments, therefore, the five following planes are alone used: See Fig. 1, in which the common (developed) plan-view is shown at *A* and the various cross-sections along any fore and aft line *ab* corresponding to the above mentioned types are shown at *B*, *C*, *D*, *E*, and *F*. The arrow gives the direction of the air current.

In each case the (developed) length is 8.485 inches and the width is 16.97 inches, giving a two-to-one ratio with an area of 143.99 square inches or practically one square foot.

These five types were experimented with, first as to their "lifts" from 0° to 20°, second as to their "drifts" from 0° to 20° and third as to their longitudinal stability, or as to the position of the center of pressure at all angles from 0° to 90°.

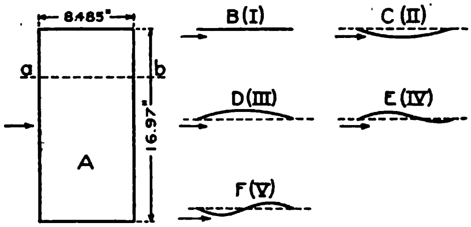

Fig. 1.

METHODS AND RESULTS.

General.

It is a well-known fact that the natural wind varies greatly (even from second to second) in intensity;[1] and it is even a difficult matter to obtain an artificial wind that has absolute constancy; in these experiments, therefore, in order that the results may be exactly comparable with one another, the writer employed a heavy 4-bladed fan-propeller to produce a constant air current and this fan-propeller was driven by an electric motor whose current was from a storage-battery of ample capacity. This fan-propeller was mounted at one (open) end of a rectangular box 6 feet long and 22 inches square and the speed of the propeller was so adjusted that the air-current at the opposite (open) end was exactly ten miles per hour at a distance of one foot from the end of the box. In the box near the propeller and for the first half of its length thin "baffle-plates" were placed to avoid rotary air currents so that a practically rectilinear air current was produced at the experimental end of the box. The box was further raised from the floor about two feet and this is a most important point in such experiments, for if the box be placed on or near the floor the intake of the fan-propeller is unequal with a consequent inequality at the opposite end of the box.

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[1] Compare Langley, "The Internal Work of the Wind," and others.

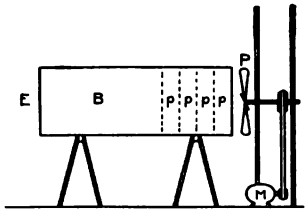

Fig. 2.

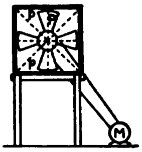

Fig. 3.

Fig. 2 shows a side view and Fig. 3 an end view of the general arrangement of apparatus in all the experiments; in which *M*, is the electric motor; *P*, the propeller; *B*, the long box; *pp*, baffle-plates and *E*, the end at which the different apparatus for "lift," "drift" and center of pressure were placed.

The air current used in all the experiments was 10 miles per hour, for it was found (as before frequently established) that the "lift" and "drift" of the aëroplanes vary as the square of the speed of the air current, so that it is a simple matter to compute the "lift" and "drift" for any speed (within ordinary limits at least) when the values for a certain speed are known. The variation (if any) for the position of the center of pressure, with different velocity of air currents, was not determined, as the system was not sufficiently flexible to obtain high speeds.

The aëroplanes experimented with were made of thin tin plate with only a sufficient number of ribs (three) on the upper side, to make the forms absolutely rigid.

The original experiments were made in February to April, 1905, and were further repeated with refined apparatus in June and July, 1906.

"Lift" Experiments.

The apparatus for obtaining the "lift" (or vertical component of air pressure) of the aëroplanes, was mounted in such a way that the centers of the aëroplanes were one foot from the free end of the box and exactly in the middle of the opening.

Fig. 4 shows a side elevation, and Fig. 5 an end elevation, of this apparatus which is simply a common balance arm, *A* (mounted on horizontal pivots, *pp*), with the aëroplane, *R*, mounted on one arm, and with a movable scale-pan, *S*, on the other arm to measure the loss of weight when the air current passes.

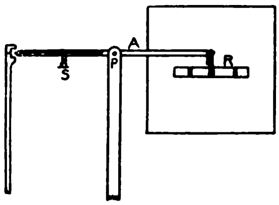

Fig. 4.

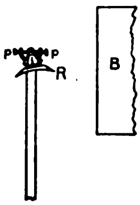

Fig. 5.

Throughout the experiments the aëroplane, *R*, was inclined at angles of 0°, 5°, 10°, 15° and 20° with the horizontal, and the corresponding settings of the scale-pan, *S*, to make a balance when the air current passes, were recorded.

In this way a series of readings were obtained which are plotted in Fig. 6, the average of several readings for each inclination of the aëroplane being used to determine several points on the curves.

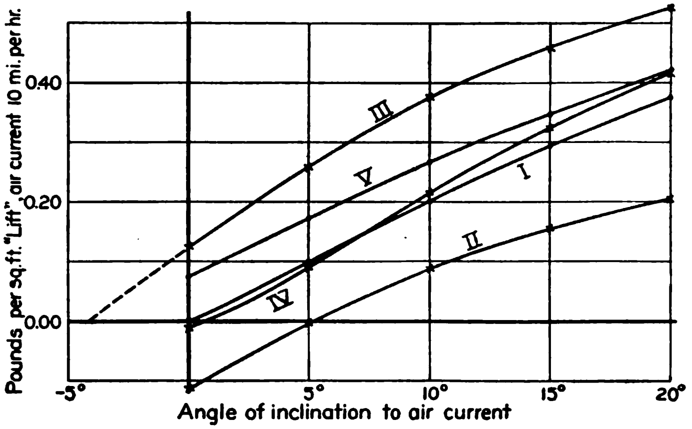

Fig. 6.

It will be noted that the *comparative*[1] lift of Type V. at small angles is slightly greater than at large angles, by comparison with any of the other four types.

It will also be noted that the best "lift" (per se) is given by the single curvature surface, concave on the under side, and the poorest by the single curvature surface, convex on the under side; but, in selecting the best form of aëroplane, for use in an aërodyne, other factors enter as will be seen when these results are considered in connection with the "drift" and center of pressure experiments.

"Drift" Experiments.

The apparatus for measuring the "drift" (or horizontal component of the air-pressure) was mounted in such a way that the center of the aëroplane was one foot from the free end of the box and exactly in the middle of the opening.

Fig. 7 shows a side elevation and Fig. 8 an end elevation of this apparatus, which is simply a balance arm, *A*, mounted on vertical pivots, *pp*, with the aëroplane, *R*, mounted on one arm and with the other arm attached to a cord, *C*, passing over a frictionless pulley, *P*. The scale-pan, *S*, in this case, being attached to the other end of the cord and the "drift" being measured by the adjustment of weights on the scale-pan.

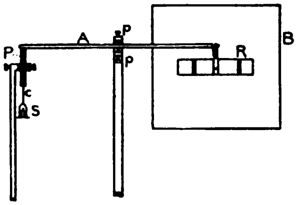

Fig. 7.

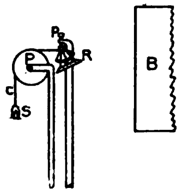

Fig. 8.

As in the case of the "lift" experiments the aëroplane, *R*, was successively placed at 0°, 5°, 10°, 15° and 20° and the corresponding weights to make a balance, when the air current was passing, were recorded. For each reading the "drift" of the arm without the plane was subtracted from the reading, I might mention here that a compound balance (such as was used by Maxim in his experiments) for obtaining both the "lift" and "drift" with the same apparatus was first tried, but the method of employing separate pieces of apparatus for each form of experiment was found to be far more sensitive and accurate.

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[1] The writer has taken out patents in the United States, Canada, Great Britain, Germany and France on Types IV. and V., but by an oversight on the part of the patent attorneys the word "comparative" in referring to the lift of Type V. was omitted from a part of the description rendering the explanation of action absurd as there is no known form of aëroplane which will give a *greater* lift at a *smaller* angle, for ordinary angles between zero and thirty degrees.

The data of the experiments on "drift" are plotted in Fig. 9, and on comparing them with the "lifts" of Fig. 6 we have some interesting results. It is noted that the single curvature surface (Type III.) concave on the under side, not only has the largest "lift," but also the largest "drift," while the "drift" for Type IV. (concave in front, convex in the rear, on the under side) actually decreases from 0° to 4°. For equal drifts we have the table given in the second following section and from this table we see that for equal "drifts" Type IV. at 10° has more than double the "lift" of Type III. at — 0.7°. This fact has more than usual interest, be-

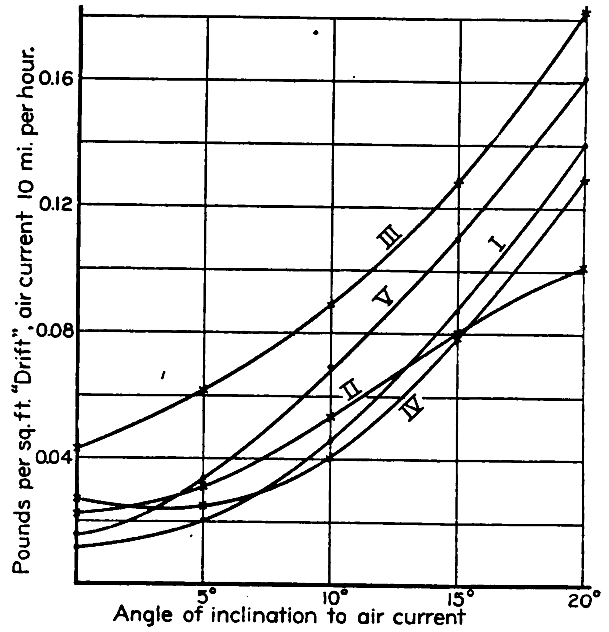

Fig. 9.

cause it will be shown in the next section of this paper that Type IV. has automatic longitudinal stability from 90° to 0°, while Type III. has longitudinal instability between the angles of 28° and 0°.

The decreasing drift of Type IV. between the angles of 0° and 4° has also something to do with the soaring of birds, in the opinion of the writer, but this will be further referred to in another section.

In the two foregoing sections the writer has measured only the vertical component of air resistance ("lift") and the horizontal component of air resistance ("drift"). I have not followed Lilienthal's[1] method of plotting a resultant force (the total air resistance) from the "lift" and "drift" and afterwards resolving this resultant into a "normal" and "tangential" force; for these latter mainly apply in the case of an aëroplane falling through the air at the same time moving forwards along a course at a certain negative angle with the horizontal.

While Lilienthal's method is interesting when applied to his assumed slope of a bird's wing during the down-stroke, it is applicable only to artificial flight when it is sought to solve that problem by beating wings and it is, in general, not applicable to the problem of artificial flight with fixed aëroplanes driven horizontally through the air at a certain positive angle of inclination.

Center of Pressure.

Thorough in many ways as Lilienthal's results on "lift" and "drift" are, he unfortunately passed all too lightly over the more important question of the center of pressure, and while Joessel, Kummer and Langley experimented on this all essential problem, their experiments, invariably stopped short of those critical angles between 25° and 0° which are by all means the important ones in the study of natural and artificial flight.

The first method used by the writer, for measuring the center of pressure, was the usual one of the eccentric plane with counterpoising weight (used by Langley and others, see "Experiments in Aërodynamics," Langley, pp. 89–93, 1902) but its decided limitations were soon apparent and a delicate counterpoising spring was substituted with excellent results, for aëroplanes whose centers of pressure advance far in front of their centers of figure.

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[1] "Der Vogelflug als Grundlage der Fliegekunst," Berlin, 1889.

The counterpoising spring is limited however to use with certain aëroplanes between about 35° and 0° so that a use of both methods is necessitated for a complete set of readings between 0° and 90°.

The aëroplanes to be tested were ruled with lines (parallel to the leading edges) at distances of ½″, 1″, 1½″, etc., in front of (and behind) their centers of figure and at the points where these lines reached the side edges of the aëroplanes screw clamps were successively placed. The clamps being provided with pivot-holes for the insertion of the pivots of the supporting frame.

Figs. 10 and 11 show front and side elevations of the apparatus as employed with counterpoising spring; *M* being the arm holding the supporting frame, *F*. *F* carries small arms which are adjustable along its length to allow of use with different widths of aëroplanes, and these small arms carry the pivots, *pp′*, which carry the aëroplane under test, *A*. The frame, *F*, also carries a vertical rod, *B*, to which is clamped at any desired height, the block, *E*. *E* houses a horizontal rod, *H* (adjustable horizontally), which carries at its end a long sensitive spring, *S*, acting as a counterpoise for the unbalanced weight of *A*.

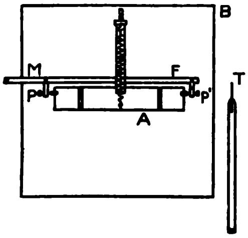

Fig. 10.

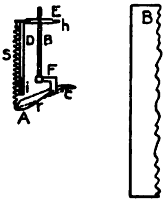

Fig. 11.

S is protected from the air-current by a shield, *D* (V-shaped in plain cross-section), and a small pointer soldered to *S* plays over the scale, *i*, near the bottom of *D*.

The dotted line, *C*, Fig. 11, shows the (imaginary) chord of the curvature of *A* and in every case the upper side of the rib, *r*, is made parallel to this chord so that the angle which *A* makes with the horizontal may be read by means of a vertically mounted transparent protractor (shown in Fig. 10 at *T*) by sighting through the protractor along the upper line of the ribs.

When using the aëroplanes with a counterpoising weight the spring with its accessories was removed and the counterpoise secured by attaching a flat leaden weight to the upper side of the aëroplane under test.

Both methods of experiment were used as the circumstances of each setting of the pivots demanded and where the results of the two methods "overlapped" the accordance of the readings was satisfactory.

I have unfortunately been unable to obtain the original papers of Joessel and Kummer so that I do not know of their methods of experimenting but Langley mounted his counterpoised eccentric planes on the arm of a large whirling table and obtained (from a pencil trace) the angle corresponding to a certain setting of the side pivots and it is just here that the superiority of the method above described (with stationary aëroplanes and artificial air current) is most pronounced, for with several tpyes of aëroplanes (viz.: Types I., III., and V.) there is *not one angle but two* corresponding to one setting of the side pivots.

To illustrate this, let us take the example of plane No. III., with pivots on the center line; by Langley's method the only reading obtained is that at 90°; by the writer's method the reading for 90° is also obtained if the plane is untouched, but if the rear edge of the plane is supported loosely between the thumb and forefinger of one hand and carried to a position nearer the horizontal we find a second angle (at $12\frac{3}{4}°$) where, if the plane be slightly depressed, the rear edge will be strongly depressed, and if slightly raised, the rear edge will be strongly raised and we here have a second angle (capable of accurate measurement) *where the center of pressure again corresponds to the center of figure.* Of course, in place of the thumb and forefinger, a metal yoke with adjustable set screws could be similarly employed and could be vertically raised until this second critical angle was reached, but the thumb and forefinger furnish a sensitive index of the approach to this angle and the accuracy of the results are excellent.

The results of these experiments are given in Fig. 12 where angles with the horizontal are plotted as abscissæ and centers of pressure (in percentage of the side of plane), both before and behind the center of figure, are ordinates.

In the case of the plane (Type I.) we note that the center of pressure steadily advances in front of the center of figure until the critical angle of 26° is reached when the center of pressure rapidly retreats to the center of figure as the angle becomes smaller and reaches 0°.[1]

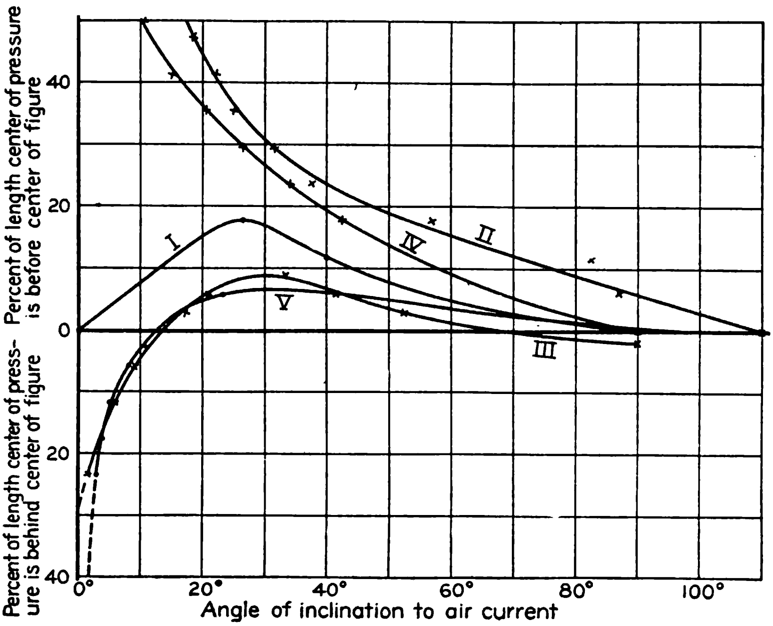

Fig. 12.

In the cases of Types III. and V. this reversal takes place at somewhat near the same angle but at a much shorter distance in front of the center of figure, the center of pressure then rapidly retreats behind the center of figure which it passes at about 13°.[2]

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[1] In the case of *a square* with 9″ side the writer found this critical angle at about 18° and it is just about this point where the curves of Joessel, Kummer, and Langley (who all experimented with square planes) cease. Joessel inferring that the law formulated by him, $c = 0.3(1 - \sin a)\ L$ held good until 0° was reached.

[2] My results for Type III. agree fairly well with those of Spratt who, according to Moedebeck's "Taschenbuch fur Flugtechniker und Luftschiffer," 1904, p. 335, has not made his experiments public. Spratts' results as given by Moedebeck are these: for a single concave curvature surface of $\frac{1}{12}$ curvature (shape not stated) the center of pressure

Type III., on account of its excellent "lifting" qualities, is the aëroplane that has been almost universally used in artificial flight experiments, but this reversal in the position of the center of pressure with its consequent utter lack of longitudinal stability is doubtless responsible for nearly all the accidents that have occured in aëroplane experiments, Lilienthal and Pilcher lost their lives through ignorance of this reversal and Langley's large machine was also probably wrecked through a lack of knowledge on this point.

For Types II. and IV. we have the remarkable result that the center of pressure steadily advances to the forward edge of the plane, where, by reference to Fig. 6, we note that the lifts are both negative. We therefore have, for these two types, *the condition of automatic longitudinal stability for all positive angles and "lifts."*[1]

SIGNIFICANCE OF THESE RESULTS FOR ARTIFICIAL FLIGHT.

In comparing the relative merits of these five types from the viewpoint of artificial flight, we must not only take into consideration their stability curves as shown in Fig. 12, but also compare the "lifts" and "drifts" as given in Figs. 6 and 9.

We then find that for the two aëroplanes of automatic stability Type IV. has a much superior "lift" to Type II. and a lesser "drift" between the angles of 2° and 15°; which are the important ones for artificial flight.

As pointed out above, Type III. has been almost exclusively used in artificial flight, but when it is compared to Type IV. we see that it has an utter lack of stability while Type IV. has automatic stability and while, at first sight, Type III. has the superior lifting qualities it also has a large "drift." As a matter of fact the "lift" of Type IV., as compared to its "drift," is superior to the "lift" of Type III., as compared to its "drift," *for certain angles* as per the table below, in which the respective values of "lift," at their corresponding angles, are given *for equal "drifts"* in each case.

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at 0° lies near the rear side, it advances as the angle increases passing the middle point at about 17° and advances to about 8½ per cent. from the center of figure at 30°. It then moves in the opposite direction until it reaches the center of figure at 90°.

[1] Ahlborn, "Illus. Aeron. Mitteilungen, 1904. States *that only slightly convex* aëroplanes (Type II. of the writer's experiments) possess unconditional automatic stability, but his statement seems to be based on incomplete observations rather than on experiments.

Type IV.		Type III.		Lift IV. ÷ Lift III.
Angle.	Lift.	Angle.	Lift.	Ratio.
20°	.417	15.1°	.459	.91
19	.400	13.7	.439	.911
18	.381	12.4	.416	.916
17	.364	11.1	.395	.921
16	.346	9.6	.368	.941
15	.325	8	.331	.982
14	.303	6.5	.297	1.020
13	.283	4.6	.249	1.136
12	.261	2.9	.204	1.280
11	.240	1	.154	1.558
10	.216	−0.7	.105	2.055

From this table it appears that *for equal drifts*, Type IV. at 10° has actually double the lifting value of Type III. at −0.7°.

Lilienthal's principle for artificial flight was certainly well taken, viz.: "Stability first, propulsion afterwards" but we cannot review the history of experimental aëronautics without insisting on this revision: *Automatic stability first and always* propulsion so soon as prime-movers become sufficiently light.

Aëroplanes of Type IV. give the essential elements of automatic longitudinal stability; the well established principles of the "dihedral angle" and keels give automatic transverse stability; and gas engines, now approaching 3 pounds per horsepower give the required lightness of prime movers.

In these three principles, therefore, the problem of aërial navigation is theoretically solved. It now remains only to so combine these three essentials, in practical form, to solve the problem from an engineering standpoint.

I could hardly insist that aëroplanes of Type IV. are the only ones possessing the all important properties of automatic stability but among simple aëroplanes (*i. e.*, those having zero, single, or double curvature), my laboratory experiments have clearly shown that *Type IV. is the only aëroplane having at the same time automatic longitudinal stability combined with excellent lifting properties and exceedingly small drifting properties*, for all angles that would probably be utilized in artificial flight.

Significance of These Results to Natural Flight.

Lilienthal in his valuable book[1] has given an able analysis of bird flight, but he has made one assumption (page 76) which is hardly tenable, viz.: that the slightly concave surface is the form most like that of the bird's wing.

Neither my experiments nor a general consideration of the structure of the bird's wing lead to this conclusion.

The average cross-section of a birds wing when extended consists essentially of a forward portion, *A*, Fig. 13, of rigid formation, with a central portion, *B*, of partial rigidity, and a rear portion, *C*, of extreme flexibility.

While this formation certainly varies from the portion nearest the body to the extreme tip of the wings, I think it will be generally conceded that Fig. 13 fairly represents an average cross-ectiosn of an average bird's wing (during the down stroke).

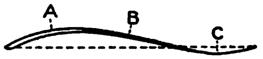

Fig. 13.

If now we conceive this wing in rapid up-and-down strokes passing through a resisting medium, such as the air, it requires only a rudimental conception of mechanics to infer that in the up stroke the cross-section assumes the form shown in *AC*, Fig. 14, while in the down stroke a form similar to that shown in *AC*, Fig. 15, would be necessitated.

In each figure, *op*, gives the direction of the wing motion in each case, while *or* represents the absolute direction of the air current. It is obvious that the *relative* movement of the air particles is roughly represented by *qo* and it is equally obvious that a wing, of the type described in connection with Fig. 12, would be forced into the approximate forms shown in Figs. 14 and 15—*corresponding to the two Types II. and IV. both of which have been shown by the laboratory experiments to have automatic longitudinal stability.*

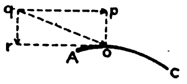

Fig. 14.

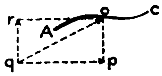

Fig. 15.

If then we accept this reasoning, it is proven that in bird flight the two forms assumed by the wing *both have automatic longitudinal stability*, that in the up stroke the "lift" (negative in this case) is a minimum (see Type II., Fig. 6) while the "drift" is small (see Type II., Fig. 9) and that in the down stroke the "lift" is excellent (see Type IV., Fig. 6) while the "drift" is a minimum (see Type IV., Fig. 9).

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[1] "Der Vogelflug als Grundlage der Fliegekunst," Berlin, 1889.

From this it is seen that the laboratory experiments are in full accord and give a most rational explanation of the wonderful properties displayed in the flight of a bird.

For soaring or sailing flight the most probable form of the bird's wing is that shown in Fig. 16 which is simply a less accentuated form of Fig. 15, due to the lessened air pressure on the under side of the wing.

The resolution of forces as shown in this figure gives a perfectly rational explanation of soaring flight, by which a bird is supported and at the same time advances against an ascending current of air, for in working out the "lift" and "drift" under these conditions we find that *OR*, the resultant, is in advance of the normal to the chord of the double curvature, and we have for the forces acting upon the surface the vertical component *ON* equal to the weight of the bird and the tangential force, *OT*, which furthers, instead of hinders, the advance against the air current.

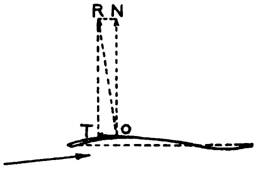

Fig. 16.

It has been frequently shown by delicate anemometer measurements, etc., that no natural wind is steady but that the natural air currents have a pulsating movement, with pulsations varying sometimes several times in a second. The soaring bird can instinctively feel these pulsations and can make excellent use of them by slightly inclining the extended wings more or less to the pulsating current and can thus maintain or increase its height while at the same time it slowly advances against the air current.

Experiments from many sources (Lilienthal and others) have shown that there is also in natural winds, near the surface of land or water, an ascensional trend of about 3° and this further facilitates the soaring of most birds.

Referring again to the automatic stability of birds, the extended tail in many species is undoubtedly a valuable *auxiliary* to the maintenance of automatic longitudinal stability; but it is quite evident from a consideration of several species in which the tail is almost lacking (*e. g.*, the wild duck) that the tail is not essential, and this further accentuates the idea that the primal and main cause of automatic stability in bird flight is to be found in the wing-forms shown in Figs. 14 and 15.

Lilienthal's close adherence to his conception of the natural wing form was the probable cause of his death, for in all of his soaring machines (and in a great majority of soaring machines that have since been built) the single concave surface has alone been used and with it of necessity (see Curve III., Fig. 12), automatic stability is impossible and "stability" is only obtainable by the agility of the operator in moving his body or an auxiliary steering aëroplane.

The Influence of an Underplane.

In considering the subject of aëroplanes used in artificial flight it occurred to the writer that the lifting properties of aëroplanes might

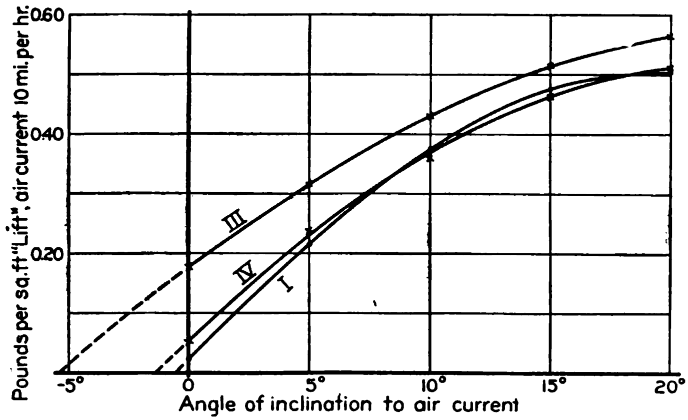

Fig. 17.

be considerably increased by allowing the aëroplanes to skim close to an underplane such as a level piece of land or close over water.

As is shown in Fig. 17, a considerable increase in "lift" really occurs (due no doubt to the wedging action and consequent compression between the aëroplane and underplane) and at first sight it would seem a valuable consideration in the design of an aërodyne; since it is just when arising and alighting, when the speed would be comparatively low, that the maximum "lift" would be desired. However, many other modifying conditions must be considered, since screw propellers must have a considerable diameter and their central line of thrust must be in line with the aëroplane resistance; and, moreover, the "drift" is also considerably increased with an underplane and the center of pressure is shifted, in general, considerably farther "aft" than it would be in free flight, introducing serious difficulties in balancing. These various drawbacks of the underplane may be overcome (*e. g.*, by superimposed planes), but it is well to draw attention to them.

In Fig. 17 the "lift" curves, with an underplane, are given for Types I., III., and IV. The readings with the underplane were obtained with the apparatus shown in Figs. 4 and 5 under the same conditions as before, but with an underplane (a large drawing board) placed horizontally one half inch below the rear edge of the aëroplane in its mean position of balance.

It is needless here to go into details concerning the influence of an underplane on the "drift" and center of pressure of aëroplanes, but the writer's experiments with "lifts" may be summarized as follows: (1) With plane surfaces (Type I.) and double curvature surfaces (Types IV. and V.) the *relative increases* of "lifts" with an underplane are greater than in the case of single curvature surfaces. (2) Under certain conditions the lift may be nearly doubled by an underplane very close to the rear edge, but in general, the influence nearly disappears at a distance above the underplane equal to about one third the length (in the direction of the air current) of the aëroplane.

In conclusion, with regard to the general design of a practical aërodyne, I can only say that the proportion of parts is an essential feature which must be worked out by outdoor experiments, preferably over a sheet of water.

The laboratory experiments, described in this paper, can give but little help in the proportioning of parts, but they clearly show the type of aëroplane to be adopted and the type to be distinctly avoided. On the one hand we have Type IV., with its great advantages of *automatic longitudinal stability, excellent lifting properties and exceptionally low "drift"* and on the other hand the long-adopted aëroplane with single concave curvature, which, while it has most superior lifting qualities, has also a most serious "drift" (for high speeds particularly to be considered) and *above all in importance, most treacherous longitudinal stability.*

A trifle in added speed gives the former type equal "lift" with the latter (since the lift and drift increase with the square of the speed), so that it should be an easy matter for a designer to choose unhesitatingly between the two.

ROTHESAY, N. B., CANADA,
October 10, 1906.

ON THE NATURE OF OPTICAL IMAGES.

BY ALBERT B. PORTER.

THERE is no doubt that the teaching of mathematics has suffered from the somewhat arbitrary division of the subject matter into what have been aptly termed "water-tight compartments." To a lesser degree the same thing is true of physics, if current teaching may be judged by some of the current texts. Thus in the treatment of light, for example, although the sharp distinction between geometrical and physical optics has largely passed with the passing of the old-fashioned (but often indispensable) ray-method of treating the formation of images, yet the wave-method which has replaced it is sometimes presented in a form almost as purely geometrical. We have abandoned the diagrams in which the so-called rays were frankly intended to represent nothing more than directions of propagation, and have substituted figures showing an impossible array of sharply limited wave-fronts primly marching toward a focus, with never a trace of diffractive fraying at the edges. Even in the few text-books with which I am acquainted in which this fraying is taken into account in an investigation of the effects of aperture on resolving power, the discussion follows so far after the study of image formation that the student must almost inevitably absorb the idea that diffraction and interference merely act to modify an image otherwise formed. Nor do I anywhere recall in the elementary books an explicit statement to the effect that all optical images arise by interference and are indeed particular cases of interference patterns. That this is, however, true appears at once from a consideration of the method of propagation of light by means of secondary wavelets, which spread in all directions from every point of the wave-front in such a manner that, were it not for interference, each point in the image would receive light from all points in the object. Dark places in the image are thus dark because of destructive interference, and light parts are bright on account of additive interference. All this, though of course sufficiently familiar to the teacher, may be far from evident to the student, and this is my excuse for describing a simple and quickly arranged experiment which shows the relation between interference and image formation in a manner so striking as not to be easily forgotten.

The experiment consists in passing a parallel beam of monochromatic light through a coarsely ruled, black-line, diffraction grating, and then through a convex lens. On the far side of the lens a system of sharply defined interference fringes is formed which can be seen by aid of an eyepiece, or intercepted on a screen, at any point over a considerable range along the axis. Somewhere in this system of fringes is the geometrical image of the grating, but it is visually quite indistinguishable from any other transverse section of the fringe system. Clearly in this case, the geometrical image is merely that section in which the geometrical condition of similarity to the object is satisfied.

The best arrangement of the experiment is the following. Light from an arc lamp *A* is focused by means of the lens *B* upon a narrow slit *C*. Thence it passes through a direct-vision prism at *D*, and the spectrum is focused by the lens *E* upon the narrow slit of a collimator *FG*. The parallel beam of monochromatic light thus obtained falls upon the mirror *H* of a microscope *KJ*, upon whose stage, at *I*, the grating is placed in such a position that its ruled lines are parallel to the projections of the two slits *C* and *F*. Using a black-line grating of 400 lines to the inch, and having both slits narrowed down to a small fraction of a millimeter so as to secure very homogeneous illumination, the field of view was examined

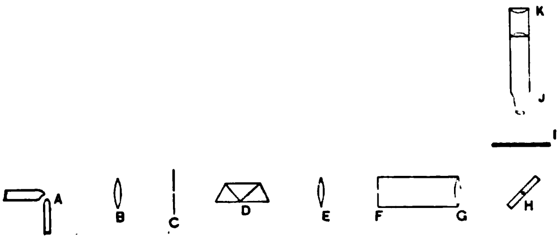

Fig. 1.

with ⅔ inch objective and 1 inch eyepiece. The interference fringes appeared in the field of the eyepiece with exquisitely sharp definition throughout the whole range of the coarse adjustment of the microscope, *i. e.*, over a distance of 58 mm., beginning with the front of the objective in contact with the grating and with its focal plane 7 mm. below the ruled surface; and the fringes could be traced through a much greater range by withdrawing the eyepiece and moving it back along the axis. As the microscope is slowly focused upward, the bands undergo curious changes in appearance, the lines showing sometimes close together and again farther apart, but the definition is almost equally sharp throughout the whole range of adjustment, so that any section of the fringe system is as good an apparent image as any other section. Similar but less perfect effects may be obtained by illuminating the field by means of sodium light passing through a slit a couple of millimeters wide at a distance of one or two meters.

If the angle of the incidence of the light on the grating is changed by moving the mirror, the whole fringe system shifts to one side or the other except in the focal plane, where it remains stationary. This shows (1) that the focal plane is the plane in which the interference fringes formed by light of all incidences coincide; (2) that, when a broad source is used, the geometrical image is really a superposition of coincident interference patterns; and (3) that the usual absence of a sharp image outside the focal plane is due to the more or less uniform illumination resulting from the overlapping of fringe systems due to light coming from various points in the source. When the grating is illuminated by a parallel beam of white light by means of a collimator with very narrow slit or, less perfectly, by a distant gas flame turned edgewise, the effects are similar except that outside the focal plane the fringes are colored. Hence (4) the focal plane is also the plane of achromatic interference, *i. e.*, the plane in which the fringes due to light of various wave-lengths coincide.

These experiments show very clearly why it is in general essential to use a condenser to illuminate the field of a microscope in order to obtain a critical image, *i. e.*, an image which comes sharply into and out of focus and which is hence as free as possible from confusion with details of structure lying above and below the focal plane. It is interesting to observe how, as the illumination is made less homogeneous and more convergent, the distance through which the interference bands can be distinguished decreases, and the precision of focusing increases.

In the case of self-luminous objects, although the geometrical image is still to be considered as an interference pattern, the effects outside of the focal plane are greatly modified by the absence of definite phase relations between the waves emanating from various points.

RADIATION FROM SELECTIVELY REFLECTING BODIES.

By W. W. Coblentz.

In his memoir on infra-red reflection spectra the author[1] shows that all the silicates examined have bands of strong selective reflection in the region of 8 to 10 μ and discusses the effect this would have upon a reflecting surface like the earth or the moon which is largely composed of silicates. Since this discussion has brought forth criticism, favorable and unfavorable, in print and in private communications, the purpose of the present paper is to summarize, in a general way, the present data bearing upon the subject, and to discuss certain points not mentioned previously.

THE EFFECTIVE TEMPERATURE OF THE SUN.

It has been shown by Poynting[2] that, when a surface is a complete radiator and absorber, its temperature can be determined at once by the fourth power law, if we know the rate at which it is radiating energy. If it is radiating what it receives from the sun, then a knowledge of the solar constant enables us to find the temperature, which will be the highest the surface can attain when it is receiving heat only from the sun. Knowing the solar constant and the radiation constant ($\sigma = 5.32 \times 10^{-5}$ ergs Kurlbaum[3]) Poynting computes the effective temperature of the sun. If s is the radius of the sun's surface, and R the radiation per square centimeter then the total rate of emission is $4\pi s^2 R$. This must equal the radiation passing through the sphere of radius r, at the distance of the earth, and with a surface $4\pi r^2$ gives

$$4\pi s^2 R = 4\pi r^2 S$$

where S is the solar constant.

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[1] Investigations of Infra-Red Spectra, Parts III. and IV. Published by the Carnegie Institution of Washington, December, 1906.

[2] Phil. Trans. Roy. Soc. London, Vol. 202 A, p. 525, 1903. Of course no surface is a complete radiator or absorber of energy.

[3] Kurlbaum, Wied. Ann., 65, p. 748, 1898.

Hence, $R = 46{,}000\, S$. Using the solar constant $S = 2.5$ calories $= 0.175 \times 10^7$ ergs per sq. cm. per sec., $R = 0.805 \times 10^{11}$.

If we equate the sun's radiation to $\sigma\theta^4$ where σ is the radiation constant, we get θ, the "effective temperature" of the sun, that is, the temperature of a full radiator which is emitting energy at the same rate.

Thus

$$5.35 \times 10^{-5}\theta^4 = 0.805 \times 10^{11}$$

whence

$$\theta = 6{,}200^\circ \text{ Abs.}$$

With each new determination, however, the solar constant is being reduced from the former high values, so that the more probable value[1] is about 2.1 gr. cal./cm.^{2} min.

Using this value of $S = 2.1 = 0.147 \times 10^7$ ergs/cm.^{2} sec., $R = 0.676 \times 10^{11}$.

Thus

$$5.32 \times 10^{-5}\theta^4 = 0.676 \times 10^{11}$$

whence

$$\theta = 5{,}980^\circ \text{ Abs.}$$

This is somewhat closer to Wilson's[2] value, 5,773° Abs. which he obtained by making a direct comparison of the radiation from the sun with that from a "full radiator"[3] at a known temperature.

The Limiting Temperature of the Surface of the Moon.

Poynting (loc. cit.) further shows that when there is no conduction inwards from the surface, the highest temperature of a full radiator is attained when its radiation is equal to the energy received. Equating the energy to the solar constant using

$$S = 0.175 \times 10^7$$

$$5.35 \times 10^{-5}\theta^4 = 0.175 \times 10^7$$

whence $\theta = 426^\circ$ Abs. (Moon's temperature).

[1] Abbot, Smithsonian Miscellaneous Collections, Vol. 45, p. 81, 1903.

[2] Wilson, Proc. Roy. Soc., Vol. 69, p. 312, 1901. Fery and Millóchau (C. R., 143, p. 570, 731, 1906) using a pyrometer found the temperature of the solar surface to vary from 5888° to 5963° Abs. with a mean value of 5620° Abs.

[3] Poynting (loc. cit.) very aptly says: "A surface which absorbs and therefore emits every kind of radiation, is usually described as 'black,' a description which is obviously bad when the surface is luminous. It is much better described as 'a full absorber' or 'a full radiator,'" *i. e.*, a complete radiator, as distinguished from a partial radiator or so-called "non-black body." In justice to Kirchhoff who was the first to give a clear discussion of this subject it should be called the *Kirchhoff Radiator*.

If part of the energy is reflected and only a fraction x of that falling on it is absorbed then the effective temperature is $x^{1/4} \times 426$ Abs. From Langley's[1] estimate that the moon absorbs $x = \frac{7}{8}$ the energy it receives, Poynting (loc. cit.) computes upper limit of temperature of the surface exposed to a zenith sun to be

$$\theta = 426 \times (\tfrac{7}{8})^{1/4} 412^\circ \text{ Abs.}$$

But this upper limit to the temperature of the hottest part of an airless planet is never attained because the moon turns the same face to the earth instead of to the sun. He shows that, if N is the normal stream of radiation from a unit of surface of the moon immediately under the sun, the normal stream from the equivalent flat disk is $N_d = 2/3\ N$. "The effective temperature of the flat disk is therefore $\sqrt[4]{2/3}$ that of the surface immediately under the sun at the same distance from it. Then the effective average temperature $= 412 \times \sqrt[4]{2/3} = 371^\circ$ Abs. The upper limit then, to the average effective temperature of the moon's disk is just below that of boilnig water." If we use $S = 0.147 \times 10^7$ instead of 0.175×10^7.

$$5.32 \times 10^{-5}\theta^4 = 0.147 \times 10^7$$

whence $\theta = 408^\circ$ Abs. Hence $x^{1/4} \times 408 = 395^\circ$ Abs. and the "effective average temperature" is $395 \times \sqrt[4]{2/3} = 355^\circ$ Abs. or 82° C. for the full moon. This assumes no conduction inwards. Evidently there is an appreciable amount of heat conducted inwards, for the results of Langley (loc. cit.) and of Very[2] show that the surface of the full moon is about 300° Abs.

FALL OF TEMPERATURE OF THE MOON DURING ECLIPSE.

Langley (loc. cit.) made observations on the eclipse of Sept. 23, 1885, which indicate a sudden and very rapid fall of energy from the moon as the eclipse commenced, with some indications of a rise nearly as rapid after its conclusion. In Fig. 1 are plotted his observed galvanometer deflections during the progress of the eclipse. The observations were interrupted by the formation of clouds just at the predicted time for the moon to leave the umbra. The curve shows that in the short time of about 1.5 hours, in passing from the penumbra to the umbra, the radiation from the west limb has fallen from a maximum to a zero value. In other words the fall of temperature is practically coincident with the change in illumination, and at first appeared to the writer[1] to indicate that the greater part is reflected energy. That the moon at mid-eclipse is still as warm as the earth is shown by the fact that the galvanometer gave zero deflections. If the moon had been cooler than the earth then the deflections would have been negative. Subsequent search of the literature on the subject shows that Very[2] found appreciable radiation from the moon during totality. The following computations show that this is to be expected. After about 11 days of insolation the temperature of the sun-lit surface of the moon will be fairly constant. From the surface inwards there will be a layer which will be at a uniform temperature, for the period of 1.5 hours, as compared with 11 days. If then the moon were suddenly eclipsed find the fall of temperature of the surface with time ($x = 0$, $\theta = f(t)$, assuming at time $t = 0$, $\theta_0 = 300°$ Abs.) is found from the equation:

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[1] Langley, Nat. Acad. Sci., Vol. 4, part 2, p. 197.

[2] Very, Astrophys. Jour., 8, p. 199, 1898. Very, however, shows that the maximum temperature may be 455° Abs. for limited regions of the moon's surface, and (p. 286) a a mean temperature of about +97° C.

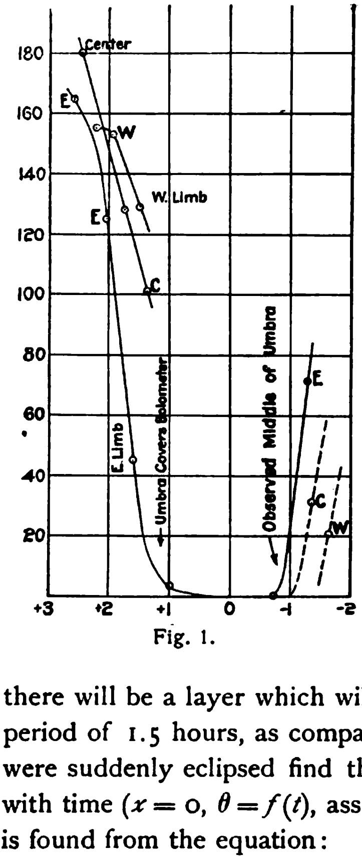

Fig. 1.

$$k\frac{d^2\theta}{dx^2} - \sigma\theta^4 = \rho s\frac{d\theta}{dt}$$

where $k =$ conductivity, $\rho =$ density and $s =$ specific heat. The dependent variable enters here as a fourth power, and makes the solution difficult. The computations are made for instantaneous eclipse, while in the actual case the shadow moves slowly across the surface. Hence the error due to neglecting conductivity is partly compensated by the slow eclipse.

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[1] Physical Review, 23, p. 247, 1906.

[2] Very, Prize Essay on the Distribution of the Moon's Heat, p. 40.

By neglecting conductivity from the interior, the temperature of the surface will fall more rapidly and we have a steeper temperature decadence curve. The solution of the equation is very simple if we neglect conduction and is

$$\theta = \frac{\theta_0}{\sqrt[3]{\frac{3\theta_0^3\sigma t}{\rho s} + 1}}$$

For t in minutes $\sigma = 76.6 \times 10^{-12}$ gr. cal./cm.^{2} min. $\rho = 2$, and $s = 0.2$ (approximate values for rock material). Hence

$$\theta = \frac{300}{\sqrt[3]{1 + 0.0155t}}$$

for a complete radiator.

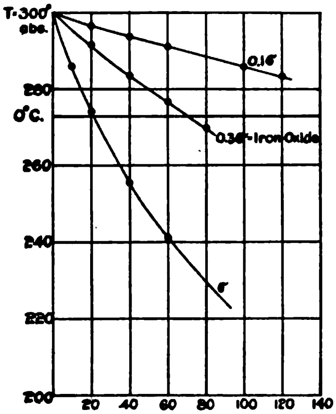

Fig. 2.

In Fig. 2 are plotted the temperature decadence curves for the radiation constant σ, for 0.3 σ (emissivity of iron oxide) and for 0.1 σ. The curves show that for a complete radiator the temperature would fall to 280° (room temperature when the bolometer would be in temperature equilibrium with the moon) in 17 minutes, for 0.3 σ in 55 minutes, and for 0.1 σ in 140 minutes.[1] By including the conductivity term these periods would be considerably prolonged. The present solution is close enough, however, to show that the temperature decadence curve is not coincident with the eclipse curve, unless the emissivity is of the order 0.3 σ. Hence, the writer's[2] previous surmise that at the beginning of totality radiation should still be appreciable is substantiated, although the observations made by Langley, Fig. 1, seemed to contradict it. Very's observations[3] show that during totality of the eclipse of January 17, 1889, the radiation from the umbra of the eclipsed moon was about 1 per cent. of the heat which was to be expected from the full moon.

A computation was also made assuming the temperature to be

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[1] The solution for 0.1 σ and 0.3 σ is, of course, only approximate since the emissivity varies more nearly as the 5th power instead of the 4th power, as here used: and the computed temperatures should be somewhat higher.

[2] Phys. Rev., 23, p. 247. 1906.

[3] Very, Distribution of the Moon's Heat, p. 40.

350° Abs. Here for σ, the temperature would fall to 288° in 38 minutes and for 0.3σ in about 130 minutes.

SELECTIVE REFLECTION OF SILICATES COMPARED WITH REFLECTION FROM THE MOON.

The writer (loc. cit.) has examined the infra-red reflecting power of a series of materials including quartz, feldspars, amphiboles and micas, which are the chief constituents of the rocks of the earth's crust and has found a uniformly decreasing reflection power of 3 to 5 per cent. at 4 μ to 0.3 per cent. at 7 μ, followed by bands of metallic reflection from 8.5 to 10 μ. In this latter region the mean reflecting power is from 40 to 50 per cent. for silicates while pure quartz has a reflecting power of 90 per cent. The reflection curves are given in Fig. 3, in a condensed form. In the lower part of the

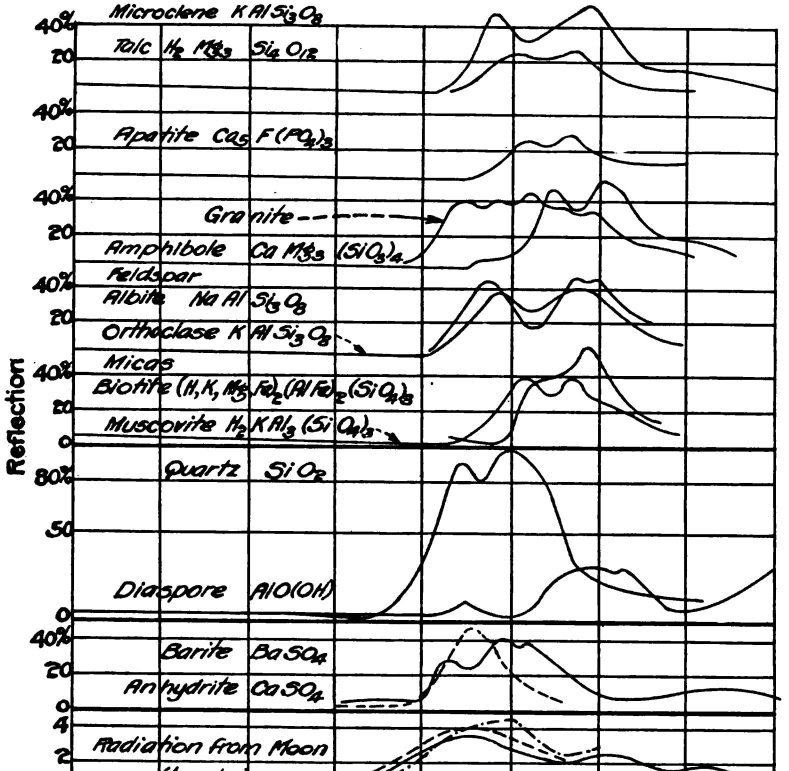

Fig. 3.

figure several of Langley's lunar radiation (reflection) curves are given, in which the ordinates are in arbitrary units. Of course, the absolute reflecting power of the moon is unknown. Zöllner has shown that, owing to the irregularities of its surface, the full moon does not reflect as a smooth sphere would do, but very nearly as a flat disk of like reflecting power and filling the same angle. Such a disk, if it diffused all the solar energy which falls on it, would send to us 1/97,300 of what the sun does provided it reflected perfectly, in all directions, the total incident solar energy. The moon, however, is not such a reflector, and in the visible spectrum the solar light is about 500,000 times moonlight. On this scale the curves of the moon given in Fig. 3 would not be visible. The depression in the lunar curves at 9.6 μ, is due to an atmospheric water band. The region from 5 to 7 μ likewise contains water bands. One can readily see from these curves that a reflecting surface composed of rocks such as granite, basalt, diorite, etc., which are mixtures of quartz, feldspar, hornblende and mica would give a curve similar to that of the moon, *i. e.*, that part transmitted by the earth's atmosphere between 8 and 12 μ.

The general assumption is that the crust of the earth and of the moon have the same constitution, viz.: a series of silicates. It follows from the present discussion that the earth and the moon may be considered selectively reflecting surfaces, with a band of metallic reflection from 8.5 to 10 μ. Hence, in the region of the spectrum from 4 to 7 μ only 3 to 5 per cent. of the radiation from the sun would be reflected from the moon, under the best surface conditions (diffuse reflection), while in the region from 8 to 10 μ the reflecting power would rise to 50 or even 80 per cent. Under actual conditions the balance will still be in favor of the region of 8 to 10 μ. In the region of 7 μ the reflecting power falls below 1 per cent.

Reflection and Radiation from the Moon.

From an examination of some of the discussions of the reflection of the moon it appears that a matte surface cannot be selectively reflecting. In fact the whole misunderstanding seems to hinge on this point. Diffuse reflection follows from directed (or scattered) radiation falling upon small reflecting surfaces, the planes of which lie in all directions. For each infinitesimal surface, the ordinary laws of reflection are obeyed in full, unless the linear dimensions of the reflecting surface, or of the rugosities or inequalities on it, are small compared with the wave-length. In diffuse reflection (see Wood's Optics, p. 36 for examples) the irregularity decreases as the angle of incidence increases, so that for a given roughness we get regular reflection.[1] The long waves will be reflected first, then the shorter. But a surface consisting of a substance like quartz, which reflects like a metal for $\lambda = 8.5$ and 9.03 μ, and like a transparent medium for all other wave-lengths, must still be selective for diffuse reflection. We are not so much concerned with the question as to where the maximum emission of the moon occurs as we are in the fact that, in the region of 8 to 10 μ, where Langley observed radiation from the moon, there is also reflected energy from the sun.

Assuming that the temperature of the moon is 350° Abs. and the temperature of the sun[2] is 5,900° Abs., the problem is reduced to the calculation of the relative intensities of the energy emitted and reflected by the moon at 8 to 10 μ. From the transmission curves of the water vapor (Fig. 6) it will be seen that in the region from 5 to 8 μ almost all the energy will be absorbed by the earth's atmosphere. The reflecting power of the moon for visible rays, according to Langley (loc. cit.), is only 1/500,000 full sunlight. Assuming that at 9 μ the reflecting power is, on an average, 10 times that at 0.5 to 4 μ (a low estimate), the value becomes 1/50,000 or 0.00002 per cent.

Using the above values for the temperature of the moon and of the sun, from Planck's[3] formula for the distribution of the energy in the spectrum of a full radiator (which, of course, the moon is not)

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[1] Apropos of this discussion, Very (Ap. Jour., 8, p. 278, 1898) records that the percentage of reflected rays, as measured by a bolometer, is "especially large (perhaps two or three times the usual proportion) in the narrow crescent moon where the rays suffer a grazing reflection at a large angle of incidence, the emission under the corresponding angle of emission being small."

[2] This value is about the mean of the observed and computed temperatures. Abbot's (loc. cit.) solar spectrum energy curves deduced from bolometric measurements give us further data for computing the temperature of the sun. The published curve is drawn very asymmetrical and is much depressed on the side of the short wave-lengths. The data given permit one to draw a more symmetrical curve (as one would expect it to be) which shifts the maximum wave-length from 0.49μ to 0.46μ. From $\lambda_{\max} T = 2920$ the latter value ($\lambda = 0.46\mu$) gives a "black body" temperature of about 6300° Abs. If the sun's radiation is purely thermal it must lie between that of a complete radiator and that of some highly refractory substance like bright platinum, for which the constant is 2630 instead of 2920. Using 2630 the computed temperature is $T = 5700°$ Abs.

[3] Planck, Verh. d. Deutsch. Phys. Ges., 2, p. 202, 1900.

$$E = c_1\lambda^{-5}(\varepsilon^{c_2/\lambda T} - 1)^{-1}$$

we can obtain the ratio of the intensities for the two temperatures $T_1 = 350°$ Abs. and $T_2 = 5{,}900°$ Abs. from the formula

$$(\varepsilon^{c_2 \div \lambda T_2} - 1) \div (\varepsilon^{c_2 \div \lambda T_1} - 1)$$

where $c_2 = 14{,}500$ and λ max. $= 9\,\mu$ the ratio is 0.00316. But the moon, not being a perfect radiator, will have a smaller emissivity at 8 to $10\,\mu$. If its surface were iron oxide[1] its emissivity would be only 0.3 that of a Kirchhoff radiator, and for the region at $9\,\mu$, judging from the drop in the emission curve (Fig. 5) the emissive power may be less than this, say 0.1. This ratio of the emissive power of the moon to that of the sun will then be 0.000316, which is 16 times (0.000316 ÷ 0.00002) the reflected energy of the sun from the moon. Computations like these, however, which require all sorts of assumptions, can be of little value[2] ultimately. As matters now stand, we know that Langley observed also direct radiation from the sun, in this region of the spectrum, and from existing data of the radiation from the moon we do not know how much of it is selectively reflected energy from the sun. In the visible spectrum the observed energy will consist of that reflected from the surface and that part which enters the particles and is totally reflected. In the region of 8 to $10\,\mu$ (for silicates) almost all of the observed energy will be reflected from the surface (no internal reflection), hence the above computations can be only rough approximations.

EMISSION FROM A PARTIAL RADIATOR.

We have now to consider a problem which has heretofore not been discussed in connection with the radiation from the moon. Planck's formula for the intensity of radiation at a given wave length, λ, in the spectrum of a full radiator is

$$E_\lambda = c_1\lambda^{-5}(\varepsilon^{c_2/\lambda T} - 1)^{-1}.$$

For any radiating body the emission

$$\varphi_\lambda = A_\lambda E_\lambda$$

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[1] Kayser, Spectroscopy, Vol. 2, p. 80.

[2] Very, Astrophys. Jour., 24, p. 353, using different data finds a larger ratio than above.

where A is the absorption coefficient. But $A_\lambda = 1 - R_\lambda$(for non-conductors) where R is the reflecting power, and hence, $\varphi_\lambda E_\lambda (1 - R_\lambda)$.

It follows therefore that a selectively reflecting body like quartz will be a partial radiator and that if we compare its radiation curves with that of a full radiator, there will be *minima* of *emission* in the region of 8.5 μ and 9.03 μ. This was predicted by Aschkinass[1] and experimentally verified by Rosenthal,[2] with quartz mica and glass. The latter found the observed emission curves of these substances to coincide precisely with the computed curves, using the observed reflecting power of these minerals and the observed emission curve of a complete radiator at the same temperature.

Since we have constantly before us examples of selective emission, such as the Welsbach mantle, and high temperature radiation of such metals as tungsten and tantalum it is of interest to consider the radiation of several substances having bands of selective reflection.

In Fig. 4 is given an illustration of the marked effect of selective reflection upon emission. The extraordinary reflection of carborundum, SiC, curve *a* has been found in only one other substance, viz., quartz, and in the latter it is the result of two well defined bands, at 8.5 and 9.03 μ. The effect of such a band on the dispersion of this substance must be very marked. In fact the unusual dispersion in the visible spectrum, found by others, no doubt is greatly influenced by this band. In this figure are given the computed energy curve, *b*, for a perfect radiator (using Planck's

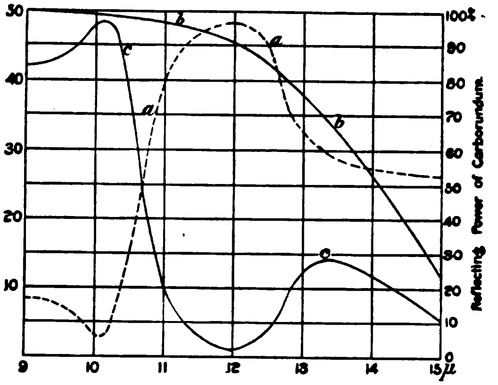

Fig. 4.

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[1] Aschinass, Verh. d. Deutsch. Phys. Ges., 17, p. 101, 1898.

[2] Rosenthal, Ann. der. Phys. (3), 68, p. 791, 1899.

formula, any temperature might have been selected instead of 300° Abs.) and the computed energy curve, *c*, of carborundum at the same temperature, using the formula $\varphi_\lambda = E_\lambda(1 - R_\lambda)$ where R is the observed reflecting power and E is the corresponding intensity of the complete radiator. It will be noticed that the radiation is highly selective, and that at 10 μ it approaches closely to that of the full radiator. If then one were to observe the emission curve of carborundum, the minimum at 10 μ would appear as an "emission band," but its explanation is different from that of emission bands of hot vapors, *e. g.*, CO_2 and H_2O in the Bunsen flame. Melted gold and copper are good examples of selective emission.

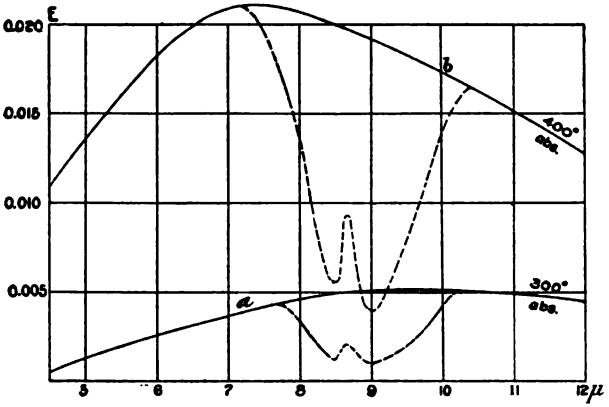

Fig. 5.

In Fig. 5 are given the computed emission curves, *a* and *b*, for a full radiator, for temperatures 300° and 400° Abs. respectively, and the corresponding emission curves[1] (dotted curves) of quartz, at the same temperature using the values of the reflecting power of quartz given by Rosenthal (loc. cit.). These values are somewhat lower than found by the writer. In general, the difference in the emissivity of the silicates and that of a full radiator, at 8 to 10 μ would not be so marked as in quartz. For mica (see Fig. 2) the emission minima would occur at 9.1 and 9.8 μ while for granite the emission minimum would extend from 8.5 to 10 μ. The effect of the combination would produce an appreciable depression in the radiation curve of a surface like that of the moon. Even, if we exclude atmospheric absorption, the emission curve will not be smooth and continuous, as some writers seem to think. If we superpose atmospheric absorption, the emission curve will be somewhat as shown in curve *b*, in Fig. 6. The transmission curve, *c*, is for a column of air 110 meters in length, containing 1.2 mm. precipitable water, and is due to Langley (loc. cit.). A thicker layer would shift the maximum at 8 μ toward 8.3 μ, found in the moon's radiation curve. In Fig. 7, curves *b*, *c*, *d*, show several of Langley's observed lunar radiation curves, and as a whole there is a close parallelism between the theoretical and the observed curve, especially at 10.7 μ where we have to consider only atmospheric absorption. From 8 to 9 μ, however, we have to consider the combined effect of atmosphere absorption, of incomplete emission of the moon (emission minima from 8.5 to 9.8 μ) and of selectively reflected energy from the sun. The latter will, no doubt, vary the most. The computed emission curve is the most intense at 10.2 μ while the observed curve is the most intense at 8.3 μ (see Fig. 7). This is to be expected if the observed energy curve is the composite of the selectively emitted energy of the moon, and the selectively reflected energy of the sun. The selectively reflected energy of the sun would, to a certain extent, fill up the minima in the lunar emission curve, and as far as our present knowledge goes would explain the observed curves, *b*, *c*, *d*, (Fig. 7), which lack a minimum at 8.5 μ. As a whole, from whatever standpoint we view this matter, we come to the same conclusion, viz.: that in the region from 8 to 10 μ the energy emitted from the moon consists of its own proper radiation and of reflected energy from the sun. This discussion of the de-

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[1] Theoretically, of course, the dotted curves cannot touch the curve for a complete radiator, as here given.

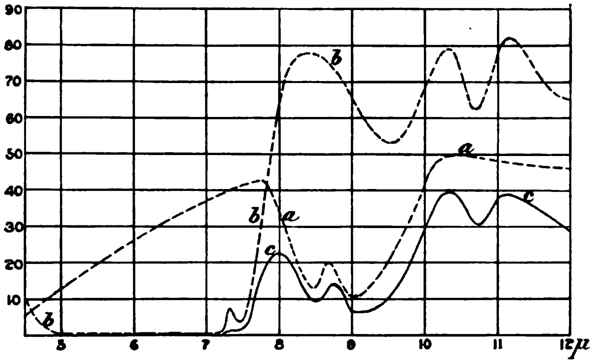

Fig. 6.

parture of the emissive power of the moon from that of a complete radiator applies equally well to the sun, which also does not radiate[1] like an "absolutely black body." Here the photospheric radiation is made up of the emission of particles at various depths and having a wide range of temperature. It is not surprising then,[2] that

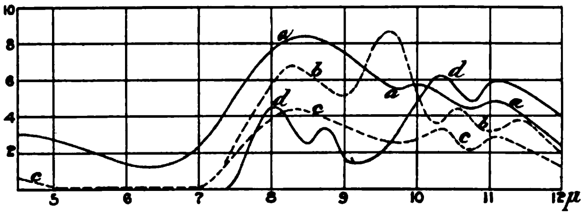

Fig. 7.

the radiation from the umbra of sun spots appears to be as intense as the still brilliant edge of the sun's disk. For, as far as our knowledge goes, the dark spots may be due to an inward convection[3] of photospheric material, thus forming an "ideal black body," the like of which, in magnitude, can not be duplicated in a laboratory.

If the brighter portions of the sun are composed of material having a higher reflecting power they may be at a higher temperature than the darker spots, and yet their emissivity may not be greater than in the latter, as in the cases just discussed.

To sum up, as matters now stand, we know that Langley observed radiation from the moon in the region of 8 to 10 μ. He observed also direct radiation from the sun in this region, but we do not know how much of this is superposed upon the direct radiation from the moon due to the latter being, without doubt, selectively reflecting in this region of the spectrum. As mentioned in a previous communication (loc. cit., p. 115), computations which require all sorts of assumptions will not settle the question. Bolometric comparison of the spectrum energy curves of the sun and of the moon made at high altitudes will be of greater service in clear-ing up the matter. Very's[1] suggestion of searching for a solar image, in the lunar image, with a delicate heat-measuring instrument, covered by a screen with a pinhole aperture also deserves a thorough trial.

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[1] Very, Monthly Weather Review, No. 254, p. 22, 1901.

[2] Abbot, Astrophys. Journ., 23. p. 275, 1906.

[3] Whether there is a difference in the emissivity of the bright and dark spots appears to be an open question. An inrush in one region means an outrush in another region. An outrush of a transparent gas, if of sufficient depth might radiate like a "black body" and could have the same emissivity as a more opaque vapor which is at a higher temperature. It is of course unnecessary to assume convection to account for the equal emissivity of two bodies which are of different brightness optically. The difference in the emissivity of a full radiator and of platinum is sufficient to demonstrate this point.

Exception has been taken to the writer's statement (loc. cit.) that, of the radiation observed by Langley in the lunar spectrum at 8 to 10 μ, we do not know how much is selectively reflected energy from the sun. But the writer maintains that just as soon as we admit the possibility of the moon being selectively reflecting in the region where it has its own proper radiation, then the use of glass[2] as an absorbing screen for testing the quality of that radiation is inadmissible, although it does serve as a rough test of the reflected energy at 0.5 to 3 μ. For this purpose it has been serviceable, and when we consider the great difficulties under which such work must be carried on, and the numerous correction factors that must be introduced (which at all times may be larger than the one for selective reflection) the use of a glass screen is not objectionable.

Furthermore, we have noticed that the silicates reflect as a transparent substance in all regions of the spectrum considered, except from 8 to 10 μ where it reflects like a metal. This means that the reflected energy in all these regions except 8 to 10 μ will consist of two parts, viz., that reflected from the outer surface, and that due to internal reflection. From 8 to 10 μ there will be but very little if any energy due to internal reflection. Hence to use the reflected energy spectrum of the moon from 0.5 to 4 μ as a means of estimating the amount of reflected energy at 8 to 10 μ is not a fair test, and, as used (for want of better data) in the present paper, can only serve as an approximation.

WASHINGTON, D. C.,
January 15, 1907.

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[1] Very, Astrophys. Journ., 24, p. 353, 1906.

[2] Various observers have compared the total radiation from the moon to that part which is transmitted by glass. Since glass is opaque beyond 4 μ it was assumed to absorb the proper radiation from the moon (which must lie in the region of 8 to 10 μ) while the part transmitted by glass was considered to be the reflected energy of the sun. Evidently, if there is selectively reflected energy of the sun beyond 4 μ, it will be superposed upon the direct radiation of the moon, and will be absorbed by the glass screen, thus giving too low values for the reflected energy.

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THE

PHYSICAL REVIEW.

ON THE MAGNETIC BEHAVIOR OF CERTAIN NICKEL ALLOYS.

By Bruce V. Hill.

THE work described in this paper was begun by the author several years ago at the University of Berlin and an abstract of the part then completed was published in the *Verhandlungen der Deutschen Physikalischen Gesellschaft.* It was not possible for me to obtain the material to finish the experiments till a few months ago. Meanwhile accounts of work by other experimenters upon kindred alloys have appeared, so that I have now collected their data together with my own.

There are two classes of alloys of the ferro-magnetic metals which are of particular importance: the steels and the alloys used in the construction of resistances. On the importance of the steels, it is unnecessary to comment. It is only recently, however, that it has been realized that the properties which render steel so universally useful are very closely connected with its magnetic characteristics. A few years ago the hardening of steel by sudden cooling was explained by supposing that, in the rapid fall of temperature, strains were set up in the metal which caused its hardness, or the hardness was accounted for by the presence of small particles of very hard carbon or of carbide. While this latter explanation may have been in part true, it is not sufficient to account for the phenomena of the tempering of steel. The electrolytic iron, which has been produced so successfully in the laboratories of the University of Wisconsin, is very hard when deposited. Dr. Taylor informs me that it behaves like hard steel magnetically also, but after annealing it is soft, both mechanically and magnetically, like Norway iron. Certainly neither sudden cooling nor temper-carbon plays any part in this case. In making malleable castings of iron the annealing process changes the glass-hard, brittle cementite Fe_3C, into ferrite, or free iron, which is soft and ductile, mixed with finely divided carbon. The hard, brittle nature of the original casting seems to have been due, not to free carbon, but to a definite carbide, Fe_3C. As to the theory of hardness caused by internal strains, it is difficult to see why other metals than iron should not be hardened in the same manner.

The ordinary carbon-steel, having been once tempered or hardened, may have this temper or hardness withdrawn by annealing from a red heat. With the alloy steels, on the other hand, the case is entirely different. When nickel and iron are alloyed together, the product acquires something of the toughness of the nickel while retaining the hardness and high elastic limit of the iron. An exhaustive study of these alloys has been made by M. L. Dumas.[1] Only a few of these need be mentioned here. The non-expansive alloy of Guillaume contains 34 per cent. of nickel and has a coefficient of linear expansion of .000001. Clock pendulums made of this material vary so little in length that a clock thus provided will run more accurately than other clocks with the ordinary compensating devices. This alloy might also be useful in making fine instruments such as transits and the like. It will take a good polish and resist corrosion as well as nickel. Screws for dividing engines might also be made of it to advantage.

The steels with from 40 to 50 per cent. of nickel have so nearly the same coefficient of expansion as glass that they may be employed when it is desired to seal a metal into glass. This material, called "platinite," had already begun to replace platinum in the manufacture of incandescent lamps in France several years ago. These alloys do not seem as yet to be much known in this country. However useful the nickel-iron alloys of high per cent. may be, the high price of the nickel prevents their very extensive employment. An alloy containing from 3 to 3.5 per cent. of nickel and about

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[1] Aciers au nickel a hautes teneurs.

0.25 per cent. of carbon combines a very high degree of ductility with a great tensile strength, hardness and a very high elastic limit. Such a steel, with the addition of a little chromium is used for the armor plate of vessels.

A small amount of manganese in steel, up to five or six parts in a hundred, renders it very brittle, but an alloy containing 12 per cent. of manganese and 1.5 per cent. of carbon is very hard but yet ductile. It is unmagnetic. Mushet discovered that, by the addition of manganese and tungsten to tool steel, a cutting edge could be maintained upon the tool at a temperature much higher than with carbon-steel. This, of course, made it possible to work these tools at a considerably greater speed. These steels, known as the self-hardening or air-hardening steels were hardened by slow cooling from a cherry-red, that is from about 900°C. If heated to a temperature much above this the quality of the steel deteriorated very rapidly. With the addition of a small amount of chromium and a little tungsten or molybdenum it was found that this deterioration at high temperatures was confined to a very narrow range, from about 850° to 950° C., while above 950° to about 1,100° the efficiency is greatly increased. These last steels are known as the high-speed steels of Taylor and White. Their alloy contains:

Carbon from 0.75 to 1.25 per cent.
Chromium from 3.00 to 4.00 per cent.
Tungsten 8.50 per cent.

The exact composition of the steel varies according to whether the tool is to be used for cutting soft steel or gray cast-iron, or hard steel. By employing tools made of such steel, the rate of cutting can be increased to three or four times that possible with a carbon-steel tool, since with a high-speed tool it may be allowed to heat till it shows a faint tinge of red. A large increase in efficiency is thus gained by the use of these tools. These alloy-steels are unmagnetic, a very small amount of manganese being sufficient to render the iron unmagnetic.

The second important group of alloys of the magnetic metals is that of the alloys used for electric resistances. These are mostly nickel alloys, the greater part of them being unmagnetic. Their resistance is higher than that of the pure metals while the temperature coefficient is lower. Manganin, for example, containing 84 per cent. of copper, 12 of nickel and 4 of manganese, retains its resistance almost constant between 0° and 30° C.

Seeing that the alloy-steels are hardened by annealing, whereas in the carbon-steels the hardening was accomplished by sudden cooling, it is evident that neither the theory of strains within the metal nor of hard carbon within its mass can account for its hardness, and the explanation was sought elsewhere.

The magnetic metals, if heated to a certain critical temperature, become unmagnetic and exhibit a much higher specific heat and a higher electric resistance together with a much smaller temperature-coefficient. The chemical properties also show differences above and below the critical temperature.

Again, if iron containing carbon be cooled from a high temperature there will be a marked evolution of heat at about 850° C. and another at 760° C. indicating some transformation in the metal itself. Osmond based his allotropic theory of the magnetic metals upon these phenomena. The iron as it exists at ordinary temperatures and up to 760° C. the lowest temperature at which the evolution of heat occurred (the temperature of recalescence is below 760° but is due to the separation of the eutectic alloy of iron and carbon, that is, the alloy containing 0.85 per cent. of carbon) Osmond called *alpha iron.* In the next interval, namely between 760° and 850° C., he applied the name *beta iron*, and above 860° *gamma iron.*

In nickel and cobalt the phenomena of transformation are very similar to those in iron, except that in these metals there are but two allotropic forms, the *alpha* and the *beta.*

Let us now return to the steels and apply the allotropic theory to explain their behavior. Let us suppose the *alpha iron* to be soft and magnetic; the *beta iron* hard, tough, rather brittle and unmagnetic; and the *gamma iron* soft and unmagnetic. At ordinary temperatures common iron and steel contain a mixture of the *alpha* and *beta* modifications. If pure iron be heated to a high temperature and cooled either rapidly or slowly, the transformations take place as usual and the metal remains soft. If the proper amount of carbon be added — 0.25 to 1.50 per cent. — and the metal be suddenly cooled from a temperature above the critical point, a large amount of the *beta* iron persists and hardens the specimen. This is, of course, an incomplete account of these phenomena since the carbides of iron, which must also undergo transformations, have not been considered.

If any substance, other than carbon, could be added to the iron and prevent its transformation, we should expect to be able to alter the properties of the alloy in a way depending not only upon the nature of the added substance, but also upon the amount of *beta* and *gamma* iron present at ordinary temperatures. Lownds and Richardson[1] found that if aluminium be alloyed with iron the temperature of transformation of the latter is depressed by an amount proportional to the mass of aluminium added. This holds more or less closely for other elements alloyed with the iron. Osmond and van t'Hoff have worked out the phase diagram for iron and carbon alloys.[2] One of the most important results of the new knowledge of the physical chemistry of steels is the greater certainty with which a certain grade of steel can be produced. A comparatively inexperienced workman is able to secure more uniform results than were formerly obtained by men of much longer practice. A phase diagram has been given by Messrs. Shepard and Upton for alloys of copper and tin.[3]

If now there were some law by which the action of any element in alloy with iron could be predicted it would be of great value in steel-making. The most ready analogy to the depression of the temperature of magnetic transformation of iron and nickel by other metals alloyed with them, is the lowering of the freezing point of a liquid by dissolved substances. The law of van t'Hoff applying to the latter case is:

$$\Delta t = \frac{.02 T^2 m}{ML}.$$

Where T is the absolute temperature of freezing of the pure solvent, .02 the thermo-dynamic constant, L the latent heat of fusion of the solvent, M the molecular weight of the solute and Δt the depression of the freezing point caused by m grams of the solute in 100 grams of the solvent. The present work was undertaken to test the applicability of this formula to the case in hand. To do this we would have to put T equal to the absolute temperature of transformation of the pure metal (nickel or iron), L the latent heat of transformation, M the molecular weight of the added metal and Δt the depression of the point of transformation caused by m grams of this metal. The derivation of van t'Hoff's formula assumes that the pure solvent freezes out. In attempting to apply this equation now it must be remembered that neither metal separates from the other at the point of transformation. Microscopic examination shows this to be true. We might hope, however, that the formula would apply approximately or in a modified form.

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[1] Phil. Mag. (6), 1, p. 601, 1901.

[2] The Metallographist, 1898.

[3] Journal of Physical Chemistry, 1905.

Nickel was chosen for these experiments rather than iron because of its low temperature of transformation, which being below 400° C. was easily reached and measured. The relations existing in alloys of iron and nickel are known to be very complicated so that, in order to have the conditions as simple as possible, the nickel could not be alloyed with any other ferro-magnetic metal. Nickel and copper can be alloyed in all proportions and such alloys are easily obtained. They were furnished me by the firm of Basse and Selve, in the form of wire. As one sees from the equation, the depression of the transformation point of the nickel in the alloy should be proportional to the percentage of the second metal and inversely proportional to the molecular weight of the latter. It was desirable therefore to obtain alloys with other metals having molecular weights as different as possible from that of copper. This was difficult however. We tried to get alloys of lead or bismuth or aluminium from the same firm who had supplied those of nickel and copper, but after several months they said that they were unable to make them. The firm of Haereus in Hanau finally succeeded in supplying some rings of alloys of nickel and tin containing 5, 10 and 15 per cent. of tin respectively.

There were four specimens of the nickel-copper alloys available. They were in the form of wires 2 millimeters in diameter and contained 4, 8, 20 and 40 per cent. of copper. To determine the relation between induction and temperature a secondary coil of bare copper wire was wound about a glass tube and over this tube a primary coil 45 centimeters long and over both of these a heating coil of nickeline wire, the whole being insulated with asbestos. The wire under examination, together with a standard mercury thermometer, was put into the glass tube and the apparatus packed about with asbestos as heat insulation. The induction was measured ballistically, the throw caused by reversing the primary current being observed in each case. The material in order to be fit for these experiments had to be free from temperature hysteresis, like that in the irreversible nickel-steels. Experiments showed that the point of transformation with rising temperature was the same as with falling temperature. The 20 per cent. alloy was then held for an hour at a temperature of 100° C., a temperature near that of transformation, without any change in the induction in a given field being observed. It appears, then, that when a given temperature is reached the corresponding magnetic condition establishes itself without appreciable lag.

As is known, the loss of susceptibility does not occur suddenly, but gradually, through a wide interval of temperature. As one sees from the curves, taken from a field near that of saturation, the magnetic intensity, *I*, decreases with rising temperature very slowly at first and then more and more rapidly. When it has fallen to a small fraction of its original value, the decrease becomes much

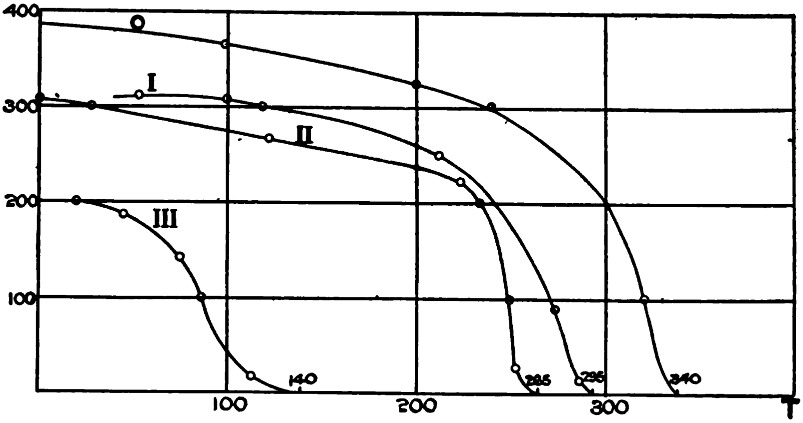

Fig. 1.

slower. The temperature at which this occurs could be determined within 5°. The intensity-temperature curves for the pure nickel and the 4, 8 and 20 per cent. alloys are given in the figure. It will be seen that the analogy to the solidification of water does not hold for the transformation of the pure nickel. When water has reached the freezing point, the temperature remains constant till the solidification is complete, while the transformation of the *beta* into the *alpha* nickel is gradual. This can be explained if one assumes that the two phases are mutually soluble. As the microscope shows no difference between them, this assumption is permissible. We may suppose that as soon as a portion of the *beta* nickel has transformed into the *alpha* form it at once dissolves in the remaining *beta* metal and so lowers the transformation point requiring a further fall of temperature before further transformation can occur. In the alloys this gradual fall of temperature with the progress of the transformation is to be expected as in the case of the freezing of a solution of salt in water.

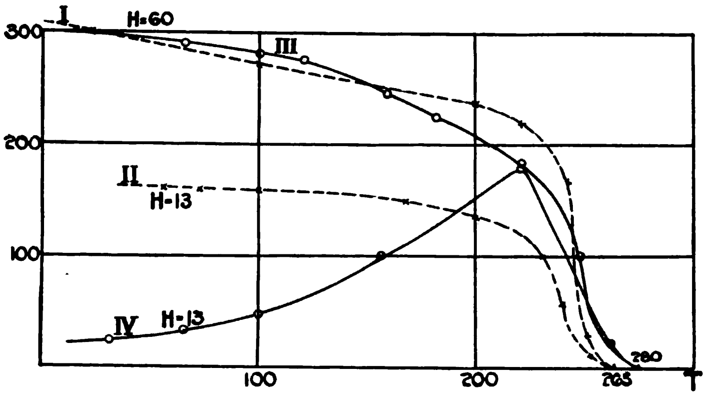

Fig. 2.

The second figure gives the intensity-temperature curves for the 8 per cent. alloy in fields of 13 and 63 C.G.S. units respectively, The full line refers to the metal in the hard-drawn and the broken line to that in the annealed condition. The temperature of transformation appears to be independent of the field, but is lowered about 15° by annealing. This was true of the nickel and of all of the nickel-copper alloys.

The table shows the temperatures of transformation of the alloys of nickel and copper:

Per Cent. Cu.	Temp. (Hard).	Δt	Temp. (Annealed).	Δt	$\Delta t/m$
0	355°		340°		
4	310	45°	295	45°	11.2°
8	280	75	265	75	9.4
20	155	200	140	200	10.0
40	−100	455			11.4
					Mean 10.5°

According to this table the depression of the point of transformation is very nearly proportional to the amount of copper, m, in the alloy. Only in the first two alloys is the concentration small enough to justify the application of the formula of van t'Hoff, still the depression due to one part of copper in one hundred is so nearly constant that we may take the mean of these values.

Before the results just given could be applied in the formula, it was necessary to know the latent heat of transformation, L. This cannot be determined directly and at best can only be approximated by indirect methods. The values obtained in this way may give an idea of the order of magnitude of the quantity sought, however. The alloy containing 40 per cent. of copper, is unmagnetic at room temperature and so contains nickel in the *beta* condition. The specific heat of this alloy, as well as that of pure copper, was found between 20° and 370° C. That for the former was 0.1049 and for the latter 0.0996. If now we assume that the specific heat of this alloy can be found from those of its two constituents by the rule of mixtures, we have for the specific heat of the *beta* nickel

$$\frac{100 \cdot 0.1049 - 40 \cdot 0.0996}{60} = 0.1104.$$

A mass, m grams, of nickel was now heated to 370° and dropped into a water calorimeter whose water equivalent was K. The initial temperature was t_1 and the final temperature t_2. If the specific heat of the unmagnetic nickel just found is now introduced we have for the latent heat of transformation

$$L = C(370 - t_2) - K(t_2 - t_1)/m.$$

In five experiments the following values were obtained for L:

4.16
4.33
4.03
5.05
4.83
Mean 4.48

The agreement among these numbers is as good as the method would lead one to expect. In the first place the rule of mixtures holds only approximately for the specific heats of alloys. The specific heat was determined for the four nickel-copper alloys between 20° and 100°. If one takes 0.1100 for the value for nickel and 0.0933 for copper as found in the tables and calculates the specific heats of the alloys, he finds:

Alloy.	C (Observed).	C (Calculated).
4 per cent.	0.1080	0.1093
8	0.1141	0.1087
20	0.1145	0.1066
40	0.1003	0.1033

The agreement is not exact, and in the case of the 40 per cent. alloy the variation is 3 per cent.

By an entirely different method Pionchon[1] found the latent heat of transformation of nickel to be 4.64, a value not in bad agreement with that just given. Too much weight is not to be given to this agreement, however, as he found the latent heat of transformation as the difference between the quantities of heat given off when the metal was cooled first from a point just above and then from one just below the temperature of transformation, the latter point so high that the transformation was by no means complete. These values must at least give the order of magnitude of L however.

Before passing to the experiments upon other alloys of nickel and the further discussion of the formula it may be of interest to show the relation of the electric resistance of the nickel-copper alloys to their magnetic properties. H. Le Chatelier[2] and P. Harrison[3] have given curves exhibiting the relation of the resistance to the temperature in nickel and iron. Fig. 3 gives similar curves for the 4, 8, and 20 per cent. alloys.

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[1] Ann. de chim. et de phys., 6, 11, p. 77, 1887.
[2] Compt. Rend., CX., p. 283, 1896.
[3] Phil. Mag. (6), 3, p 177, 1902.

In all the specimens studied the temperature coefficient increases more or less rapidly till a temperature near to that of transformation is reached. It then becomes smaller and nearly constant. This change is quite sharply marked though not so distinct as the magnetic transformation. It is plainer in the alloys than in the pure nickel. In the 20 per cent. alloy the change of direction of the curve is sharp and corresponds very closely to the transformation point.

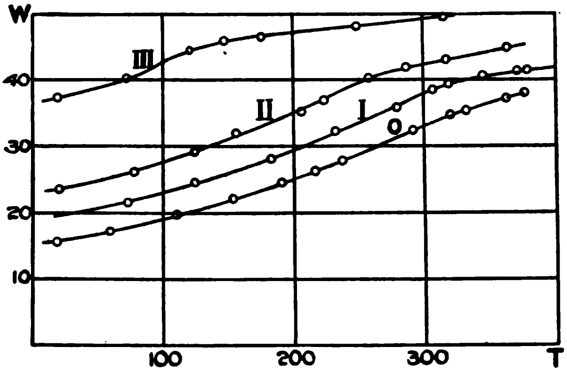

Fig. 3.

I had three alloys of nickel and tin, containing 5, 10 and 15 per cent. of tin respectively. A microscopic examination showed them to be chemically homogeneous with no trace of free tin. Physically they were not uniform, however, as the metal was full of pores. The holes were the largest in the 5 per cent. and smallest in the 15 per cent. alloy. Alloys containing more than 20 per cent. of tin could not be cast and those with that amount were so porous that they could not be used. The specimens were in the form of rings. To determine the induction they were wound with insulated wire and heated in an oil bath, the temperature being measured with a mercury thermometer. With the ring containing 5 per cent. of tin the insulation gave way and it had to be rewound with bare wire and asbestos. The results for these alloys are given in the table:

Per Cent. of Tin.	Temp.	Δt	$\Delta t/m$
0	340°		
5	303	37°	7.4°
10	268	72	7.2
15	230	110	7.3
			Mean 7.3°

Here, as with the nickel-copper alloys, the depression of the transformation point is proportional to the amount of tin in the alloy. I will now compare the results found for the two series of alloys. The temperature of transformation for the pure nickel was 340° C. or 613° absolute. The latent heat of transformation of the nickel was 4.5 calories per gram. $\Delta t/m$ was equal to 10.5° for the copper and 7.3° for the tin. Introducing the above values for L and T into the equation of van t'Hoff, we have $\Delta t/m = 1670/M$. The molecular weight must therefore be known. If it were possible, the most direct way to determine the molecular weights of tin and copper in their solutions in nickel would be to apply the van t'Hoff formula to the lowering of the point of solidification of nickel by these metals dissolved in it. To do this, the latent heat of fusion of nickel would have to be known and this would be very difficult to determine as the melting point of nickel lies so high, in the neighborhood of 1600° C. Heycock and Neville [1] have made a series of experiments upon the lowering of the melting point of tin, thallium and sodium by alloying other metals with them. They found the "atomic lowering" of the metal point of tin by the solution of other metals in it, that is the depression caused by the alloying of one atomic weight of the given metal in 100 atomic weights of the tin, to have the following values.

For Nickel	2.94
Silver	2.93
Gold	2.93
Copper	2.91
Thallium	2.86
Sodium	2 84
Mean	2.90

If these metals in their alloys with tin are monatomic, the formula of van t'Hoff gives, since $T = 504.5°$, $L = 14.25$ and the atomic weight of tin is 118.8 (later determinations give 118.5),

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[1] Jour. Chem. Soc., 55, 666, 1889; Chem. News, 68, 305, 1893.

$$\Delta t = \frac{.02 \cdot T^2 m/118.8}{Lm} = .02 \cdot T^2/118.8 \cdot L = 3.01.$$

The agreement between this calculated value for the *atomic lowering* and the observed values seemed sufficiently close to warrant the conclusion that these metals are monatomic in their alloys with tin.

G. Meyer[1] found, from the electromotive force of cells whose electrodes were amalgams of different concentrations of the same metal, that the metals with which he experimented (Zn, Cd, Pb, Sn, Cu, Na) are monatomic in their solutions in mercury. The natural conclusion, then, seems to be that the metals have but one atom to the molecule. The probable correctness of this supposition will be discussed more fully later.

If then we assume that tin and copper are monatomic in their alloys with nickel and put the values of the atomic weights in the formula above given, we find (Cu = 63.6 and Sn = 118.5):

For	$\Delta t/m$ (Calculated).	$\Delta t/m$ (Observed).
Copper	26.3°	10.5°
Tin	14.1°	7.3°

The observed values are much smaller than the calculated. The lowering of the point of transformation is not inversely proportional to the atomic weight for, if this were the case, the depression caused by one per cent. of tin, as calculated from that observed to be due to one per cent. of copper would be 5.6° instead of 7.3°. The agreement would be better if we assumed two atoms to the molecule rather than one, but there is no reason for doing this. While the methods of Heycock and Neville and of Meyer show that the metals with which they experimented are monatomic at the melting point or when in solution in mercury, it is possible that in a solid condition the molecules may be highly associated. In fact the great departure from the law of Dulong and Petit at high temperatures might, perhaps, be accounted for in this way.

For other alloys of nickel the depression of the point of transformation does not obey the law of van t'Hoff even so well as do copper and tin. The next series of alloys which I tested were of nickel and silver prepared at the University of California. The nickel was melted in a graphite crucible lined with a mixture of calcined magnesia and alumina like that used by Messrs. Allen and Day as insulation in the furnace which they employed to standardize the normal platinum — platinum-10 per cent. rhodium thermocouple,[1] and the formula for which was given me by Dr. Day. The graphite crucibles so lined were found satisfactory. In this way a series of nickel-cobalt alloys were made and two alloys of nickel and silver containing 4 and 8 per cent. of silver respectively. On testing these alloys, the temperature of transformation was found to be 390° C. irrespective of the amount of silver. The permeability of the alloys was found to be lower than that of the nickel, however, by more than the percentage of the silver present.

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[1] Zeitschr. f. Phys. Chem., VII., 477, 1889.

M. Levin studied a series of nickel-gold alloys with from 0 to 90 per cent. of gold.[2] The temperature of transformation was independent of the amount of gold in the alloy and was near 310° C. The fact that the different specimens of nickel show so widely varying points of transformation is probably due in large measure to the impurity of some of them. The presence of iron or cobalt would raise this temperature slightly while carbon would lower it considerably. For example, the commercial nickel used in my alloys and melted in the lined graphite crucible transformed at 390° C., while a specimen of the same metal which had been melted in the unlined graphite crucible became unmagnetic at 300° C.

Alloys containing equal parts of nickel and gold and of nickel and platinum were found by Lampadius[3] to be as permeable as pure nickel. It may be that in the cases of the nickel-silver and nickel-gold alloys we have to do with mixtures resembling chryohydrates rather than solid solutions.

Osmond, and later Dumas, determined the transformation points of nickel-iron alloys and found that this point rises with the amount of iron till the latter reaches 30 per cent. and then falls again, so that the temperature of transformation may have the same value for two values of the iron content. Dumas tested a small number of alloys of nickel and chromium and of nickel and manganese. 2.7 per cent. of chromium depressed the point of transformation 125°, or 42° per per cent.; 4.3 per cent. depressed it 225°, or 52° per per cent., while greater amounts of chromium did not cause further lowering of the temperature of transformation, indicating probably that the chromium was no longer forming a solid solution with the nickel.

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[1] Physical Review, XIX., p. 177, 1904.

[2] Zeitschr. f. anorgan. Chem., 45, p. 238, 1904.

[3] Wiedemann, "Elektricitaets Lehre," III., p. 784.

In the nickel-manganese alloys, the depressions of the transformation point were:

Per Cent. Mn.	Depression.	Depression per Per Cent.
3.7	80° .	21°
8.8	110	12
13.0	200	15

In both the nickel-chromium and the nickel-manganese alloys, the lowering of the point of transformation is not proportional to the amount of chromium or of manganese in the alloy but is irregular. The alloys of Dumas contained as high as 1.5 per cent. of iron which would vitiate the results.

Alloys of nickel and cobalt were studied by Guertler and Tammann.[1] The transformation point rose regularly from that of nickel to that of cobalt (1100° C.). Moisson[2] prepared borates of iron, cobalt and nickel but the temperatures of transformation were not given, though the compounds were magnetic. Alloys of nickel and antimony were made by K. Lossern.[3]

Metal with Nickel.	Atomic Weight.	Depression per Per Cent.	Observer.
Chromium.	52.1	45° C. Irregular.	Dumas.
Manganese.	55.0	16° C. Irregular.	Dumas.
Iron.	56.0	Point rises.	Osmond-Dumas.
Cobalt.	59.0	Point rises.	Guertler-Tammann.
Copper.	63.6	10.5°. Regular.	Hill.
Silver.	107.9	Independent of alloy.	Hill.
Tin.	118.5	7.3°. Regular.	Hill.
Gold.	197.2	Independent of alloy.	Levin.

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[1] Zeitschr. f. anorgan. Chem., 42, p. 353, 1903.

[2] Ann. de Chim. et de Phys. (7), 9, 272, 1905.

[3] Zeitschr. f. anorgan. Chem., 49, p. 58, 1906.

The experimental results already given for the effect of various metals upon the transformation point of nickel are collected in the table.

There appears, then, to be no simple relation between the atomic weights of the metals named in the table and their effect in changing the temperature of transformation of nickel when in alloy with them. As already stated, we probably do not know the molecular weights of the metals at ordinary temperatures. Professor Howe, of Columbia University, says in his book on "Iron, Steel and Other Alloys" that in alloy-steel two parts of molybdenum replace one part of chromium as might be expected from their atomic weights. In the light of the experiments upon nickel alloys just described, I do not think that we can recognize any so general law. In the cases of manganese and chromium alloyed with nickel, a concentration is reached where a further addition of the second metal causes no greater lowering of the transformation point, and with gold and silver this point of saturation, if we may call it so, is reached at a very low concentration if there is a true solid solution of these metals in the nickel at all. It might be of value to know to what extent the properties of the alloy-steels are due to changes in the nature of the iron itself and how much to the nature of the other constituents of the alloy. It is certain that the addition of a small amount of manganese or chromium to steel produces an effect greatly out of proportion to the amount added if we consider the properties of the alloy as in a sense the mean of those of the components. It is only therefore in the most general way and on physical, rather than on chemical grounds, that the properties on an alloy of iron or nickel with any other metal can be predicted. We might expect an alloy to present properties like those of the metals composing it, but this is not necessarily the case as is seen in the copper-aluminium alloys. Both of these metals are soft and ductile but the alloy is very hard and somewhat brittle. It is probable that the properties of any alloy will always have to be found by experiment though some surmises may be made beforehand as to their character.

PHYSICAL LABORATORY,
UNIVERSITY OF KANSAS,
January, 1907.

ROTATION AND ELLIPTIC POLARIZATION PRODUCED BY IRON FILMS IN A MAGNETIC FIELD.[1]

By W. D. Harris.

THE rotation of plane polarized light passing through metallic films in the direction of a magnetic field has been determined by Kundt,[2] DuBois,[3] Lobach,[4] Righi,[5] and others. Righi also discovered that an iron film produced at the same time a small ellipticity in the transmitted ray, which, measured for red light, gave a ratio of $\frac{1}{225}$ for the two axes of the ellipse. Righi's discovery of the effect of iron under the influence of a magnetic field led Cotton[6] to investigate certain optically active solutions for the presence of the same phenomenon, and in these he discovered and measured the ellipticity of the transmitted light. By means of a Brace half-shade elliptic polarizer and compensator McDowell[7] succeeded in extending the work of Cotton considerably. This suggested the possibility of making with the same system a more thorough examination of the elliptic polarization produced by iron films under the influence of a magnetic field. The object of this investigation was primarily therefore to determine the ellipticity produced by iron films as affected by wave length of light and strength of magnetic field; to find the connection, if any, between this and the corresponding rotation of the plane; and to test for ellipticity of the reflected ray, the presence of which theoretical considerations might lead us to expect. The experimental results led finally to a study of films produced under different conditions, as there were indications that the presence of absorbed gases affected materially the magnitude of the results obtained. In addition to iron, cobalt and nickel films were also tested.

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[1] Read before joint meeting of the Am. Phys. Soc. and Section B of the A. A. A. S., Ithaca, June, 1906.

[2] Wied. Ann., 23, p. 223, 27, p. 191.

[3] Wied. Ann., 31, p. 941.

[4] Wied. Ann., 39, p. 347.

[5] Mem. R. Accad. Sc. d. Bologna, 1886, p. 443.

[6] L' eclairage Electr., T. 8, pp. 162, 198. Ann. de Chem., et de Phys., Series 7, T. 8, p. 347.

[7] Phys. Rev., XX., p. 293.

Apparatus.

Fig. 1 shows the arrangement of the optical system. Sunlight reflected from a heliostät is focused by lens *N* on slit *B* placed at the principal focal distance from the achromatic lens *C*. The colors dispersed by the prism *D* are focused by *E* in the plane of the slit *F*. This light is directed by a total reflecting prism through polarizer *P*, the film *X* (placed normal to the field between poles of the magnet), and the half-shade *H* which is observed by means of the lens *L*, through the compensator *K* (mounted on a graduated circle) and analyzer *A*.

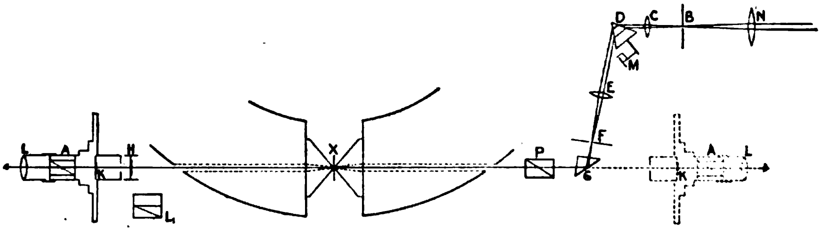

Fig. 1.

Measurement of Ellipticity.— For adjusting the system, the half-shade and compensator are removed, the polarizer and analyzer "crossed"; then the half-shade (an exceedingly thin strip of mica) is placed to cover half the field and so that its principal plane makes an angle of 45° with the plane of polarization, as indicated by maximum intensity. The compensator *K* (also a thin strip of mica, of about twice the order of the half shade) is next introduced (covering the entire field) and this rotated to produce a match of intensity in the two halves of the field. A small ellipticity produced in the originally plane polarized light is then measured as proportional to the angle through which the compensator must be rotated to reproduce the match. The order of the one used is given by McDowell as 1/40 for yellow-green light[1]). By rotating the compensator from a "match" through 0.1° a marked contrast in the two parts of the field became evident. When, as in the present case, the medium tested produces also a small rotation, the polarizer may first be rotated to produce minimum intensity of field — the condition for maximum sensibility — after which the compensator is set for a match, the angle through which it is rotated being a measure of the ellipticity as before.

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[1] Since Righi used for this purpose a quarter wave plate, the system employed here should, for the same ellipticity, give about ten times the rotation he observed, namely, 2.5° where he had 15′. As may be seen by referring to curve *H*, Fig. 3, this magnitude was obtained with films deposited in hydrogen.

Measurement of Rotation. — To measure the rotation of the plane of polarization, the half-shade and compensator were removed while the analyzer *A* combined with a Lippich prism *Li* was mounted in the graduated circle. With good light a rotation of 0.05° could readily be detected by destroying the "match" of the two halves of the field of view. In the observations with films a variation of more than 0.1° from the mean of several was unusual except in the violet where the intensity of the transmitted light was very low.

The Electromagnet.—This consisted (Figs. 1, 2) of two semi-circular cores (forming a circle about 56 cm. in diameter), wound uniformly over the whole length with 110 pounds of No. 14 magnet wire. Two accurately planed soft iron wedges *O* and *S*, Fig. 2, permitted the distance between pole pieces to be varied with ease. Cold water circulating through a hollow copper tube wound next to the core reduced the heating when working with the higher currents.

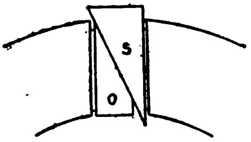

Fig. 2.

Two sets of pole pieces were provided, one pierced by an aperture of one millimeter diameter, and the other five. With the former a current of 1 ampere gave through an air gap of 3.3 millimeters a field strength of 10,000 C.G.S. units; 8.5 amperes, 15,000; 23 amperes, 17,000. By using the pole pieces of smaller aperture with a shorter air gap a much stronger field could be produced, but this proved unnecessary in the present work. The field strength was determined by the rotation it produced in a tube of carbon bisulphide of given length, its value being calculated from Verdet's constant as determined by Rayleigh.

A 500-volt power circuit furnished the higher currents, a storage battery the smaller. Suitable rheostats permitted the current to be varied by steps up to 24 amperes.

PRODUCTION OF FILMS.

Righi used a film deposited by the electrolysis of ferrous ammonium sulphate on half-silvered glass. Kundt used the same solution, depositing on platinized glass. He also deposited by a cathode discharge in vacuo.

All films used in this investigation were deposited from the cathode under the glow discharge after the manner of Wright,[1] Longdon,[2] Kundt,[3] Patterson[4] and others. This was done with a gas pressure of less than a millimeter of mercury and a current of about 25 milliamperes furnished by a small 1,000-volt generator. The character of the deposit varied greatly with the distance of the plate from the cathode and some practice was necessary to obtain a metallic mirror deposit of sufficiently uniform thickness, this being necessary since an irregular or prismatic film may exhibit double refraction as pointed out by Kundt.[5] The most satisfactory distance from the cathode was found to be 2 or 3 millimeters outside the Crooke's dark space.

Iron films were deposited in hydrogen, nitrogen and oxygen. Only those with even mirror surface were used. Films deposited in hydrogen and nitrogen show by transmitted light a darkseal brown color when thin and transparent, and a steel gray appearance when translucent. Those in oxygen appeared a distinct reddish brown.

As the success of the experiment depended on the purity of the deposit, much care was exercised in eliminating impurities in the cathode. The best grade of soft Swedish iron, terminating in a circular disc of 2 millimeters diameter was used.

To remove the foreign gases givenoff during action, the discharge tube was washed out repeatedly with the gas used during the deposition. Before starting the current the system was allowed to stand in connection with a phosphorus pentoxide dryer to remove any water vapor present. Thin sections of the fresh films obtained in the different gases exhibited strong polarity when suspended by silk fibers in a magnetic field, while the older ones showed it but feebly if at all. The difference was ascribed to gradual oxidation of the films after exposure to the atmosphere, since after a time all trace of both magnetic rotation and ellipticity disappeared. Nickel and cobalt films deposited from chemically pure cathodes were also obtained by the same process.

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[1] Am. Jour. Science, XIII., pp. 49–169, 1877.

[2] PHYS. REV., XI., pp. 40–84.

[3] Wied. Ann., 27, p. 59.

[4] Phil. Mag., 4, p. 652.

[5] Wied. Ann., 27, p. 59. [The effect of lack of uniformity could, however, be obviated to a large extent by confining observations to a small area of sufficient evenness.]

PRELIMINARY TRIALS.

As the ellipticity produced by iron was known to be very small and perhaps peculiar to the manner of deposition used by Righi, various refinements in method were at first attempted, to eliminate, if possible, all factors but that arising from the iron alone.

Since the glass on which the films are deposited may become doubly refracting by the heat during process of deposition, the effect increasing with the thickness, the plan was tried of mounting very thin films of blown glass on small glass frames and depositing the film of metal on the thin section of glass. This method proved unsatisfactory. The plan was also tried of depositing the metal on different polished crystals and then by dissolving the crystal leave the metal film to be mounted, but, though iron is very tenacious ordinarily, films could not be obtained thin enough to be transparent and yet strong enough to be mounted. Finally thin microscope coverglasses, selected free from double refraction before deposition and then with part of the film removed by acid, also tested after deposition, were found to work very satisfactorily. Although in a nearly saturated field these produced a rotation of the plane large enough to be detected, yet it was smaller than the fluctuation in observations made on the film, and hence ignored in the results.

Another scheme was tried with the hope of eliminating the platinized surface of Kundt's and the silvered of Righi's experiment. This was to deposit first the exceedingly thin conducting layer of iron on the glass by the cathode method, and use this to deposit films electrolytically with the Varrentrap bath method, but no satisfactory films could be obtained.

The first iron films tested had been deposited in nitrogen a few months before. They revealed no ellipticity though a very small rotation not caused by the glass was present. These films imparted to transmitted light the reddish brown tint of the oxide rather than that of metallic iron. In order to avoid oxidation, films were therefore deposited in an atmosphere of hydrogen and left in this until they could be removed for an immediate test. These were found to possess to a marked degree the power of producing both rotation and elliptic polarization, thus indicating that the same was destroyed in the old films probably by oxidation. In fact, as will be shown, these films gradually though slowly lost these properties.

STUDY OF IRON.

Films Deposited in Hydrogen. — Fresh films, deposited in hydrogen, were now tested for the rotation and ellipticity produced on various wave-lengths of transmitted light in a magnetic field of definite intensity. These films were as thick as was consistent with reliable measurements.

In Table I. are recorded two series of such observations taken on successive days with a film which had been left unchanged in its position during that time.[1] The strength of field was about 7000 C.G.S. units; such as to produce a nearly maximum ellipticity.

TABLE I.

Film Deposited in Hydrogen. Strength of Field, 7,000 *C.G.S.*

Wave-Length.	First Series.		Second Series.	
	Rotation.	Ellipticity.	Rotation.	Ellipticity.
466 $\mu\mu$	0°	.03°	0°	.12°
521	0.21	1.98	0.22	2.00
537	—	2.29	—	2.34
550	0.47	—	0.40	—
556	—	2.88	—	2.88
561	0.46	—	0.42	—
603	0.62	—	0.65	—
612	—	3.00	—	2.97
621	0.68	2.89	0.64	2.90
644	—	2.84	—	2.83
657	0.75	2.67	0.76	2.45
684	0.84	2.50	0.82	2.41
712	0.84	2.20	0.82	2.20

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[1] It proved to be a difficult matter to return a film, after removal, to its original position with sufficient accuracy to reproduce observations within the limits of error of the optical settings, although it was carefully diaphragmed down to the size of the aperture in the pole-pieces.

The fluctuation in the observed values was generally largest at both extremities of the spectrum where the absorption was greatest, but this error was in part compensated for by making a larger number of settings for a given wave-length. In all cases the double value was obtained by reversing the magnetic field; however, only the single values are recorded.

The results in Table I. are represented graphically by curves *H* in Fig. 3, the upper plot indicating the rotation of the plane of polarization with wave-length, and the lower the amount of elliptic polarization produced (in terms of the rotation of the compensator).

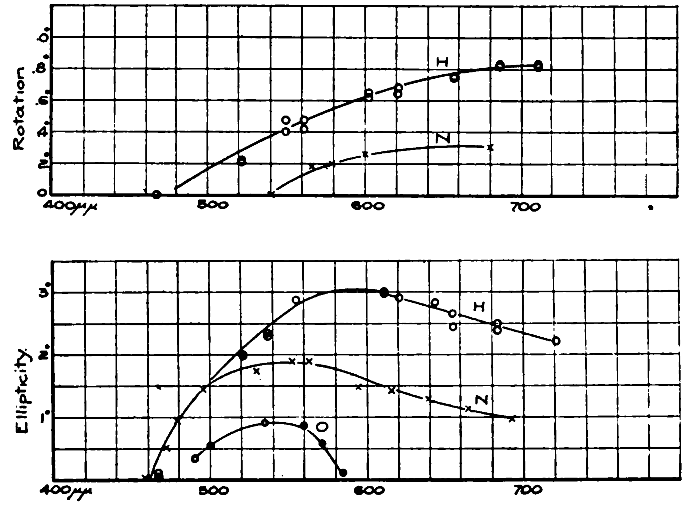

Fig. 3.

Comparing the rotation here with that given by Lobach (*l. c.*), it is noted that in agreement with his values it is anomalous, *i. e.*, it increases with wave-length; but though he observed that the rotation for a wave-length of 430 $\mu\mu$ was about half that for 670 $\mu\mu$ the value here drops to practically zero at about 470 $\mu\mu$. While the accuracy of the settings in this region is not at all good, yet these values were obtained repeatedly by two observers, so that we are led to believe in their correctness. The curve here is also distinctly concave toward the axis of abscissas, while that of Lobach approaches closely to a straight line. In the main, however, the rotation of the plane in this case indicates that we have to do with a normal iron film.

The ellipticity of the transmitted light, given in the lower plot, rises rapidly from a zero value, at the same wave-length as the curve of rotation, 470 $\mu\mu$, to a maximum at about 590 $\mu\mu$ and then drops slowly towards the red end of the spectrum. The point of maximum ellipticity lies in the yellow-green part of the spectrum, which qualitative tests with a spectroscope showed was also the region of least absorption.

When compared with results given by optically active solutions in which an absorption region is present, *the ellipticity produced by iron also appears anomalous*, in the sense that, as shown by Cotton and by McDowell, the latter reveal a maximum ellipticity in the region of greatest absorption.

Films Deposited in Nitrogen. — As stated above the first films tested had been deposited in nitrogen some time previously. The fresh films from hydrogen revealed such a marked increase in action over the old that it was questionable if this arose entirely from age. To test this, films were deposited in nitrogen and subjected to an immediate test. Table II. represents the results obtained from one of these which to all appearances did not differ from that obtained with hydrogen. It appeared equally susceptible to magnetism and absorbed about the same amount of light.

TABLE II.

Film Deposited in Nitrogen. Strength of Field, 12,000 *C.G.S.*

Wave-Lengths.	Rotation.		Ellipticity.	Wave-Lengths.	Rotation.	Ellipticity.
458 $\mu\mu$	—		0.03°	564 $\mu\mu$	—	1.90°
464	0.10°	caused by the glass.	—	567	0.18°	—
473	—		0.52	579	0.20	—
479	—		0.95	595	—	1.48
496	—		1.45	600	0.23	—
500	—		1.50	616	—	1.43
503	Trace.		—	639	—	1.31
512	Trace.		—	664	—	1.23
530	—		1.73	680	0.30	—
538	Trace.		—	693	—	0.94
553	—		1.92			

The rotation produced by this film in a magnetic field of 12,000 C.G.S. units is represented by the curve *N* in Fig. 3, upper plot;

and the ellipticity by N in the lower plot. The rotation is less than half that of the hydrogen film — starting from a zero value at about 540 $\mu\mu$ and rising to about a constant maximum between 600 and 680 $\mu\mu$. The ellipticity rises from a zero value at the same point as the hydrogen film, reaches a smaller maximum value at about 560 $\mu\mu$ instead of 590, then drops to a value less than half that of the hydrogen film at 690. On comparing the two films for transmitted light by a spectroscope, the absorption at the extremities of the spectrum appeared somewhat greater in the nitrogen film, than in the one deposited in hydrogen.

Films Deposited in Oxygen. — Films deposited in oxygen were next tested in a magnetic field of the strength used in the previous case, 12,000 units. A film was chosen of about the same optical density as the others and tested, as before, for both rotation and ellipticity. Only a trace of rotation could be detected and this increased with decreasing wave-length, which indicated that the effect was caused by the glass and not the film. A measurable ellipticity was however present. This is given in Table III. and represented by

TABLE III.

Film Deposited in Oxygen. Strength of Field, 12,000 *C.G.S.*

Wave-Length.	Ellipticity.	Wave-Length.	Ellipticity.
490 $\mu\mu$	0.33°	572 $\mu\mu$	0.60°
500	0.55	585	0.10
535	0.90	600	Trace.
560	0.84	705	No trace.

curve O in the lower plot of Fig. 3. The ellipticity rises from a zero value very likely in the same vicinity as the other curves, but owing to the great absorption in this region by the oxygen film no observation there could be made. The maximum value of this curve compared with the others appears to be shifted toward the shorter wave-lengths, and possesses a magnitude only about half that of the nitrogen film. The curve drops comparatively rapidly also toward a zero value at about 590 $\mu\mu$. This film absorbed much more light at both ends of the spectrum than either of the others giving to transmitted light a very different tint. Here, too, the maximum ellipticity was evidently in the region of least absorption.

The results from the films deposited in the different gases leads to the interesting question as to how the gas affects the qualities of the film. As with the others, the oxygen film showed magnetic susceptibility, yet it is probable that much of it was in reality an oxide of iron. It is very likely also that a certain amount of nitrogen combined with the iron during the process of deposition in that gas, as a continuous decrease of gas pressure was noted, though this may have arisen from anode absorption.[1] As to whether the film deposited in hydrogen was free from any effect arising from the presence of absorbed hydrogen[2] remains at present an open question, as the writer was unable to carry the investigation farther.

Deterioration of Hydrogen Films. — In Table IV. are given comparative results of the ellipticity produced by a fresh film deposited in hydrogen and the same film tested a month later. The observations were made with a magnetic field of about 7,000 units. The results are plotted as before in Fig. 4. Curve H represents those from the fresh film, H' those from the same film one month later. It is evident that with age the qualities of the hydrogen film approach those of the nitrogen and oxygen films, the change being greater with the longer wave-lengths.

TABLE IV.

Showing Deterioration of Film Deposited in Hydrogen. Field Strength, 7,000 *C.G.S.*

Wave-Length.	Ellipticity.		Wave-Length.	Ellipticity.	
	Fresh Film.	Same, 1 Month Later.		Fresh Film.	Same, 1 Month Later.
470 $\mu\mu$	0°	—	621 $\mu\mu$	—	2.60°
504	—	1.81°	626	2.80	—
524	1.98	—	632	—	2.48
535	—	2.40	642	—	2.48
538	2.30	—	650	2.78	—
561	—	2.64	658	2.65	2.03
567	2.80	—	687	—	1.59
589	—	2.79	691	2.49	—
612	3.00	2.64	712	2.20	1.58

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[1] See Skinner, PHYS. REV., XXI., p. 169.

[2] W. Heald (PHYS. REV.) has found that freshly deposited iron absorbs a considerable amount of hydrogen at pressures as low as those used in this work.

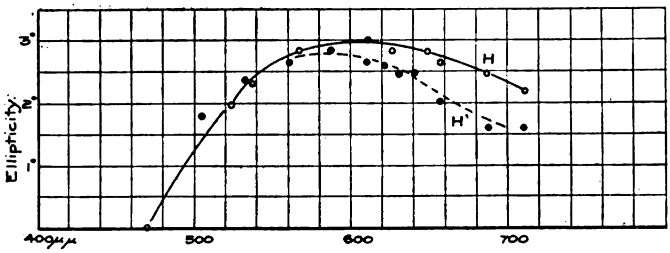

Fig. 4.

Variation of Ellipticity with Intensity of Field. — In Table V. are recorded the results of observations of the ellipticity produced on a wave-length of 548 μμ by a film deposited in hydrogen, in a magnetic field of different intensities. The film used was the one which served for the results given in Table I. Fig. 5 gives a graphical representation of the same, in which the abscissas represent the field strength and the ordinates the corresponding ellipticity it produced. The curve shows that up to an intensity of about 7,000 the ellipticity is proportional to the intensity, but beyond that the rate of increase quickly drops to a very small quantity. Its form is similar

Table V.

Ellipticity with Strength of Field.

Field Intensity (C. G. S.).	Ellipticity.	Field Intensity (C. G. S.).	Ellipticity.
3,000	1.09°	13,400	3.15°
5,600	2.11	13,950	3.18
9,800	3.03	14,300	3.22
12,000	3.13		

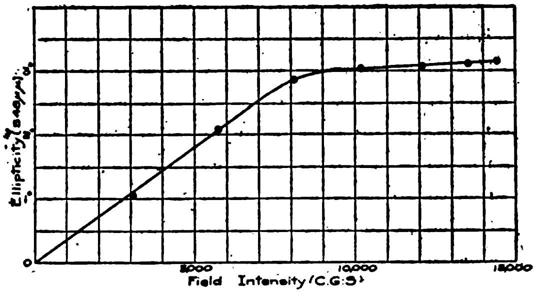

Fig. 5.

to that given by Kundt's results (l. c.) on the rotation of the plane under similar conditions.

Test of Reflected Light. — Righi suggested that the ellipticity of the transmitted light arises from the unequal reflecting power of the iron for the two opposite circular vibrations into which the incident plane vibration is resolved. To test this theory the observing system was removed to the opposite side of the film (position dotted in Fig. 1) and the reflected light observed under the same conditions as the transmitted had been. With a film which produced most marked ellipticity in the transmitted ray, repeated observations, with the strongest fields and the best of light, revealed no ellipticity of the reflected ray. The test was made under conditions such that a rotation of 0.1° of the compensator produced a very clear change in the relative intensity of the two parts of the field of view. A test of the transmitted component required a rotation of the compensator of about 6° to reproduce a match on reversing the field. If on account of large absorption in the film the effect on reflection from the front surface only be taken into account, there should have been an ellipticity in the reflected light of half that observed in the transmitted, or a rotation of the compensator of 3° should have been required to reproduce a match after reversing the magnetic field. This was however certainly less than 0.1°. Hence it is to be concluded that the ellipticity arises from unequal absorption of the two circular components in passing through the film rather than from unequal reflection at its surfaces.

Test of Nickel and Cobalt Films.

Both nickel and cobalt films deposited in nitrogen were tested for ellipticity of the transmitted light under the same conditions as iron. Though films of these metals produce a rotation of the plane in a magnetic field, no ellipticity of either transmitted or reflected light could be detected for any color, under the best working conditions. The films tested were optically as dense as the iron.

The writer is indebted to the late Dr. Brace under whose direction this research was undertaken, and to Dr. C. A. Skinner for assistance which made its completion possible.

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THE PERCENTAGE BRIDGE.

By A. C. Longden.

ABOUT six years ago, a paper entitled "A Percentage Bridge" was presented to the American Institute of Electrical Engineers by Mr. Herschel C. Parker, of Columbia University.

The method which this paper advocated does not seem to me to have received the recognition which it deserves. It may be partly because we are satisfied to "let well enough alone," and partly because of the position of commanding authority occupied by the Carey-Foster method. Be that as it may, the conclusion has been gradually forced upon me, as I have been using the two methods during the past few years, that the Percentage Method, with a few modifications is in no way inferior to the Carey-Foster method, and that it is in some ways a little the better method.

The percentage bridge may be briefly described as an ordinary four gap slide wire bridge, such as is shown in Fig. 1, in which the auxiliary resistances A_1 and A_2 are equal, and have such a relation to the resistance of the bridge wire that a change of one hundredth of 1 per cent. in the ratio of R_1 to R_2 shall produce a change of one millimeter in the position of the balancing point, p, on the bridge wire.

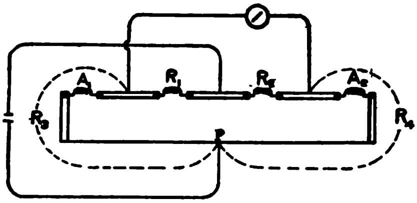

Fig. 1.

In using the bridge, one of the resistance gaps, R_1 for example, is used as the substitution gap. A standard resistance is placed in this gap and balanced against R_2. R_2 should be approximately equal to R_1, so that the balancing point shall be at or near the middle of the bridge wire. The exact value of R_2, however, does not need to be known. When the bridge is balanced, the test coil is substituted for the standard in the substitution gap. If this disturbs the balance, the bridge is again balanced and the distance in millimeters on the bridge wire between the two balancing points indicates the difference of resistance in hundredths of 1 per cent., between the test coil and the standard. If the slider is provided with a vernier, so that its movements can be read in tenths of millimeters, these readings correspond to differences in thousandths of 1 per cent.

This method, aside from its usefulness in standardizing rheostats, has a number of real advantages over the Carey-Foster method for comparing standard resistance units. Mr. Parker says: "For the comparison of standard resistance units of the same value, the Carey-Foster method, using the 'commutator Bridge,' furnishes a very precise and satisfactory means of measurement." Later, however, he refers to the simplicity of the Percentage Bridge, with its four mercury cups, as compared with the complicated commutating device of the Carey-Foster bridge, with its twenty mercury cups. This is a hint which I think we ought to take, for I fear we sometimes rely upon our measurements because we made them with the Carey-Foster bridge, when they are really not as reliable as we suppose. The fact that we cannot repeat them with the degree of precision which the theory of the method warrants, if we use another bridge, or even sometimes with the same bridge, ought to be evidence that there is room for a suspicion that something is wrong. The simplicity of the percentage method and of the apparatus used in connection with it, ought at least to entitle it to a serious consideration.

Two of the strongest points in favor of the Carey-Foster method are that the resistance of the end connections of the bridge is eliminated, and that the coils may be interchanged without handling them. The importance of the first of these claims is considerable when the bridge is used in the ordinary way and when all of the resistances in the bridge circuit are low; but when the end resistances amount to as much as twenty ohms each, as they usually do in the percentage bridge, the importance of exact equality of the end connections is not as great. Suppose, for example, the resistance of one of the end connections should exceed that of the other by one hundredth of an ohm, and this is an inexcusably large excess, even then the *difference* is only one twentieth of one per cent., and the error in the *result* of the measurement is only one twentieth of one per cent. of one hundredth of one per cent., or one part in twenty million for each millimeter of the bridge wire! Of course, it is needless to say that such an error is quite negligible even in measurements of the highest attainable precision. It is evident then, that spurious resistances which would be fatal to success in the Carey-Foster method, if not eliminated, are quite negligible when the percentage method is used.

The other claim, that the coils may be interchanged without handling, has, I am inclined to believe, lost much of its importance with recent improvements in the construction of resistance standards. The Carey-Foster commutator was invented when the British Association form of resistance standard was the only one in vogue. When the wire of which the coil was made, was securely packed away in a thick mass of heat insulating material in order that changes of temperature might be retarded as much as possible, it was important that the coils to be compared should be placed in oil baths several hours before the final tests were made; and it was equally important that they should not be disturbed during the tests. The present method of constructing resistance coils requires that the wire shall be exposed to the oil as fully as possible, so that a coil at any temperature, within reasonable limits, may be placed in an oil bath and it will assume the temperature of the oil in a few minutes. The only important change resulting from such a course will be a slight change in the temperature of the bath; and as this can be easily determined and readjusted, the matter is not at all serious. Furthermore, the temperature of the coil may be brought to the temperature of the bath by means of an auxiliary bath, so that no perceptible change of temperature need occur when one coil is substituted for another. It must also be remembered that even if the temperature of the bath be changed, it is perfectly legitimate to compare the resistance of a test coil at one temperature with the resistance of a standard at another, and finally, it must also be remembered that the case to which I have referred is an extreme one. It is far more likely that the two coils will be in oil baths at the same temperature, or perhaps they will be in the same bath. Of course, it is important that the conducting rods and the metal parts of the mountings should not be heated by contact with the hand.

I am not arguing that it is just as well to handle the coils as not to handle them, but I do argue that the importance of the matter is not as great with the Reichsanstalt form of coil as with the B. A. form. I will also add that if the interchange without handling requires a commutator with twenty mercury cups, the operation is at least questionable.

If it is really important to substitute one coil for another without moving or touching the coils, this can be accomplished much more easily in the percentage bridge than in the Carey-Foster bridge. It is only necessary to extend the terminals of the substitution gap a short distance, as at *A*, in Fig. 2, placing a copper block carrying three small mercury cups or one large one at *B*. The two coils to be compared, R_x and R_s may then be placed side by side in the same oil bath and may remain undisturbed, and either one of them may be short circuited by means of the heavy copper loop, *L*, while the other is being balanced against R_2. The resistances of the loop and of the extension bars may be very small, but it is not necessary that they should be negligible for they do not enter into the measurement at all, as the substitution occurs at their extremities.

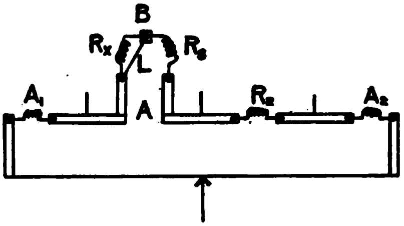

Fig. 2.

Another distinct advantage of the percentage method is its sensitiveness. In the Carey-Foster bridge, the bridge wire must be comparatively large, in order that small differences of resistance may be easily detected and measured. Of course, there is no special reason why the wire should *not* be large, except that a large wire means a small *range;* but there must be a limit somewhere, beyond which we could scarcely expect to go. It may be said in general, whatever the size of the wire, that with the same bridge wire, the same galvanometer and the same current flowing through the standard coils, it is about as easy to measure thousandths of one per cent. with the percentage bridge as to measure hundredths of one per cent. with the Carey-Foster bridge.

The auxiliary coils A_1 and A_2 in Fig. 1, not only serve the purpose of adjusting the percentage ratio to one hundredth of one per cent., but, having a resistance of about twenty ohms each, they protect the standard coils from heavy currents, making it unnecessary to introduce added resistance into the battery circuit, outside of the bridge circuit, as is done in measuring low resistances by the Carey-Foster method.

The battery and galvanometer should be arranged as in Fig. 1, connecting one of the battery wires between R_1 and R_2 and the other to the slider. This puts the auxiliary coils in series with the standard coils and also with the bridge wire, protecting the entire system of standard resistances.

Another advantage of the percentage bridge which is at least worth mentioning, is its range. As has already been remarked, the Carey-Foster bridge is an instrument of low range, usually one per cent. or less, because of the low resistance of the bridge wire. In the percentage bridge adjusted to a percentage ratio of one hundredth of one per cent. per millimeter, the range, for a bridge having a wire a meter long, is ten per cent., minus a small fraction of each end of the wire, upon which the slider cannot be operated.

One of the special fields in which the percentage bridge is very useful and exceptionally convenient is the measurement of temperature coefficients. It is not necessary to compare the resistance under test with a standard or even to know its resistance at all. It is simply placed in the substitution gap and balanced against R_2. Its temperature is then changed and a new balance obtained. Since the distance in millimeters between the two balancing points on the wire is numerically equal to the change in resistance in hundredths of one per cent., it is only necessary to read this distance and divide it by the change in temperature in degrees to obtain the temperature coefficient.

The real object of this paper is yet to be stated. I have thus far considered only the virtues of the percentage bridge. It has *one* fault, — just one I think. It is the purpose of this paper to point out the fault and to present a simple method of eliminating the errors resulting from it.

If the auxiliary resistances have such a relation to the resistance of the bridge wire that a change of one hundredth of one per cent. in the ratio of R_1 to R_2, produces a change of one millimeter in the position of the balancing point on the bridge wire *at any stated position*, this will not be strictly true at any other position. Mr. Parker has taken an *average* value for the ratio of the resistance of the auxiliary coils to that of the bridge wire, and has assumed that it is a sufficiently close approximation for all positions of the slider. He states the value of this ratio as 19.6 to 1. That is, the resistance of each auxiliary coil must be 19.6 times the resistance of the bridge wire. Now as a matter of fact, if the bridge is balanced exactly in the middle, this ratio should be 19.5 to 1. It is 19.6 to 1 if the balancing point is ten centimeters from the middle of the wire.

It is quite obvious that for a small range near the middle of the wire, the errors arising from Mr. Parker's assumption are not very large, but it is also quite obvious that this is true only for a very small range, and that the results are strictly correct, only when the bridge is balanced at a point ten centimeters from the middle of the wire. The errors may, as Mr. Parker suggests, be minimized by adding compensating resistances to the auxiliary resistances when the balancing point is a considerable distance from the center. This, of course, means different ratios for all different positions of the slider.

Instead of thus complicating the apparatus and only *minimizing* the errors, we propose that the errors be wholly eliminated by using a single set of auxiliary coils, correct for the middle point on the wire, and applying a suitable scale of corrections for other points. The following table furnishes the corrections, worked out for points ten centimeters apart on the wire.

A	*B*	*C*	*D*
10^{cm} toward R_4	$\frac{201}{199}$, or 101.005%	1%	.005%
20 "	$\frac{202}{198}$, or 102.020	2	.020
30 "	$\frac{203}{197}$, or 103.046	3	.046
40 "	$\frac{204}{196}$, or 104.082	4	.082
50 "	$\frac{205}{195}$, or 105.128	5	.128
10^{cm} toward R_3.	$\frac{199}{201}$, or 99.005	1	.005
20 "	$\frac{198}{202}$, or 98.020	2	.020
30 "	$\frac{197}{203}$, or 97.044	3	.044
40 "	$\frac{196}{204}$, or 96.078	4	.078
50 "	$\frac{195}{205}$, or 95.122	5	.122

In column A, is found the balancing point on the bridge wire, and in column B, the true ratio of the test coil to the standard for that balancing point. Column C presents the uncorrected difference between the test coil and the standard, assuming each millimeter of the wire as .01 per cent. The corrections are given in column D.

These corrections are a trifle larger for points on the half of the wire running toward R_4 than for corresponding points on the other half; R_1 being the substitution gap. This is because in that case the ratio of R_3 to R_4 increases as the balancing point runs toward R_4.

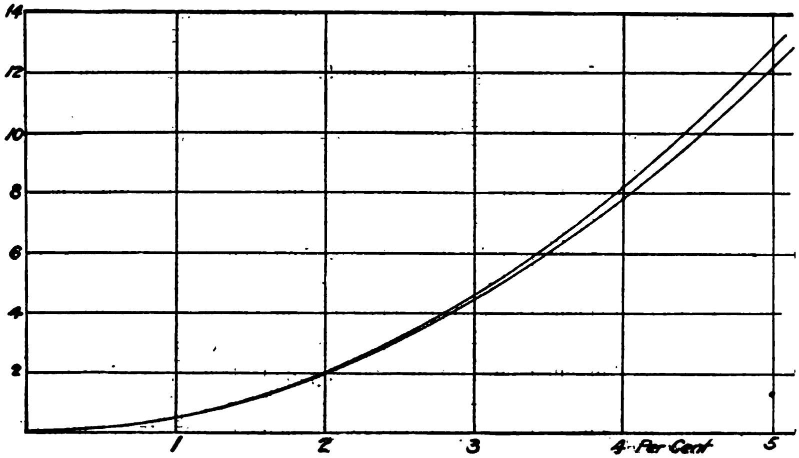

Fig. 3.

Fig. 3 presents curves from which may be taken the correction for any point on the wire. The uncorrected differences in column C, are plotted as abscissæ and the corrections in column D, as ordinates. The upper curve is for the first half of the table and the lower curve for the second half. A mean between the two would be within .003 per cent. of either, even at the extremities of the wire, where the errors are maximum. The corrections are given in hundredths of one per cent. and are to be *added* to the uncorrected results, using the upper curve, if the bridge readings run toward R_4, and the lower curve, if the readings run toward R_3; R_1 being the substitution gap.

With these corrections properly applied, the percentage method of comparing standard resistances becomes a precision method of the highest rank.

I have said nothing about errors due to the irregularities in the bridge wire, but of course the wire may be calibrated for the percentage bridge as well as for any other.

Knox College Physical Laboratory,
Galesburg, Ill., November 23, 1906.

A SPECTRO-PHOTOMETRIC STUDY OF THE ABSORBING POWER AND THE FLUORESCENCE OF RESORUFIN.

By Frances G. Wick.

IT has frequently been observed that the color of the light emitted by a fluorescent solution is altered by a change in the concentration of the solution. A dilute solution of fluorescein, for example, gives a green fluorescence, while the light emitted by a concentrated solution of the same substance shows a distinctly yellow tinge. In other words an increase in concentration causes the maximum of the fluorescence spectrum to shift toward the longer wave lengths.

Such a shift might result from a real change in the period of vibration of the fluorescent molecules, *i. e.*, the form of the typical fluorescence spectrum might depend upon the concentration of the solution. But the observed change in the fluorescence spectrum might equally well result from a change in the absorbing power of the solution. The light emitted by portions of the active substance in the interior of the solution must pass through the solution before emerging, and is therefore weakened by absorption. Since the maximum of the fluorescence spectrum does not occur at the same wave length as the maximum absorption, being always slightly displaced in the direction of the longer waves, it is clear that different portions of the fluorescence spectrum will be affected by absorption in varying degree; the shorter waves will always be absorbed most strongly. The maximum of the fluorescence spectrum is therefore always shifted toward the red to some extent, and the increased absorbing power of concentrated solutions causes the shift due to this cause to increase with the concentration.

The experiments described in the present paper, the object of which was to determine whether the typical fluorescence spectrum changes with the concentration or whether the shift referred to above is caused by absorption only, were undertaken at the suggestion of Professors Nichols and Merritt, to whom the author wishes to acknowledge her indebtedness. To their advice and kindly criticism is due whatever may be of value in the results.

The substance used was resorufin, whose strong fluorescence in the yellow-red is conveniently excited and readily observed. The experimental work naturally falls under three heads as follows:

1. The relation between absorption and thickness, to test whether or not the absorption of a fluorescent solution obeys the exponential law of optically perfect substances.

2. The relation between absorption and concentration, to test the application of Beer's[1] law, *i. e.*, that an increase in the concentration of a solution is equivalent to an increase in thickness.

3. The fluorescence spectra of six dilutions of resorufin were observed to determine whether or not there is any shift in the maximum of fluorescence as concentration changes.

The first and second of the problems under consideration have been investigated, to some extent, by B. Walter[2] and by Nichols and Merritt.[3] The results here given for resorufin are entirely consistent with those obtained by these authors for other substances.

HISTORICAL.

The history of the investigations upon fluorescence, as related to the above problems, is briefly as follows:

In 1888 B. Walter,[4] using the Vierordt spectrophotometer, found that the ratio of the fluorescent light emitted to total incident light absorbed, Fl/A, increases with dilution. This law had been previously conjectured by Lommel,[5] whose mathematical theory Walter[6] used in verifying his own results. In connection with this work Walter tested Beer's[7] law of absorption for fluorescein by four measurements. He found the law to hold true for dilute but not for concentrated solutions.

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[1] Beer, Pogg. Ann., 86, p. 78.

[2] B. Walter, Wied. Ann., 34, 1888.

[3] Nichols and Merritt, Studies in Luminescence, PHYSICAL REVIEW, Vols. 18–21.

[4] B. Walter, Wied. Ann., 34, 1888.

[5] Lommel, Pogg. Ann., 160, p. 76, 1877.

[6] B. Walter, Wied. Ann., 36, p. 502, 1889.

[7] Beer, Pogg. Ann., 86, p. 78.

Walter briefly states his conclusions as follows; "Ability to excite fluorescence, Fl/A, in the most concentrated solutions, is infinitely small or zero. After Fl/A obtains a measurable value it increases in proportion to dilution to a certain dilution called the 'critical point.' For greater dilutions Fl/A remains constant."

In explanation of the above, Walter[1] advances a molecular group theory which is of interest, preceding as it does, the present theory of ionization given by Buckingham.[2] Walter summarizes his theory in the following statements:

1. Every separate molecule of a given substance in solution absorbs, so long as it remains in the separate condition, the same fraction of the light falling upon it no matter how great its distance from other molecules in the solution may be.
2. Every separate molecule of a given fluorescent substance in solution, so long as it is in the separate condition, changes the same fraction of absorbed light into fluorescent light, no matter how great its distance from neighboring molecules may be.
3. As soon as molecules begin to combine in groups the validity of these statements ceases. Fluorescence entirely ceases in such a group and absorption extends over wave lengths which a separate molecule is not able to absorb.

Walter's idea of the separate molecule seems to correspond closely with what is now known as the ion.

In 1894 E. Buckingham[2] performed a series of careful experiments to discover whether or not fluorescent substances are in a state of ionization. His investigations led to the conclusion that, at least in some cases, fluorescence is due to ionization and produced only by that part of a solution which is ionized. Certain ions, along with their other well known optical properties, possess this property of fluorescence.

The spectro-photometric work of Nichols and Merritt deals with many of the phenomena of fluorescent substances, only a few of which need be mentioned here. The following facts observed by them[3] for fluorescein are also true for resorufin as will be shown in the third part of this paper:

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[1] B. Walter, Wied. Ann., 36, p. 518, 1889.

[2] E. Buckingham, Zeitschrift fur Physikalische Chemie, 14, p. 129, 1894.

[3] Nichols & Merritt, PHYSICAL REVIEW, Vol. 18, No. 6, 1904.

1. The characteristic fluorescence band is situated near the edge of the absorption band and is steeper on the side toward the violet.

2. As fluorescent light passes through greater thicknesses of liquid, the position of the maximum in the fluorescence spectrum is shifted toward the red.

3. The effect of diluting a fluorescent solution is similar to that of diminishing the distance through which the fluorescent light travels.

METHOD OF OBSERVATION.

The instrument used for this work was the spectro-photometer of Lummer and Brodhun. The position of the collimator tubes *A* and *B* (Fig. 1), was adjusted until the image of the sodium line from the slit *c* coincided with the same line from slit *D* This line was set to division 10 of the vernier attached to the telescope, *T*, and the instrument clamped in this position.

The source of light was an acetylene flame *S* from which light was diffusely reflected by a block of magnesium carbonate, *n*, mounted at an angle of 45° with axis of the collimator *A*. For observations on transmission a similar block of magnesium carbonate was placed in front of the slit *D*.

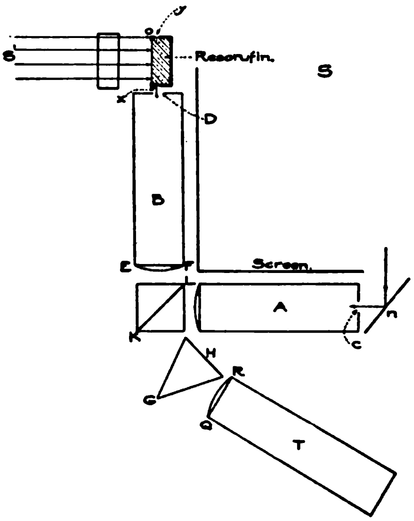


Fig. 1. Shows the apparatus arranged for measurements of fluorescence.

The work was performed in a room with black walls, the acetylene flame being the only source of light. Screens were adjusted so that no stray light could strike any part of the instrument and thus produce error.

The solution under consideration was placed in front of slit *D*, the width of which was kept constant during a single set of observations. The ocular lenses of the telescope were removed, the eye being focused upon the face of the prism magnified by the objective of the telescope *T*.

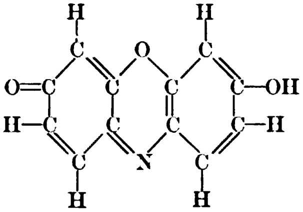
In making observations the width of slit *c* was adjusted by means of a micrometer screw until the central band of light reflected from *KL* was brought into the best possible coincidence with that transmitted through the upper and lower parts of the cube from *D*.

The results given represent an average of from 5 to 10 settings, the zero point of the screw attached to *c* being carefully determined and caution being taken to avoid error due to lost motion.

The hydrogen lines of a vacuum tube, together with a few of the Frauenhofer lines, were used for calibration.

Composition of Resorufin.

Weselski's diazo-resorufin,[1] the fluorescent substance used in this investigation, has the formula



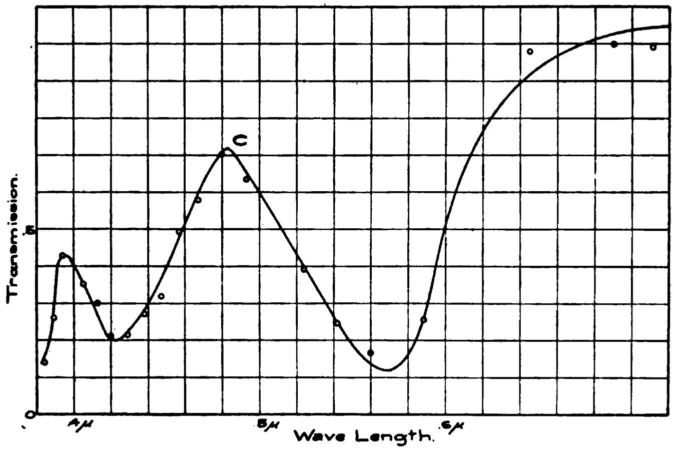


Fig. 2. Transmission spectrum of layer 1.075 cm. thick. Concentration ⅛.

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[1] Berichte, 22, 3036. Acknowledgment should be made to Professor W. R. Orndorff for his kindness in preparing the resorufin used in this work.

Dissolved in absolute alcohol it is bright red in color with a sharply defined fluorescence band extending from .54 μ to the limits of the visible spectrum in the red. The transmission spectrum given in Table I. and Fig. 1 shows the absorption band corresponding to this, and also a smaller absorption band in the region of the extreme violet.

TABLE I.

Transmission through layer of resorufin 1.075 *cm. thick.*
Concentration 1/8.
I_0 = *Intensity of light before transmission.*
I = *Intensity of light after transmission.*
λ = *Wave-length.*

λ	I_0	I	$\frac{I}{I_0}$ = **Transmission.**
.384 μ	45	6	.111
.39	46	12	.261
.394	46	20.25	.438
.407	46	16	.347
.413	46	13.625	.297
.42	46	9.75	.213
.429	46	9.95	.217
.438	45.5	12.1	.266
.448	45.5	14.15	.32
.457	45.05	22.275	.497
.468	45.05	25.975	.579
.48	45.725	32.05	.70
.492	45.725	28.875	.63
.508	44.7	24.4	.538
.524	44.7	17.575	.394
.542	46.27	11	.241
.562	46.41	7.525	.162
.589	46.25	11.64	.253
.614	46.55	32.275	.706
.646	46.623	45.625	.981
.69	44.525	44	.99
.72	41.25	41	.99

For this work different concentrations of resorufin were used. These are merely relative and are indicated as "concentration $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$ and $\frac{1}{128}$," the original solution being taken as $\frac{1}{1}$. All curves corresponding to the above concentrations are indicated by the letters *A*, *B*, *C*, *D*, *E*, *F* and *G* respectively.

ABSORPTION AND THICKNESS.

The first problem under consideration was, as stated above, to find whether a fluorescent solution acts like an optically perfect substance in obeying Lambert's law of absorption, *i. e.*, whether

$$I = I_0 e^{-\beta x}$$

where I_0 and I are the intensities before and after transmission, and x is the thickness of the absorbing layer.

To test this law the coefficient of absorption, β, was calculated from observations taken with cells of different inside thickness, the measurements of these cells being made with great care.

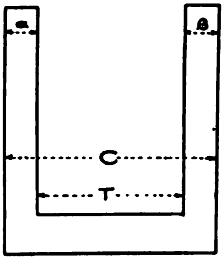

Fig. 3.

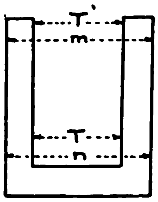

Fig. 4.

The larger cell was measured with micrometer calipers. Thickness of glass = α, β (Fig. 3). Outside thickness of that part of the cell used for transmission = C. Inside thickness = T.

$$T = C - (\alpha + \beta) = 2.05 \text{ cm.}$$

In the case of small cell (Fig. 4), calipers could not be used to measure the thickness of glass. The cell was placed under a travelling microscope to measure the average inside thickness of the top. The outside thickness at the points indicated in the figure was measured with calipers.

$$T = T' + (n - m) = 1.075 \text{ cm.}$$

It was assumed that thickness of glass from top to bottom of cell remained constant since broken pieces of the same glass varied not more than .01 mm.

To determine the transmission, the cell containing the resorufin was placed in front of slit D (Fig. 1), care being taken that no direct light from S should strike the cell, and light from the block of magnesium carbonate, in front of slit D (not shown in the figure), was transmitted through it. The reading of C was taken to get the relative intensity, I, of the light transmitted for different wave lengths. After each observation the cell containing resorufin was removed and an identical one filled with absolute alcohol put in its place. The reading of C in this case gave the value of incident light before absorption, I_0. From these observed values of I_0 and I, together with the thickness of the absorbing layer, the coefficient of absorption was computed.

As a check upon these results a different method of observation was used. Keeping D (Fig. 1) constant, the reading C_1 was taken for the light alone. A reading of C_2 was then taken with alcohol in front of C and resorufin in front of D.

$$\frac{C_1}{D} = \text{Incident light } (I_0),$$

$$\frac{C_2}{D} = \text{Transmitted light } (I),$$

$$\frac{\frac{C_1}{D}}{\frac{C_2}{D}} = \frac{C_1}{C_2} = \frac{I}{I_0}.$$

As a further check the positions of alcohol and resorufin were reversed, the alcohol being placed in front of D and the resorufin in front of C. Then

$$\frac{D}{C_1} = I, \qquad \frac{D}{C_2} = I_0,$$

$$\frac{I}{I_0} = \frac{C_2}{C_1}.$$

From these data the value of the coefficient of absorption for each wave length was found to be practically the same as that found by the first method and independent of the thickness of the cell. Absorption curves plotted from average values obtained by both methods, using cells of two thicknesses are identical, as shown in Fig. 5.[1] The

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[1] Figures 5, 6, and 7 give only relative values of coefficients since they are drawn from data computed upon the basis of ordinary instead of natural logarithms. To get from the curves the absolute value of coefficients given in the tables multiply by 2.3.

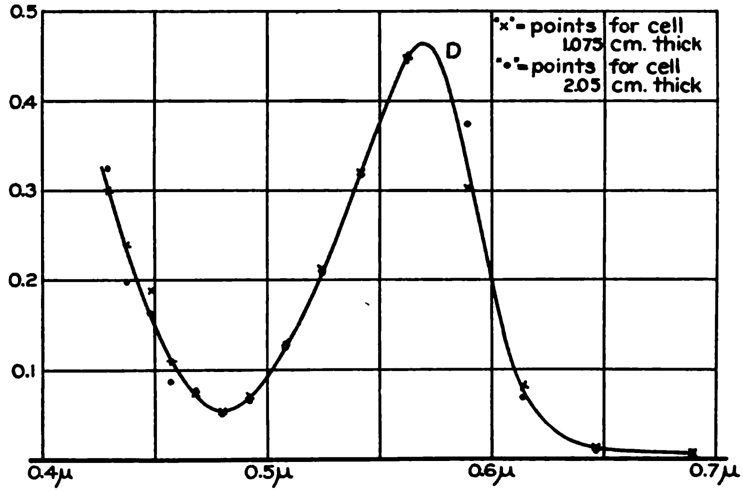

Fig. 5. Coefficient of absorption as computed from cells of different thickness.

absorption coefficients for different concentrations and also the data from which the computation was made are given in Table II.

Table II.

Coefficient of absorption computed from cells of different thickness.
Concentration = 1/16.
I_0 = *Intensity of light before transmission.*
I = *Intensity of light after transmission.*
β = *Coefficient of absorption.*

λ	Thickness of Cell = 1.075 cm.			Thickness of Cell = 2.05 cm.		
	I_0	I	β	I_0	I	β
.429 μ	79.9	35.76	.7476	81.85	19.95	.6884
.438	78.82	48.36	.4539	80.	25.93	.5495
.448	80.2	53.58	.3746	82.45	33.7	.4363
.457	79.49	62.55	.2223	80.6	48.3	.2495
.468	81.58	66.12	.1805[1]	80.36	54.5	.1702[1]
.48	78.94	70.01	.1175[1]	82.28	61.35	.1267[1]
.492	76.14	64.78	.15	80.5	57.05	.1678
.508	76.55	57.55	.2653	79.68	43.13	.2993
.524	76.8	45.83	.4808	78.93	29.58	.4786
.542	75.64	34.56	.7283	78.48	17.32	.7369
.562	75.91	25.37	1.02[1]	78.68	10.	1.0304[1]
.589	75.84	30.92	.8354	79.37	18.97	.6979
.614	75.19	63.41	.1579	79.85	54.08	.1899
.646	73.57	72.21	.0169	80.89	75.8	.0316
.69	64.93	63.85	.0148	71.33	69.2	.0144

[1] These values are average results from a number of observations.

It appears therefore that the absorption of resorufin is in accordance with Lambert's Law. Deviations from this law, if present, are at least too small to be detected by the experimental methods used.

ABSORPTION AND CONCENTRATION.

The second set of experiments was undertaken to study the relation between absorption and concentration and thus test the validity of Beer's law for a fluorescent substance, *i. e.*, "Increasing the concentration of a solution is equivalent to a like increase in thickness."

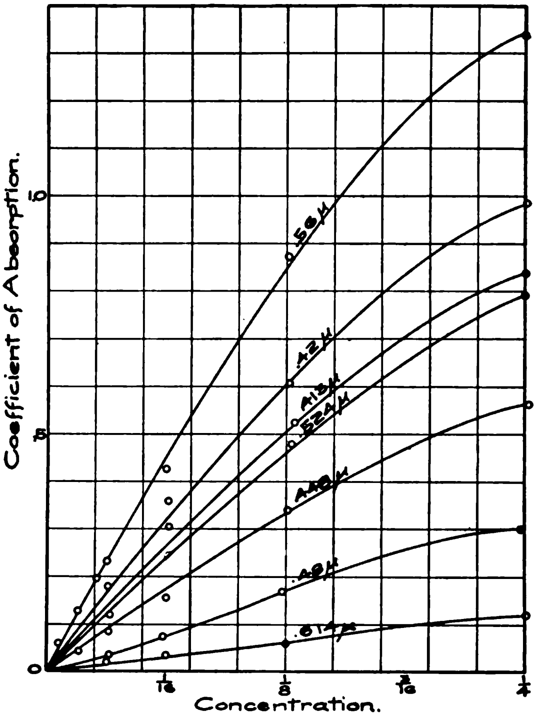

Fig. 6. Curves showing effect of change in concentration upon coefficient of absorption[1] of wave-lengths marked on curves.

The same method of observation was employed for this as for the preceding work, no changes in the apparatus being necessary. The coefficient of absorption was found for different concentrations, the solution being placed in a cell 2.05 cm. thick if dilute, and in cell 1.075 cm. thick if concentrated. This change of cell is permissible since, according to the results obtained above, Lambert's law of absorption applies to resorufin. The absorption coefficient of different wave lengths was thus obtained for seven concentrations.

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[1] See footnote on page 363.

Figs. 6 and 7 give the results of these observations. It will be noticed from these curves that the coefficient of absorption, in the case of dilute solutions, increases in direct proportion to concentration. For concentrated solutions, this proportion fails, the concentration increasing more rapidly than the absorption. This is true for all the wave lengths; in every case the curve starts out as a straight line for dilute solutions and bends downward as higher concentrations are reached.

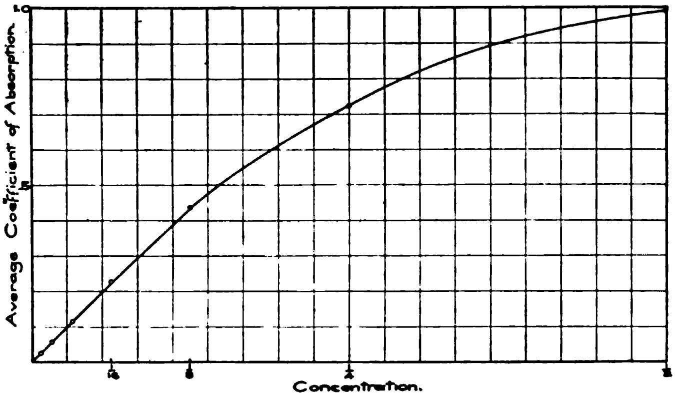

Fig. 7. Average coefficients of absorption[1] for different concentrations taken from Table III.

TABLE III.

Coefficient of absorption for corresponding wave-lengths in different concentrations.

λ	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$
.413 μ	2.254	1.8814	1.2132	.7125	.3107	.1444	.0276
.42	2.806	2.2793	1.4237	.8533	.4360	.1984	.0775
.448	2.53	1.5364	.7935	.3565	.1860	.0897	.0368
.48	1.336	.6934	.3864	.1734	.0749	.0586	.0367
.524	3.162	1.8584	1.1247	.5349	.2783	.1453	.0692
.562	3.473	3.0797	2.0171	.989	.5441	.3036	.1442
.614	.3864	.3082	.1550	.0995	.0484	.0524	.0265
Average	2.2782	1.6624	1.0162+	.5313	.2683	.1417	.0597

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[1] See footnote on page 363.

Fig. 7 shows the average coefficient of absorption for the seven wave-lengths as a function of the concentration. This curve is used later for finding the coefficient of absorption for different dilutions, especially careful measurements for the concentration 1/16, being taken as a basis of calculation. The values thus obtained, given in Table IV, were used in all subsequent computations involving the coefficient of absorption.

TABLE IV.

Coefficients of absorption computed from average of careful observations for concentration 1/16 and average absorption concentration curve. (*Fig. 7.*)

λ	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$
.524 μ	2.1986	1.60471	.9841	.51313	.2599	.12995	.0651
.542	3.266	2.385	1.455	.76245	.38398	.19251	.09637
.562	4.3608	3.175	1.946	1.0177	.5154	.25783	.1288
.589	2.7324	1.995	1.219	.63733	.3227	.16146	.0807
.614	.49335	.36087	.22157	.115322	.05842	.0292	.0146
.646	.95588	.0698	.0428	.0223	.0113	.00575	.00283

The above results agree, in every respect, with those given by Walter.[1] Beer's law is true for dilute solutions but fails for greater concentrations, as is indicated by the deviation of the curve from a straight line. The straight form of the curve (Fig. 7) up to concentration 1/8 corresponds to what Walter calls "complete solution" in which, he says, the molecules are in a "separate state." Concentration 1/8 may be regarded as his "critical dilution" where a change seems to take place in the condition of the fluorescent substance. Solutions more concentrated than 1/8 correspond to those called by Walter "incomplete" in which "molecular groups" exist.

Interpreted according to the ionization theory the curve (Fig. 7) indicates that in dilutions less than 1/8 a state of complete or nearly complete ionization has been reached. At this point a change takes place, more resorufin being contained in the solution than is ionized. As the concentration increases, more and more of the solution remains undissociated. It appears that the undissociated resorufin is not only incapable of fluorescence, but is also much less effective in causing absorption than is the dissociated substance.

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[1] B. Walter, Wied. Ann., 36, pp. 502 and 518, 1889.

Fluorescence and Concentration.

For observation of the fluorescence spectrum the spectro-photometer was adjusted as before. In front of slit *D* (Fig. 1) was placed a glass cell 5.4 cm. long containing the resorufin solution. This cell was entirely covered with black paper except for a space about 1.5 cm. high across the bottom of the face next to the exciting light *S′*, and two narrow strips *x* and *y*. The opening *x* was to allow light to enter collimator slit, while *y*, directly opposite, was used only for adjusting. The source of illumination used to produce fluorescence was a bank of four acetylene flames *S′*. Between these and the fluorescent solution was placed a glass cell filled with water to prevent the heating of the resorufin. The comparison source was another acetylene burner, *S*, light from which was reflected into the slit *C*, by a block of magnesium carbonate, *n*. The cell containing the resorufin was so placed that light from the whole layer of solution next the inside surface of glass came through the slit *D* of the collimator.

It is clear that a portion of the fluorescent light is absorbed by the solution before reaching *D;* and attention has already been called to the fact that this absorption will be different for different parts of the fluorescence spectrum. If the fluorescence is measured in the manner indicated above it is therefore necessary to apply a correction for absorption before the typical fluorescence spectrum can be determined. Two methods of making this correction have been used. In the first method the necessary correction was computed as follows :

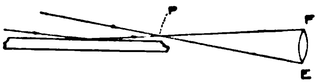


Fig. 8.

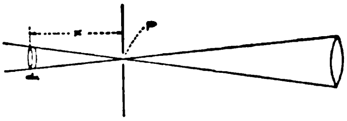


Fig. 9.

Let *P* be any point in the slit (Fig. 8). The source of the light reaching *P* is a cone in the fluorescent liquid converging toward *P*. The angle of this cone is determined by the aperture of the object glass, *EF*, of the collimator. Any part of this cone cut off by the glass walls of the cell or by the upper surface of the solution is supplied by total reflection.

Consider a disk bounded by two circular sections of this cone at distances x and $x + dx$ respectively from the apex, P (Fig. 9). The amount of fluorescent light reaching P from such a disk as this will be independent of x, provided no absorption takes place, for, while the area of the section varies as x^2, the intensity of the light reaching P from each small portion of the disk varies as $1/x^2$. The fluorescent light that would reach P from such a disk if there were no absorption is therefore kdx, where k is a function of the wave-length, λ.

Since the light emitted by each section of the cone is in part absorbed, we have for the light reaching P from one of the disks

$$di = kdx \cdot e^{-\beta x}$$

and the total light reaching P from the whole cone is

$$i = k\int_0^a e^{-\beta x}\,dx$$

where a is the thickness of the fluorescent solution. Hence

$$i = k \cdot \frac{1 - e^{-\beta a}}{\beta}$$

and

$$k = \frac{\beta i}{1 - e^{-\beta a}}.$$

While the i in this expression represents the light reaching one infinitesimal portion of the slit, it is clear that the total light reaching the slit is proportional to i. It is also evident that k is proportional to the total amount of light, f, of the wave-length considered that is emitted by the fluorescent substance per unit volume. Hence

$$f = \frac{k\beta I}{1 - e^{-\beta a}}.$$

The curve showing the relation between f and λ is the "typical fluorescence spectrum."

TABLE V.

Observed fluorescence intensity before correction is made for absorption. Measurements made according to Method I, fluorescence excitation extending over whole face of cell.

λ	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$
.524 μ	1.77	2	2.1	2	2.08	2.4
.542	1.86	2.1	2.2	2	2.13	2.41
.562	3.46	4.67	5.61	7.66	7.43	7.266
.589	30.29	46.44	56.64	34.87	23.21	16.17
.614	79.46	95.49	95.27	34.03	17.63	11.62
.646	86.6	76.3	66.27	17.73	8.9	6.88
.69	59.27	42.42	46.23	12.67	7.28	

TABLE VI.

Observed fluorescence intensity reduced to such a scale as to make intensity of wavelength .589 μ. The same as observed fluorescence in Table VIII.

λ	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$
.524 μ	5.38	4.08	3.192	3.32	3.578	3.116
.542	5.65	4.284	3.344	3.32	3.66	3.229
.562	10.52	9.53	8.53	12.715	12.78	9.736
.589	92.17	94.64	86.06	57.95	40.41	21.68
.614	241.558	194.8	144.81	56.49	30.32	15.57
.646	263.26	155.65	100.73	29.43	15.308	9.22
.69	180.18	86.54	70.27	21.03	12.52	

To investigate the effect of concentration upon the position of the fluorescence band observations were made in six concentrations. Table V. gives values of observed fluorescence before any correction is made for absorption. Table VI., graphically represented in

TABLE VII.

Corrected fluorescence. Value of f calculated from Table V. according to formula $f = I\beta/(1 - e^{-\beta a})$.

λ	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$
.524 μ	2.841	1.968	1.148	.67	.53	.52
.542	4.436	3.056	1.74	.86	.632	.566
.562	10.98	9.088	5.734	4.196	2.558	1.85
.589	60.438	56.61	37.2	13.62	6.35	3.665
.614	33.34	29.757	23.55	7.27	3.455	2.203
.646	19.31	15.55	12.83	3.339	1.65	1.217

TABLE VIII.

Intensity of fluorescence of different concentrations at wave-length .589 μ.

	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$
Observed fluorescence.	75.5	92.17	94.64	86.06	57.95	40.01	21.68
Corrected fluorescence, *i. e.*, value of *k* as obtained from formula.	206.115	183.879	115.366	54.85	22.53	11.044	4.86

Fig. 10, gives the same sets of observations so reduced in scale as to be comparable in intensity with each other. Thus reduction was made by multiplying the data for each concentration by such a factor as to give wave-length .589 μ the same value as that given as the observed fluorescence of the corresponding concentration in Table VIII.

The curves in Fig. 10 show the observed values of I as a function of λ for six different concentrations of the solution. The shift in the maximum caused by a change in concentration is here well marked.

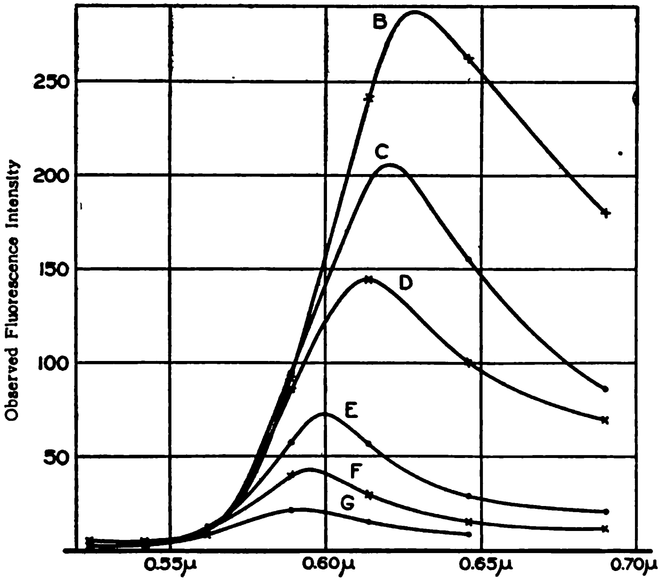


Fig. 10. Observed fluorescence intensity, showing shift of maximum, with change in concentration.

Table VII. gives the result of correcting values in Table V. for absorption according to the above formula, the value of f, which

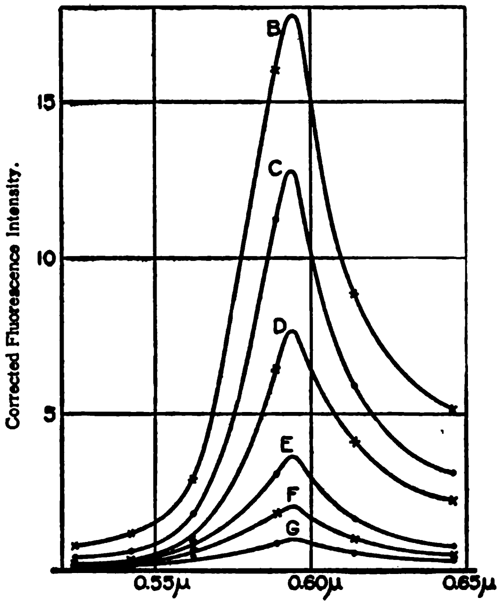


Fig. 11. Fluorescence curves corrected for absorption.

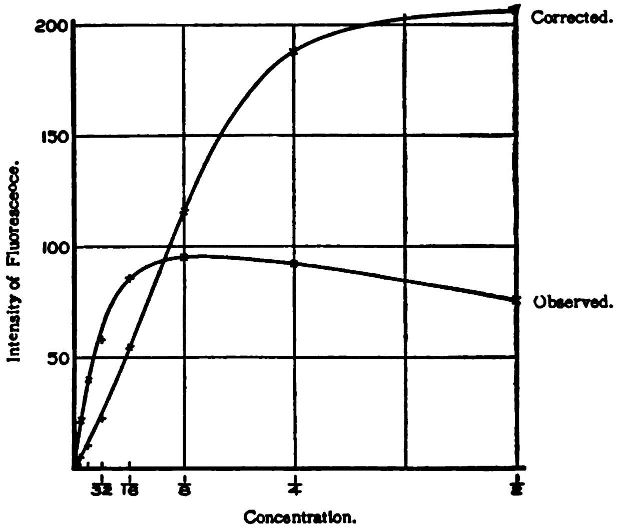


Fig. 12. Intensity of observed and corrected fluorescence for different concentrations at wave-length .589 μ.

is proportional to actual fluorescence intensity (see page 369), being found in each case. These values are plotted in Fig. 11 according to an arbitrary vertical scale which makes the height of each curve approximately proportional to intensity of fluorescence for the corresponding concentration. It will be observed that these curves are similar in form, the maximum wave-length being approximately the same, about .595 μ.

For purposes of comparison a series of measurements was made to determine the relative intensity of different concentrations at wave-length .589 μ, keeping all conditions constant. Table VIII. shows the results. In Fig. 12 the curves marked "observed" gives the actual observed fluorescence. The "corrected" curve has for ordinates the value of f computed from the formula (page 369) and shows the relative intensity of actual fluorescence in different concentrations.

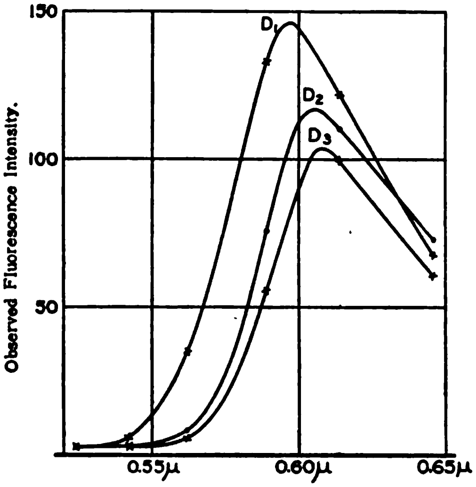


Fig. 13. Observed fluorescence produced by light from 1 mm. section passing through different thicknesses of solution.

Thickness of D_1 = 1 mm.
Thickness of D_2 = 1 cm.
Thickness of D_3 = 2 cm.

The following method of obtaining the corrected fluorescence curves was used as a check upon the first, the final results being obtained yb a graphical, rather than mathematical procedure.

The apparatus was set up as before except that the light exciting fluorescence, instead of striking the whole face of the glass, was allowed to strike only a vertical section 1 mm. in width. The side of the cell next to the exciting light *S'* was covered with a black paper screen having a vertical opening 1 mm. wide, through which light was admitted. This opening was set at different distances from the end of the cell and the fluorescence intensity was measured for three or four different positions. In order to make possible accurate measurements of the distance from the edge of the glass, a scale was etched upon the face of the cell just above the level of the collimator slit.

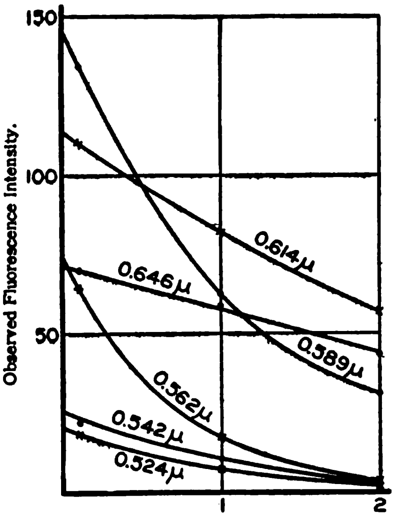


Fig. 14. Variation in intensity of observed fluorescence from 1 mm. section transmitted through different thicknesses of solution.

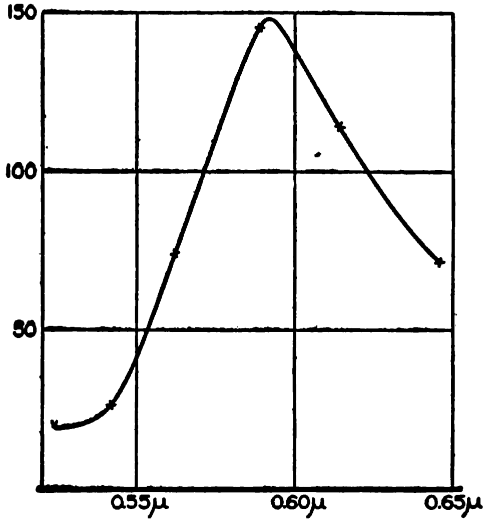


Fig. 15. Fluorescence corrected for absorption by graphical method. Concentration = ⅛.

The effect of absorption upon the position of the maximum of the fluorescence spectrum is well shown in Fig. 13, in which the three curves were taken with the opening at different distances from the face of the cell.

In order to correct for absorption, curves similar to those shown in Figs. 14 and 15 were plotted for each solution. These curves show the variation in the intensity of the fluorescent light with the thickness of the liquid through which the light passes. From the intercept of one of these curves upon the *y* axis the intensity of fluorescence can be found when the thickness becomes zero. In such a case there is no absorption. The error of extrapolation was reduced to a minimum by taking one point very close to the edge of the glass.

The corrected values of fluorescence intensity thus obtained were used in plotting the accompanying fluorescence curve in Fig. 15. The same procedure was followed in the case of each solution.

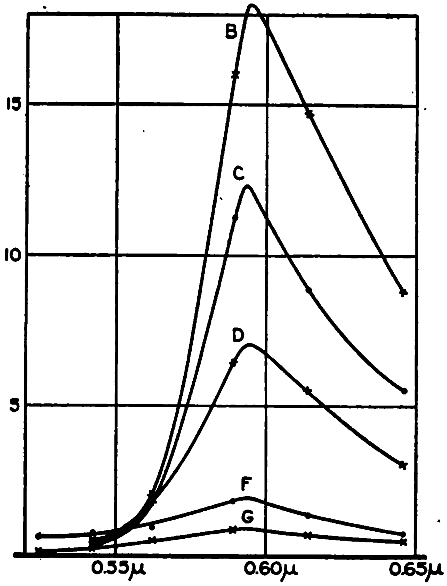


Fig. 16. Fluorescence curves corrected for absorption by graphical method. Vertical scale same as for Fig. 11.

The observations and results obtained by this graphical method are given in Table IX. The corrected fluorescence spectra obtained in this way are plotted together in Fig. 16, the vertical scale being such that values for wave-length .589 μ are the same as the values for the corresponding concentration in Fig. 11.

Table IX.

Observed fluorescence of section 1 mm. wide from which light is transmitted through different thicknesses of solution. The column marked 0 thickness gives fluorescence intensity corrected for absorption by graphical method, i. e., by extrapolation from curves similar to those in Fig. 14.

Concentration 1/2.

λ	1.5 mm.	5 mm.	1 cm.	0
.524 μ	1.75		1.1	1.5
.542	2.525	1.6	1.2.	2
.562	14.3	3	1.95	12.5
.589	66.24	41.5	29.63	114
.614	94.49	70.73	61.16	103
.646	42.4	42	42.5	43

Concentration 1/4.

λ	1.5 mm.	5 mm.	1 cm.	0
.542 μ	2.83	2.1	1.3	3
.562	9.075	4.1	2.325	13.8
.589	71.375	45.88	21.38	103.5
.614	88.07	73.73	64.66	95
.646	51.46	48.7	54.375	52.3

Concentration 1/8.

λ	1.5 mm.	5 mm.	1 cm.	0
.524 μ	2.65	2.1	1.73	2.5
.542	3.9	2.6	2.66	4
.562	34.15	12.083	7.525	50
.589	95.67	69.678	49.56	109
.6	101.75	79.5	63.33	117
.614	79.725	68.4	56.5	90
.646	34.68	33.6	31	36

Concentration 1/16.

λ	1 mm.	1 cm.	2 cm.	0
.524 μ		2.83	3	3.5
.542	5.92	2.97	2.83	8
.562	34.87	8.62	5.56	42
.589	132.7	75.95	55.43	144
.614	121.42	110.3	99.6	124.5
.646	67.36	72.7	60.27	70

Concentration 1/64.

λ	1.5 mm.	5 mm.	1 cm.	0
.508 μ	3.47	3.81	3	4
.524	4.83	3.75	3.75	4
.542	6.16	4.9	4.23	7
.562	61.77	38.5	25.55	76
.589	145.85	138	125.05	150
.614	110.45	104.86	96.15	113.5
.646	58.1	50.25	31.7	60

Concentration 1/128.

λ	1.5 mm.	5 mm.	1 cm.	0
.524 μ	4.17	4.47	3.45	4.5
.542	6.53	6.1	5.33	6.8
.562	25.2	23.66	17	26
.589	49.17	49.5	47.38	49.5
.614	39.66	37.66	38.33	39.5
.646	27.83	25	27.5	28

A comparison of the curves of Figs. 11 and 16 shows that the results obtained by the two methods are the same. The typical fluorescence curves are all similar in form and the position of maximum fluorescence (about .595 μ) remains constant for all dilutions. The shift in the observed fluorescence maximum is therefore due entirely to absorption.

This fixed position of maximum fluorescence seems to be consistent with the theory of ionization. According to Buckingham, ions may have some of the same optical properties as other substances and fluorescence is one of these properties. If only the ions fluoresce the position of maximum fluorescence would evidently remain constant whether the material of the solution were all ionized or not.

This fixed position of maximum fluorescence is also consistent with the theory of Nichols and Merritt[1] that fluorescence is caused by an unusual kind of dissociation similar to that produced in a gas by the action of Röntgen rays. That part of the solution which, during the process of change, produces fluorescence does not give evidence of any effect due to material not dissociated beyond that of absorption.

---

[1] Nichols and Merritt, PHYSICAL REVIEW, Vol. 19, No. 1, 1904; Vol. 22, No. 5, 1906.

SUMMARY.

The results of the above investigation upon resorufin, as a typical fluorescent substance, taken in connection with the work of B. Walter and Nichols and Merritt, seem to establish the truth of the following statements :

1. Fluorescent solutions are optically perfect substances, *i. e.*, they obey Lambert's Law.

2. Beer's law, *i. e.*, that increase of concentration is equivalent to increase in thickness, is true for dilute but not for concentrated solutions.

3. A change in the concentration of a fluorescent solution has no effect upon the typical fluorescence spectrum.

PHYSICAL LABORATORY
OF CORNELL UNIVERSITY.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE THIRTY-SIXTH MEETING.

A REGULAR meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York City, on Saturday, March 2, 1907.

In the absence of the president and vice-president Mr. W. C. Sabine was nominated as chairman.

Professor O. Lummer, of the University of Breslau addressed the Society on The Temperature of the Sun and recent Solar Theories.

The following papers were then presented:

1. On the Forces on the Interior of a Conductor Carrying Current. E. F. NORTHRUP.
2. The Radiation from, and the Melting Points of, Palladium and Platinum. C. W. WAIDNER and G. K. BURGESS.
3. The Viscosity of Certain Isomeric Ether Compounds. F. M. PEDERSON.
4. The Effect of Frequency upon the Capacity of Absolute Condensers. J. G. COFFIN.
5. The Physical Nature of Meteor Trains. C. C. TROWBRIDGE.
6. On Atmospheric Drifts above fifty Miles from the Surface of the Earth. C. C. TROWBRIDGE.
7. The Equations of the Fog Chamber. (By title.) CARL BARUS.
8. The Current carried by Canal Rays in a Discharge Tube. C. C. PERRY.
9. Note on the Hissing Metallic Arc. W. G. CADY.
10. The Relation between Luminosity and Electrical Conductivity. F. L. TUFTS. Adjourned.

ERNEST MERRITT,
Secretary.

NOTE ON THE FLUORESCENCE OF SODIUM VAPOR.[1]

BY ERNEST MERRITT.

THE recent experiments of R. W. Wood have shown that the fluorescence spectrum of sodium vapor, which exhibits great complexity when excited by white light, is relatively simple when suitable monochromatic exciting sources are used. The complex spectrum produced by white light appears to result from the superposition of a number of groups of lines, the lines of each group forming a spectral series. Excitation by light corresponding to one line of a group produces a fluorescence spectrum which contains only lines of the same group; but the distribution of intensities among the lines of a group appears to depend upon which line is directly stimulated by the exciting light.

---

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

Although none of the numerous attempts to devise a mechanical model of an atom have proven successful, it can scarcely be doubted that the lines of a given group or series correspond to the different natural periods of vibration of one connected system. But while it is natural to expect that the line corresponding to one mode of vibration of such a system will be stimulated by exciting light of the same frequency, it is at first difficult to see how such monochromatic excitation can also set up vibrations of different periods. In general stimulation of a mechanical system by resonance is effective only in the case of exact or nearly exact unison; even a harmonic overtone cannot be stimulated by its fundamental, or by another tone of the same series.

It has occurred to the writer that this difficulty may be met by taking into consideration the effect of collisions between atoms. Between collisions a considerable amount of energy will be absorbed in the form of vibrations of the particular type that is in resonance with the exciting waves. Whatever may be the nature of what we call a collision, it seems certain that forces then come into action which will tend to disturb the condition of vibration of the atoms. In general the energy stored between collisions will be partly changed into translational energy and partly transferred to other modes of vibration. The manner in which it is distributed among the various possible types of vibration may readily depend upon which type of vibration was originally exerted.

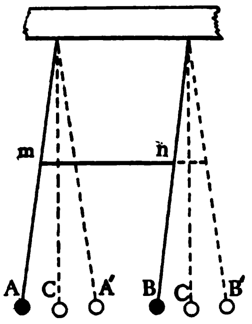


Fig. 1.

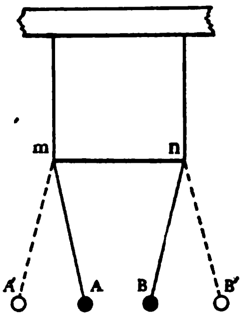


Fig. 2.

The following will serve as a simple mechanical illustration of the principle involved. Imagine two simple pendulums, of equal mass and length, connected by a rigid massless rod *mn*. The two principal modes of vibration of such a system are shown in Figs. 1 and 2. In one case *A* and *B* vibrate in the same phase and with the same period as though *mn* were absent. In the second case the rod *mn* remains at rest and *A* and *B* vibrate in opposite phase, the period being that of a pendulum of length *mA*. Let the system be set into vibration in the manner shown in Fig. 1, and as the mass *A* passes through the vertical let it collide with a heavy elastic body, so as to have its motion reversed. After such a collision the system will vibrate in the type of vibration of Fig. 2, with a period shorter than before. In such a case the energy of one type of vibration is completely transformed into that of another type. In general the collision would be less complete and the subsequent motion would be complex, involving vibrations in both of the principal modes.

It might be urged against this explanation of the phenomena that the effect of collision is merely to add certain vibrations, which in the subsequent motion are superposed upon the original vibration: but the effect of collisions alone, without excitation by light, is insignificant, as is indicated by the small temperature radiation of the gas; hence no effect as great as required to explain Wood's results can be expected. This line of reasoning leads to the suggestion that the effect of collision between *vibrating* atoms may be quite different from the effect of ordinary collisions. In a vibrating atom the electrons will at times be far removed from their equilibrium positions and may in consequence be greatly disturbed by the field of a neighboring atom.

The Hissing Point of the Metallic Arc.[1]

By W. G. Cady.

THIS paper is a preliminary report dealing chiefly with the arc between iron and copper terminals. Mr. H. D. Arnold, Assistant in Physics, is associated with the writer in this work, which is being carried on with aid from the Elizabeth Thompson Science Fund.

The fact should not be overlooked that the so-called iron arc is, in air, an arc between molten globules of magnetic oxid of iron. By letting the arc burn until it has become normal, observations can be made that compare in accuracy with those in the case of the carbon arc.

Observations were first made to verify the generally accepted linear equation $E = a + bl$ connecting volts across arc with length of arc for iron electrodes. When curves were drawn showing the potential drop as function of the length for different currents, it was found that the curves had approximately a parabolic form, suggesting that the equation should have a term involving $\sqrt{I}$.

---

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 29, 1906.

The difficulty encountered in obtaining reliable observations in the neighborhood of one ampere led Mr. Arnold to the detection of an abrupt change in the iron arc, analogous to the hissing point of the carbon arc. For currents below one ampere, the arc burns quietly and there is no well-defined spot of light on the globule at the anode. As the external resistance is decreased so that the current increases beyond a value depending largely on the curvature of the globule, the arc suddenly contracts, a bright spot appears on the anode, hissing commences, potential difference drops and current increases slightly. Experiment showed that the effect is confined mainly to the anode. When the current is decreased, the change back to the quiet stage does not take place until the current has become smaller than it was when hissing commenced, due doubtless to the high temperature at the anode. From the point of view of spectroscopy, it would be of considerable interest to find out whether the spectrum of the iron arc suffers a change at the hissing point. A brief examination showed nothing further than a general brightening of lines, but the question whether all lines are brightened to the same degree deserves attention.

When the current is increased to about two amperes, the iron arc emits a whistling sound, and the spot at the anode has a tendency to travel around rapidly, describing a circular ring. This recalls Trotter's observation of the rotation of the hissing carbon arc. It is planned to examine this phenomenon more closely, with the aid of an oscillograph.

With copper electrodes the hissing point can also be found. It occurs, with electrodes 6 mm. thick, when the current is about 0.5 amp., with about 110 volts across the arc. The arc between aluminium electrodes was too unsteady for the detection of the effect. Zinc has thus far shown no hissing point. Something resembling a hissing point was found with an arc in air between a mercury anode and carbon cathode, but this may have been due to disturbing causes.

Lecher's assertion that the iron arc is a discontinuous discharge, while the copper arc is perfectly continuous, was put to the test for both the quiet and the hissing stages. A bolometer was substituted for the hot wire used by Lecher, but it soon became evident that the capacity and self-induction of the bolometer circuit tended to set up oscillations, causing a "singing" iron arc. This fact tends to discount Lecher's conclusions and to make the method inapplicable. Good results were obtained by connecting a self-inductance of low resistance in series with the arc. The bolometer and condenser in series were then connected in parallel to the self-inductance. In this way the bolometer became heated by oscillatory currents set up in the arc, without danger that the bolometer circuit could disturb the conditions in the arc. The results are briefly these: the carbon and iron arcs, when quiet, are continuous; when hissing, they are oscillatory. The copper arc seems always to be oscillatory, and the smaller the current, the more is the bolometer heated. This point, as well as bolometric tests with other metals, is soon to be investigated.

THE EQUATIONS OF THE FOG CHAMBER.[1]

BY CARL BARUS.

IN my earlier papers, greater symmetry in the final results was obtained by the use of $p - \pi$ (where p is the total pressure and π the vapor pressure of the saturated gas) both in the isothermal and the adiabatic equations. But for the purposes of the fog chamber the correction thus made necessary is too large. Hence the following equations should be used.

Let p, π, ρ, τ, denote the total pressure, the vapor pressure, density and absolute temperature of the saturated gas, c and k the specific heats. When the variables are primed let them refer to the vacuum chamber, if unprimed to the fog chamber. Let subscripts distinguish the different thermodynamic states. Isothermally we have throughout $p - \pi = R\rho\tau$ in eight different cases; adiabatically $\tau/\tau_1 = (p/p_1)^{1-c/k}$ in two different cases.

Let the initial states of the fog chamber (before exhaustion, chambers separated) be given by p, ρ, τ, and of the vacuum chamber by p', ρ', τ; their final states when in communication after exhaustion by p_3, ρ_3, τ. Let their volume ratio be $[v/V]$.

If the two chambers had been separated immediately after exhaustion and before any condensation or heating ensued (subscript 1 referring to adiabatic conditions)

$$(p_1-\pi_1)/p_1^{1-c/k} = \frac{(p_3-\pi)(1+[v/V])}{p'^{(1-c/k)}(1+(\pi_1'-\pi_1)/(p-\pi_1)) + [v/V]p^{1-c/k}}$$

with a similar equation for the vacuum chamber. The volume expansion is given by

$$v_1/v = \frac{p}{p_3-\pi}\,\frac{(p'/p)^{1-c/k}(1-\pi_1'/p_1^{1-c/k}) + [v/V](1-\pi_1/p_1^{1-c/k})}{1+[v/V]}.$$

If each chamber is now allowed to reach atmospheric temperature (subscripts 2 denoting isothermal conditions)

$$p_2 - \pi = p^{1-c/k}(p_1-\pi_1)/p_1^{1-c/k}$$

with a similar equation for the vacuum chamber.

---

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

If the temperature of the fog chamber is heated by the condensation of water alone from the adiabatic temperature τ_1 before condensation, to $\bar{\tau}_1$ after condensation, if d_1 is the density of water vapor at τ_1 and $\bar{d}_1$ at $\bar{\tau}_1$ we may write for the determination of $\bar{\tau}_1$

$$\bar{d}_1 = \left(d/(p - \pi) - \frac{\rho c}{L} \frac{\bar{\tau}_1 - \tau_1}{p - \pi} \right) (p_2 - \pi)$$

where c is the specific heat of air at constant volume and L the latent heat of evaporation. Hence the water precipitated per cubic centimeter is

$$m = \frac{\rho c}{L} \frac{p_1 - \pi_1}{p - \pi} \left(\frac{p}{p_1} \right)^{1-c/k} (\bar{\tau}_1 - \tau_1)$$

where p_1 may usually be replaced by p_2.

If the chambers are separated immediately after condensation takes place and before other appreciable heating occurs the p_1, ρ_1, τ_1, of the fog chamber will change to $\bar{p}_1$, $\bar{\rho}_1$, $\bar{\tau}_1$, solely from condensation, and this in turn to $\bar{p}_2$, $\bar{\rho}_2$, $\bar{\tau}_2$, when isothermal conditions are regained by heating from without. We then obtain for the fog chamber

$$\bar{p}_2 - \pi = (\bar{\rho}_1/\rho_1)(p_1 - \pi_1)(p/p_1)^{1-c/k}$$

where

$$\bar{\rho}_1/\rho = \frac{\tau_1}{\bar{\tau}_1} \frac{1 + [v/V](\tau_1'/\tau_1) + (\bar{\pi}_1' - \bar{\pi}_1 - (\pi_1' - \pi_1))/(p_1 - \pi_1)}{1 + [v/V](\tau_1'/\bar{\tau}_1)}$$

with similar expressions for the vacuum chamber. Since $\bar{\tau}_1$ and $\bar{\rho}_1$ are given, $\bar{p}_1$ may also be computed.

Finally we have in general

$$\begin{aligned}(p_2' - \pi) + [v/V](p_2 - \pi) &= p' - \pi + [v/V](p - \pi) \\ &= (1 + [v/V])(p_3 - \pi)\end{aligned}$$

or

$$[v/V] = (p_3 - p')/(p - p_3).$$

CARL BARUS.

BROWN UNIVERSITY,
PROVIDENCE, R. I.

Volume XXIV. *May, 1907.* *Number 5.*

THE

PHYSICAL REVIEW.

THE TEMPERATURE COEFFICIENTS OF GAS VISCOSITY.

BY WILLARD J. FISHER.

I. SUTHERLAND'S EQUATION, PURE GASES.

THE researches of Graham [1] were among the first from which it could be with certainty deduced that gases do not flow as perfect fluids, but have a definite internal friction. This was explained on the basis of the Kinetic Theory by J. Clerk-Maxwell,[2] who showed that it was to be anticipated that the coefficient of viscosity for a given temperature, would be independent of the pressure, and for a given pressure, would vary as the square root of the absolute temperature. The first of these deductions was verified by Clerk-Maxwell himself,[3] O. E. Meyer,[4] and others,[5] using the vibration method, which is best adapted to this purpose; and the limits within which it is true were indicated by Crookes,[6] and Kundt and Warburg, who showed that the statement fails for very low pressures. Warburg and Babo [7] showed the same for CO_2 at high pressures.

The second deduction was tested by O. E. Meyer,[8] Stefan,[9] E. Wiedemann,[10] Puluj,[11] von Obermayer,[12] S. W. Holman,[13] and C. Barus,[14] and decided variations from it were found to be true. In fact, Holman showed that for air and CO_2, the variation of viscosity was not proportional to any numerical power of the absolute temperature.

At this point W. Sutherland [15] showed that " the whole discrepancy between theory and experiment disappears if account is taken of molecular force, which brings about collisions which would have been avoided in its absence," and justified his conclusions by examining the data of preceding experimental investigations. His theory led to a relation of the form.

$$\eta \propto \frac{\theta^{\frac{1}{2}}}{1+\frac{C}{\theta}}$$

in which η is the viscosity-coefficient, θ the absolute temperature, and C a constant, which is the potential energy of two molecules of the gas at their position of nearest approach, "in contact"; C has the dimensions of temperature; the formula is given for conditions such that the temperatures are above the critical and the pressures within the range of approximate exactness of Boyle's Law.

The latter reaches of Rayleigh[16], Breitenbach[17], Schultze[18], Markowski[19], Bestelmeyer[20], Kleint[21], and Tänzler[22], have further justified Sutherland's theory for the ranges of temperature and pressure for which it was proposed.

Taking η_0, the viscosity-coefficient at 0° C., as a second constant, all writers hitherto have thrown Sutherland's relation into the form

$$\eta = \eta_0 \left(\frac{\theta}{273}\right)^{\frac{1}{2}} \frac{1+\frac{C}{273}}{1+\frac{C}{\theta}}$$

and determined η_0 and C by the method of least squares. It can, however be equally well written with a proportionality-constant in the form

$$\eta = \frac{K\theta^{\frac{1}{2}}}{1+\frac{C}{\theta}}.$$

The constant K has not hitherto received any notice, no one having even determined its numerical value for any gas; but both it and C have interesting properties. Its dimensions are $[M]\,[L]\,[T]^{-1}$ $[\theta]^{-\frac{1}{2}}$; and if the dimensions of θ are the same as those of kinetic energy, then $[K] = [M]^{\frac{1}{2}}$.

Sutherland's equation may be thrown into the form

$$\theta = K\frac{\theta^{\frac{3}{2}}}{\eta} - C,$$

which is linear in terms of θ and $\theta^{\frac{3}{2}}/\eta$, with the slope equal to K, and the ordinate-intercept equal to $-C$. This suggests the plotting of the fundamental data, and the determination of the constants from the plots. For such of the investigations as have already been reduced by least squares the value of K is perhaps better deduced from the published values of η_0 and C; for some which have not, the writer has used the graphical method, finding by trial that it gives a very good agreement with the other. But the curves in themselves are interesting, and are shown, on small scale, for elements in Fig. 1, for compounds in Fig. 5. Table I. gives the fundamental data for the figures, and covers all the gases which have been studied through a sufficiently wide range above their critical temperatures. Table II. gives a summary of certain properties of these gases, arranged in the order of their molecular weights.

Table I.

Fundamental Data on Gas Viscosity.

von Obermayer,[11] *note 1.*

	Temp. C. (Means.)	$\eta \cdot 10^7$ (Means.)	θ	$\frac{\theta^{\frac{3}{2}}}{\eta} \cdot 10^{-7}$
Carbon Monoxide, CO.	17.7	1703	290.7	2.905
	53.45	1857	326.45	3.175
Nitrous Oxide, N_2O.	−21.5	1249	251.5	3.192
	14.05	1421	287.05	3.423
	53.55	1606	326.55	3.675
	100.3	1829	373.3	3.944
Ethylene, C_2H_4.	−21.5	851	251.5	4.686
	20.6	989	293.6	5.085
	53.5	1096	326.5	5.381
Carbon Dioxide, CO_2.	−21.5	1278	251.5	3.120
	14.25	1449	287.25	3.354
	53.5	1638	326.5	3.601
	102.3	1858	375.3	3.914
	162.4	2143	435.4	4.239
	222.0	2385	495.0	4.617
	240.3	2458	513.3	4.732

P. Breitenbach,[16] *note 2.*

Hydrogen.	−20.6	819	252.4	4.907
	15.0	889	288.0	5.498
	99.2	1059	372.2	6.781
	182.4	1215	455.4	8.000
	302.0	1392	575.0	9.907
Ethylene, C_2H_4.	−21.2	891	251.8	4.49
	15.0	1006	288.0	4.86
	99.3	1278	372.3	5.625
	182.4	1530	455.4	6.355
	302.0	1826	575.0	7.56
Carbon Dioxide, CO_2.	−20.7	1294	252.3	3.100
	15.0	1457	288.0	3.355
	99.1	1861	372.1	3.861
	182.4	2221	455.4	4.380
	302.0	2682	575.0	5.140
Methyl Chloride, CH_3Cl.	−15.3	936	257.7	4.415
	15.0	1052	288.0	4.64
	99.1	1384	372.1	5.185
	182.4	1706	455.4	5.70
	302.0	2139	575.0	6.455

H. Schultze,[17] *note 3.*

Argon. Series I.—2⅓ % Nitrogen.	17.9	2241	290.9	2.215
	99.8	2740	372.8	2.645
	183.7	3178	456.7	3.075
Series II.—½ % Nitrogen.	14.7	2208	287.7	2.208
	99.7	2733	372.7	2.638
	183.7	3224	456.7	3.025
Helium + 1⅔ % Ne (Vol.).	15.3	1969	288.3	2.487
	99.6	2348	372.6	3.062
	184.6	2699	457.6	3.627

H. Markowski,[18] *note 4.*

Oxygen.	16.75	2023	289.75	2.438
	99.74	2485	372.74	2.896
	185.8	2885	458.8	3.406
Hydrogen.	14.5	877	287.5	5.558
	100.5	1046	373.5	6.901
	184.2	1212	457.2	8.066
Nitrogen, chemically prepared.	15.4	1747	288.4	2.806
	100.08	2123	373.08	3.388
	182.7	2458	455.7	3.956

E. Wiedemann,[9] *note 5.*

	8°	100°	184.5°
Air	100	123.1	141.1
CO	96.87	—	136
CO_2	80.48	104.8	123.4
N_2O	80.41	105.6	124.1
C_2H_4	56.24	73.89	87.38

S. W. Holman,[12] *Note 6.*

Carbon Dioxide, Series II.

Temp. C.	18.10	40.98	59.14	79.65	100.17	119.30
η/η_0	1.0679	1.1458	1.2126	1.2848	1.3506	1.4148
Temp. C.	141.89	158.28	181.32	224.0	17.07	
η/η_0	1.4843	1.5371	1.6188	1.7474	1.0630	

Empirical Formula for the above:

$$\eta/\eta_0 = 1 + 0.003725t - t^2 \cdot 264 \cdot 10^{-8} + t^3 \cdot 417 \cdot 10^{-11}.$$

A. Bestelmeyer,[19] *Note 7.*

Nitrogen, Chemically Prepared.

Temp. C.	300.4	98.41	−78.66	−190.63
η/η_{17}	1.6279	1.2064	0.7207	0.3204

NOTES TO TABLE I.

The values of $\theta^{\frac{3}{2}}/\eta$ were computed with a very good ten-inch slide-rule, which gives sufficiently accurate results for Fig. 1. In case swhere least square values were not available, the slide-rule values were corrected with four-place logarithms as noted below, to insure the accuracy of the large-scale plots from which K was determined. The observations given have been selected from the large mass available (of which most is presented in Landolt and Börnstein, 1905) with reference mainly to the range of temperature; this necessarily excludes the results obtained by vibrations, and most of the data of Graham[1] and the other earlier observers who used the transpiration method. In a few cases where the temperature range was small but the results unique the value of C was computed to see whether it was possible, and if not, the observations were not included. Observations like those of Graham on chlorine and S. Koch[23] on mercury vapor, and others in which the temperature range was not sufficiently elevated above the critical temperature, have also been rejected for this purpose.

Note 1, von Obermayer's Data.—The transpiration apparatus used has been criticized on account of insufficient accuracy in the calibration of the capillaries, and lack of certainty that the gas had assumed the temperature of the capillaries before entering the same. The temperatures were obtained by liquid and vapor baths, and measured by means of mercury thermometers. Constancy of pressures was secured by a special form of apparatus. Many runs at different temperatures and with different capillaries were made; the means for the table were computed from these by averaging the (nearly equal) temperatures and viscosity values. These values are not considered as good as those of Graham, or some of the investigators of later years. The data given for CO are nearly unique, and are here quoted for that reason, in spite of the fact that the range of temperature is too small to allow final conclusions on the values of the constants. The values of $\theta^{\frac{3}{2}}/\eta$ given have been determined throughout from von Obermayer's results by logarithms, and plotted on cross-section paper to a scale of 1/20 inch to 1°C. for ordinates, 1/20 inch to 1/100 unit of $\theta^{\frac{3}{2}}/\eta \cdot 10^{-7}$ for abscissas. In drawing the curves temperatures below the critical were neglected. This leaves only two points for N_2O and C_2H_4, but gives a good series for CO_2.

The points for CO_2 do not lie very exactly on the straight line, but vary on the two sides somewhat. The two points for the temperatures —21°.5 and 14°.25 lie above the line, the lower more than the upper, indicating below the critical temperature a relative increase in the viscosity over that expressed by the formula. From the slope and intercept computed from two widely separated points on the line the constants are found to be $K = 165.8 \;\; 10^{-7}$, $C = 271.5$.

Breitenbach's values plotted on the same sheet give a very accurate straight line, which intersects the line for von Obermayer's data at 182°.4 C., and has a less slope, hence also a less value of C. The comparison of the various determinations of the constants of CO_2 is continued below, Note 6.

For N_2O the constants computed are $K = 173.8 \cdot 10^{-7}$, $C = 312$.

For C_2H_4, $K = 111.1 \cdot 10^{-7}$, $C = 271.6$.

For CO, $K = 135.1 \cdot 10^{-7}$, $C = 102$.

The temperature ranges for these three gases are hardly sufficient to settle any of the values.

Note 2, Breitenbach's Data.—These important researches include, beside those on pure gases given in the table, others on mixtures of H with CO_2 and CH_4 with air, also on the slipping of gases on the walls of the capillary. The apparatus was after the model used by E. Wiedemann, and, was evidently, with additions to increase symmetry and convenient use, the pattern followed by H. Schultze, mentioned in the next note. No full descriptions of the methods of preparation of the gases are given, but reference is made to the dissertation of which the Annalen article is an abstract. The results agree very well with the Sutherland formula, so far as it is applicable. Following are Breitenbach's Least Square values for the constants:

Hydrogen, $\eta_0 = 857 \cdot 10^{-7}$, $C = 71.7$; from which $K = 65.5 \cdot 10^{-7}$.
Ethylene, $\eta_0 = 961 \cdot 10^{-7}$, $C = 225.9$; from which $K = 106.2 \cdot 10^{-7}$.
CH_3Cl, $\eta_0 = 989 \cdot 10^{-7}$, $C = 454$; from which $K = 159.4 \cdot 10^{-7}$.
CO_2, $\eta_0 = 1388 \cdot 10^{-7}$, $C = 239.7$; from which $K = 157.8 \cdot 10^{-7}$.

The results on CH_3Cl are especially interesting, as they include a temperature within ten degrees of the boiling-point of the substance, and cover the large range of 317°.

Note 3, H. Schultze's Data.—The apparatus used was of the form probably best adapted to absolute determinations of viscosity, being symmetrical about a single capillary, mercury being used to force the gas through in either direction; the pressures were adjusted by screws raising or lowering the mercury reservoirs, and read by compound mercury-water manometers; temperatures were kept constant by vapor-baths of steam or aniline; volumes were determined by calibration of the two gas-pipettes, and the time of flow was determined by the rising mercury automatically ringing an electric bell. The capillary was calibrated by a method, apparently original, which takes account of irregularities in the tube, and allows of but few improvements. The ratio of length to diameter of tube was 3469 6, which is above the minimum, 3000, set by O. E. Meyer for such work. In computing, account was taken of the different temperatures of the air in the different parts of the apparatus, and also of the coefficient of slip.

The second series for argon is no doubt the better, on account of the greater purity; from it the constants deduced are $\eta_0 = 2104 \cdot 10^{-7}$, $C = 169.9$; from which $K = 206.5 \cdot 10^{-7}$; $C/K = 8.23 \cdot 10^{6}$.

The helium used was made from monazite sand, and carefully purified from CO_2, H, N and hydrocarbons; the inert Ne remaining could not be removed for lack of liquid H. The results are to be accepted with some reservations, which are pointed out by the investigator himself; chief among these are the mixture with Ne, which may have upon He the same effect that heavier gases have upon H, small per cents. increasing the viscosity abnormally. The Ne certainly produced some effect of importance, as the observations made after the apparatus had stood for some time gave higher results than those in which one run followed immediately upon another. Schultze has observed the same with mixtures of N and H.

His constants for this impure helium are, $\eta_0 = 1891.10^{-7}$, $C = 80.3$; from which $K = 148.2 \cdot 10^{-7}$.

Rayleigh's values of C, 150.2 for argon and 72.2 for helium, differ considerably from the above. He had the advantage of purer gases, but his apparatus may be criticized, as by Schultze in his article on helium.

It should be noted that the portion of the work of P. Tänzler[22] which deals with pure helium leads him to results in exact agreement with these of Schultze, namely, $C = 80.3$, $\eta_0 \cdot 10^7 = 1891$. On pp. 224–5 he says: "The density determined gave

for argon 19.946 referred to $O = 16$,
for helium 2.066 referred to $O = 16$.

As spectrometric investigations in our Institute have shown that helium usually contains argon as an impurity (even 0.573 per cent. by volume), I have consequently in the calculation of the percentage content of mixtures of helium and argon assumed the density of pure helium to be 1.98." Hence it may be doubted that the uncertainty regarding the values for helium is entirely removed, as the gas which he calls "He 100 %" in his table may have contained sufficient impurity to give the same results as the mixture used by Schultze.

Note 4, Markowski's Data. — The apparatus was that constructed by H. Schultze, and so the results are in absolute measure. The gases were prepared with great care, but it would seem with more successful results in the cases of oxygen and nitrogen than in that of hydrogen. On the agreement of the oxygen series with that of von Obermayer, the latter is not probably the better. Rayleigh's value of C for oxygen, 128.2, was found by experiments with apparatus which is not above criticism both on theoretical and on practical grounds (see H. Schultze,[17] second article).

From his experiments Markowski deduces for oxygen the values $\eta_0 = 1926.10^{-7}$, $C = 138$. From these the writer computes $K = 175.5 \cdot 10^{-7}$, $C/K = 7.865 \cdot 10^6$.

While the hydrogen used was electrolytically prepared, the results obtained do not agree with Sutherland's formula, as is shown graphically by the curve of Fig. 1, where the points are the vertices of a triangle and not in a straight line. The writer would conclude that the results of Breitenbach are to be preferred to these.

Markowski gives as the constants for hydrogen, $\eta_0 = 841 \cdot 10^{-7}$, $C = 83.0$, which yield $K = 66.4 \cdot 10^{-7}$.

On the remarkable agreement of the results for nitrogen with those of A. Bestelmeyer, see note 7. The least square constants are $\eta_0 = 1674 \cdot 10^{-7}$, $C = 113$; from which $K = 143.3 \cdot 10^{-7}$, $C/K = 7.89 \cdot 10^6$.

Note 5, E. Wiedemann's Data. — The transpiration apparatus used was apparently adapted to give relative results only; one of the two tables of data (both of which are derived from the same observations) is presented here. From it are computed the following values for C, due weight being allowed in the computation to the longer temperature intervals, and temperatures below the critical not being used :. CO, 87.0; C_2H_4, 242; N_2O, 219.5; CO_2, 177. These differ materially from the results obtainable from other investigations, as seen in Table II.

Note 6, S. W. Holman's Data. — This observer's results on air and CO_2 are derived from one of the longest and most careful investigations in the history of the subject, and may be considered standard. The transpiration apparatus with tandem capillaries was of the best form for the derivation of relative results, the pressure regulator of very ingenious design and great efficiency, the mercury thermometers carefully calibrated, and all the work of observation, computation and criticism carried through with a finish worthy of its author. He conclusively proved that constant powers of the absolute-temperature could not be used to express his results commensurately with their precision; and it was especially on the above quoted series for CO_2 that Sutherland based his first value of C. The method of computation then used does not appear to the present writer to be as good as the following.

Assuming Holman's empirical formula, and substituting in it 100° C., we obtain for the ratio η_{100}/η_0 the value 1.3503. Equating this to Sutherland's form for the same ratio, and solving for C, we have $C = 274.2$, as against Sutherland's value derived from the same data, 277. It is unfortunate that the series is insufficient for the determination of K.

Of the various determinations of the constants of CO_2, it is safe to discard at once that of E. Wiedemann, unless the original observations are to be recomputed. C from Holman's data, 274, agrees very well with 271.5 derived from von Obermayer's; but the plot of the latter shows a greater deviation series than that of Breitenbach, so that on the whole the Sutherland formula represents the latter better. Quite possibly the real values for CO_2, both of K and C, are to be regarded as still somewhat in doubt, and subject to further experimental investigation.

Note 7, Bestelmeyer's Data. — The transpiration apparatus used was designed after that of Holman, with tandem capillaries, thus rendering unnecessary the calibration of the tubes. Holman determined experimentally the ratio of the tube-constants; Bestelmeyer also calibrated his tubes by the method of H. Schultze, both before and after the experiments on viscosity. The pressure-regulating apparatus seems to have worked very well, and the temperatures were very accurately determined over the great range of 491° from that of boiling diphenylamine to that of liquid air,— below the critical temperature of nitrogen. This is the largest range of temperature ever used in viscosity work, and by its means a relative increase of viscosity below the critical temperature is discovered in the case of nitrogen as in other gases and vapors below their critical temperatures.

The table quoted is one of mean values, giving averages of the experimental data and comparing with them the values computed by least squares. From four sets of runs Bestelmeyer derives for C the values 110.6, 110.3, 114.4 and 103.6, the first three of which agree well among themselves and with Markowski's value, 113, obtained from absolute measurements with a very different apparatus. If we compute from Markowski's least square data the value of η at 15° C., and with this and Bestelmeyer's experimental values of η/η_{15} compute absolute values of η, the plotting to very large scale of these and Markowski's absolute values gives a remarkable coincidence, the two series forming one curve. Hence it may be concluded that the viscosity coefficient of nitrogen is now known more accurately and over a larger range of temperature than that of any other gas, and that further investigation of it will be unnecessary for a long time to come.

TABLE II.

Constants of Gases whose Viscosity has been Studied through a Wide Range of Temperature.

Formula.	H_2	A	He	A	N_2	A	O_2	A	Ar	A
Density, Air = 1, 760 mm. (L)	0.0693	Rt	0.1368	R&T	0.967	Ldc	1.105	Ldc	1.379	R&T
Molecular weight, $O = 16$. (L)	2.016		4		28.08		32		39.9	
Boiling point, 760 mm. (L)	−253	T	−267	T	− 195.7	F&A	− 182.8	T	−186.1	R&T
Critical temperature. (L)	−234.5	O	<−264	D	− 146	W	− 118.0	W	−117.4	R&T
	−243 to				− 146.5	W	− 118.8	O		
	−241	D			− 146	O				
$\eta \cdot 10^7$, 0°C.	861	v.O	1891	Sch	1659	v.O	1873	v.O	2104	Sch
(Quoted from Authors.)	857	B			1674	M	1926	M		
	841	M								
Viscosity constants. K · 10^7.	65.5	B	148.2	Sch	143.3	M	175.5	M	206.5	Sch
	66.4	M				.				
C (Quoted from Authors).	79	S	72.2	R	113	M	127	S	169.9	Sch
	71.7 •	B	80.3	Sch	110.6	Br	138	M	150.2	R
	72.2	R			110.3	"	128.2	R		
	83.0	M			114.4	"				
					103.6	"				
Mean C.	76.5				110.4		131.1			
Extreme temperature range	−20.6 to		15.3 to		−190.6 to		15.4 to		14.7 to	
for η.	302		184.6		300.4		185.8		183.7	Sch

Formula.	CO	A	C_2H_4	A	CO_2	A	N_2O	A	CH_3Cl	A
Density, Air = 1,760 mm. (L)	0.967	Ldc	0.985	Ss	1.5291	R	1.527	Thd	1.731	D
Molecular weight, $O = 16$. (L)	28		28.03		44.0		44.08		50.47	
Boiling point, 760 mm. (L)	−190	O	−153	(B)	−78.2	Rt	−87.9	Rt	−23.7	(B)
Critical temperature. (L)	−141.1	W	9.2		30.92		35.4	D	141.5	V&C
	−139.5	O	to	*	to	*	36.4	Jn		
			13.0		31.9		38.8	Vd		
$\eta \cdot 10^7$, 0° C.	1625	v.O	922	v.O	1387	v.O	1353	v.O	989	B
(Quoted from Authors.)			961	B	1388	B				
			966	G						
Viscosity constants. K · 10^7.	135.1	v.O	106.2	B	157.8	B	173.8	v.O	159.4	B
			111.15	v.O	165.8	v.O				
C (Quoted from Authors).	102	v.O	272	S	274	H	219.5	Wn	454	B
	87	Wn	225.9	B	239.7	B	312	v.O		
			242	Wn	177	Wn				
					271.5	v.O				
Extreme temperature range	17.5 to	v.O	21.2 to		20.7 to		−21.5		−15.3 to	
for η.	53.5		302	B	302	B	to 100.3	v.O	302	B

* Selected from many, too numerous to quote.

TABLE II. — AUTHORITIES.

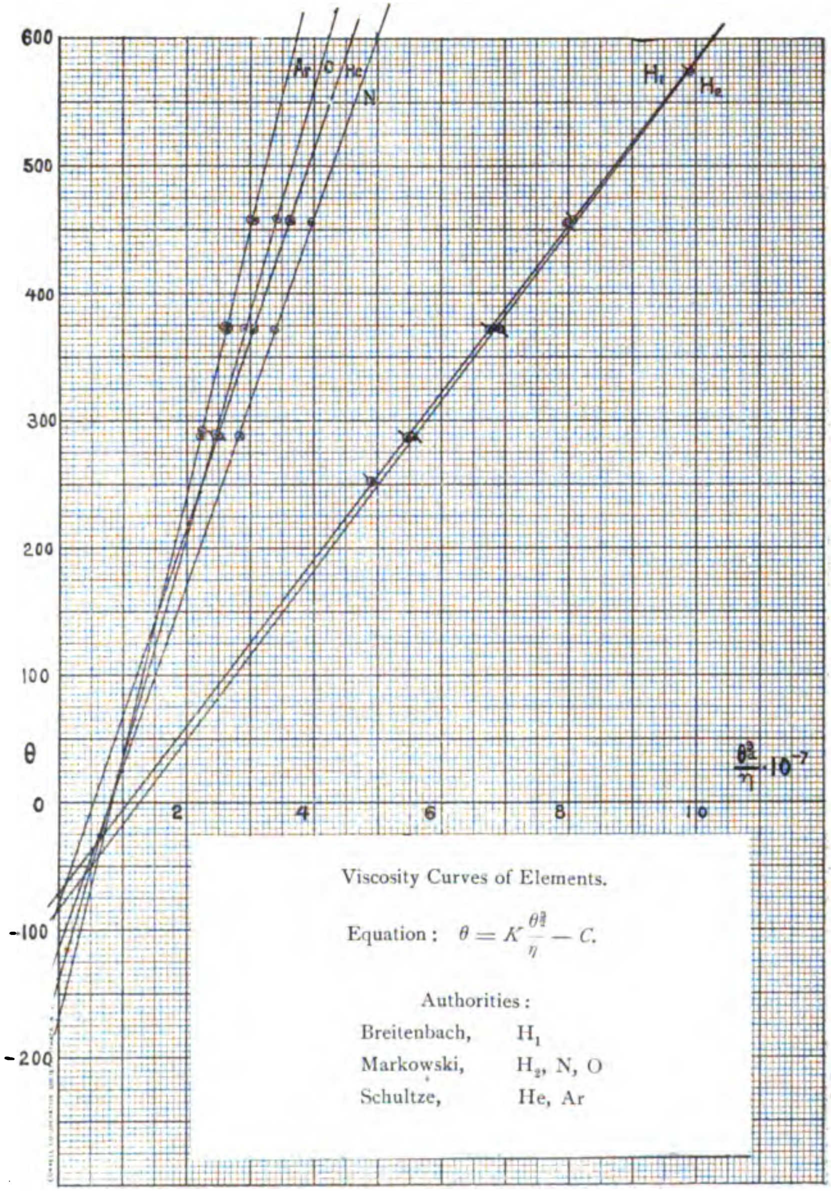
B, Breitenbach.
(B), Biedermann's Chemiker Kalendar, 1906.
Br, Bestelmeyer.
D, Dewar.
F & A, Fischer and Alt.
G, Graham.
H, Holman.
Jn, Janssen.
Kn, Knietsch.
L, Lussana.
(L), Landolt und Börnstein, 1905.
Ldc, Leduc.
M, Markowski.
O, Olszewski.
R & T, Ramsay and Travers.
Rt, Regnault.
S, Sutherland.
Sch, Schultze.
Ss, Saussure.
T, Travers.
Thd, Thenard.
V & C, Vincent and Chappuis.
Vd, Villard.
v.O, von Obermayer.
W, Wroblewski.
Wn, E. Wiedemann.

Values of K computed by present writer, from data of authors indicated.

In plotting Figs. 1 and 5 the lines were first drawn from the values of *K* and *C* given in Table II., then the points representing observations were marked; so that the lines represent least square computations in almost all cases, von Obermayer's results alone having been reduced graphically. In Figs. 2, 3 and 4, the points were marked first from the same data and standard values of molecular weights, and the lines then drawn among the points in the positions deemed to be most suitable.

With regard to the relative value of least square as compared with large scale graphical reductions, it is no doubt true that the ordinary methods of least square work assume that one of the coördinates is measured without error, or else that the measurements of both coördinates are of equal weights. It is hardly to be supposed that either of these conditions is common; in the study of gas viscosity, for example, the number of sources of error affecting the temperature values is not nearly as large as that affecting the viscosity coefficient, and it is not to be supposed that values of θ and η are by any means of equal weight; while it is certain that θ can not be properly assumed without error. Unless some account is taken of these principles, the extra labor involved in a least square reduction is mostly thrown away; if authors use least squares, they should at least state what relative weights they assign their coördinates.

From the tables and these curves certain deductions may be drawn, though no final conclusions; and indications for profitable research work.



Viscosity Curves of Elements.

Equation: $\theta = K\frac{\theta^{\frac{3}{2}}}{\eta} - C.$

Authorities:

Breitenbach,	H_1
Markowski,	H_2, N, O
Schultze,	He, Ar

Fig. 1.

Fig. 1 shows that for the elements the curves have a general converging tendency, not toward the origin but a point in its neighborhood, and that both the slope and the *y*-intercept increase with increasing molecular weight. The presence of a positive, not a zero, *x*-intercept, shows that the ratio $\theta^{\frac{3}{2}}/\eta$ approaches a definite limit as θ approaches zero, or would do so if the law did not change at or about the critical temperature. The convergency of the lines is best shown by N, O and Ar, which come together almost on the *x*-axis, with a common abscissa nearly equal to $8 \cdot 10^6$. While H and He show the same convergent tendency, the discrepancy of the lines representing different investigations on H, and the differences of the numerical values in the tables, tend to show that its constants are not as yet well known; and further investigation might possibly show that the absolutely pure gas would give a plot crossing the axis nearer the point for N, O and Ar. No certain conclusion

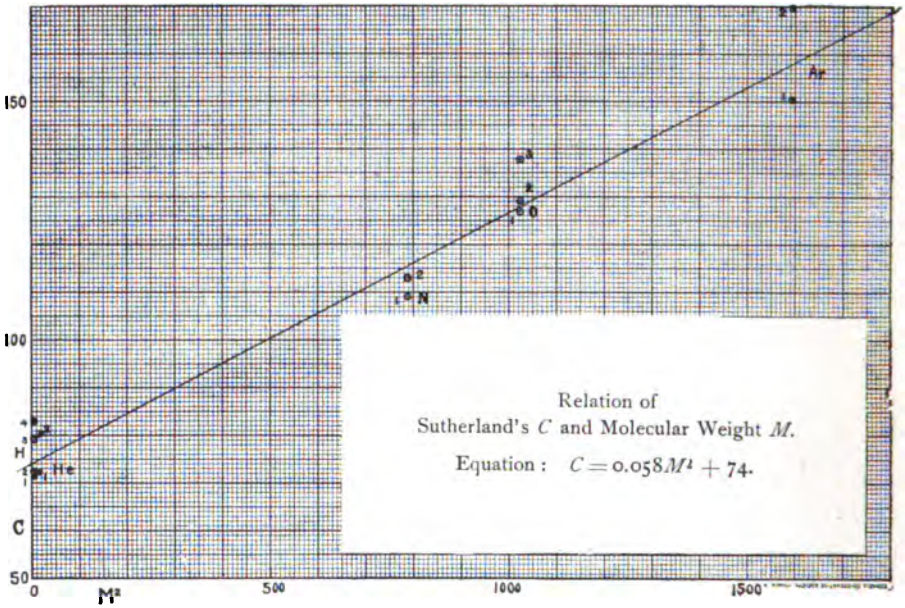


Fig. 2.

Authorities:

H. (1) Breitenbach, (2) Rayleigh, (3) Sutherland, (4) Markowski.
He. (1) Rayleigh, (2) Schultze.
N. (1) Bestelmeyer, (2) Markowski.
O. (1) Sutherland, (2) Rayleigh, (3) Markowski.
Ar. (1) Rayleigh, (2) Schultze.

can be drawn from the data or curve for He, as it was the result of work with impure gas; though it would not be surprising if He, in this respect as in others, should prove to be different from other elements. Judging from its molecular weight alone it would be expected to give a plot much nearer to the H-line than to the N-line, and lying between them.

Fig. 2 shows the relation between C and M^2, in which M is the molecular weight of chemistry. It is seen to be roughly a straight line, with equation $C = 0.058\ M^2 + 74$. The line has been drawn low among the points in order to give weight to the smaller values, which are perhaps the better. The coördinate M^2 was suggested by Sutherland's expression for C;

$$C = \frac{1}{c} m^2 f\left(\frac{1}{2a}\right)$$

This formula would not lead one to expect an intercept. If, however, Sutherland's value be equated to the empirical relation just given, we have

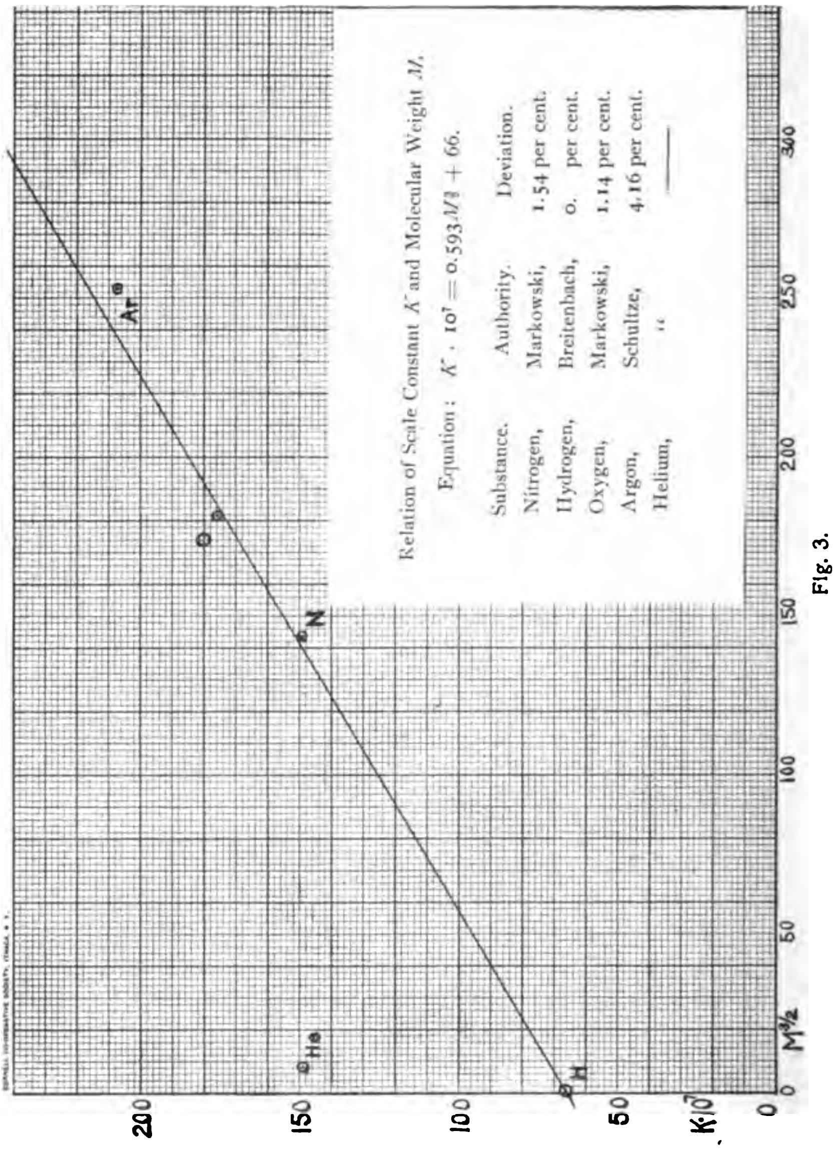
$$f\left(\frac{1}{2a}\right) = \frac{c}{h^2}\left(0.058 + \frac{74}{M^2}\right)$$

in which *h* is the mass in grams of a hydrogen atom, $c/2$ the conversion factor for kinetic energy and temperature, *a* the radius of a molecule and *f* the symbol for the law of potential energy due to the attracting forces.

Fig. 3, the relation between the coefficient K and the molecular weight, also shows a deviation from Sutherland's theory. According to that K should have the value

$$\frac{.064 c^{\frac{1}{2}} m^{\frac{1}{2}}}{(2a)^2}$$

while from the curve we obtain $K . 10^7 = 0.593 M^{\frac{1}{2}} + 66$. Argon deviates about 4 per cent. from the line, and helium evidently does not form part of the series.



Relation of Scale Constant K and Molecular Weight M.

Equation: $K \cdot 10^7 = 0.593 M^{3/2} + 66$.

Substance.	Authority.	Deviation.
Nitrogen,	Markowski,	1.54 per cent.
Hydrogen,	Breitenbach,	0. per cent.
Oxygen,	Markowski,	1.14 per cent.
Argon,	Schultze,	4.16 per cent.
Helium,	"	——

Fig. 3.

Fig. 4 is presented to show graphically the ratio between K and C, which is, as already shown, nearly constant for N, O and Ar.

The researches which may be supposed to be of promise as to the laws of viscosity of the elements would all be difficult; but if the uncertainty relating to hydrogen and helium were cleared up, and information were secured as to the constants of neon, M = 20, chlorine, M = 71, and mercury vapor, M = 200, over large ranges of temperature, advances might result in our knowledge of the internal structure of elementary molecules.

Fig. 5, showing the curves for compounds, displays no such regularity as Fig. 1. The marked tendency for convergence shown by the curves for the elements is lacking, though there is a roughly outlined tendency for both K and C to increase (*a*) with increasing molecular weight, (*b*) with increasing molecular complexity and asymmetry. The comparative system of the curves for the elements may be due to the fact that their molecules are both more simple and more symmetrical than those of compounds. Since in the list of compounds presented no two can be said to be of either equal complexity or like symmetry, it is clear that among them we can expect to find no law indicated.

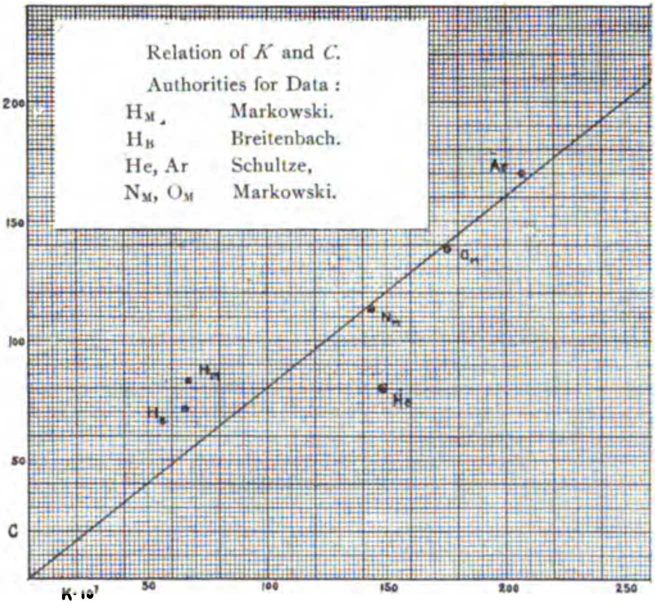


Fig. 4.

It is along these lines, then, that further research on the viscosity of gases should direct itself. Apparatus should be used which is

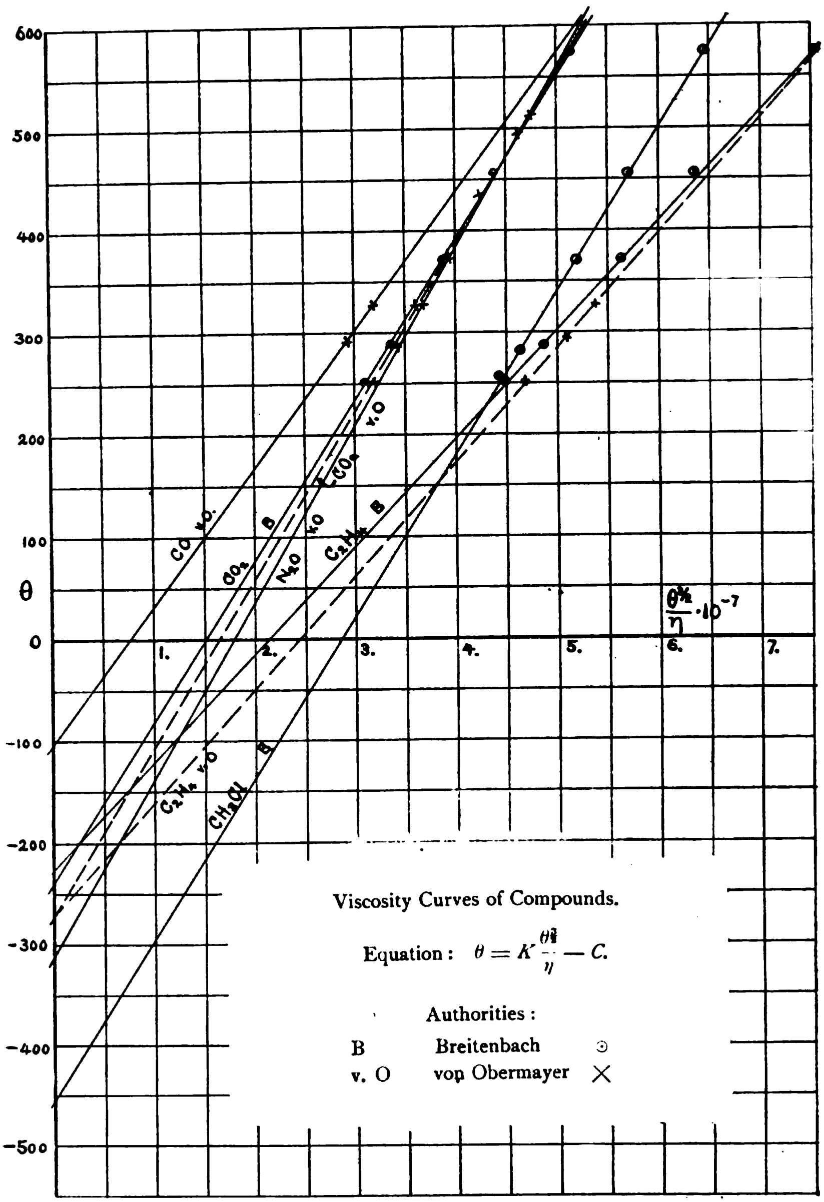


Fig. 5.

capable of giving results in absolute measure, so that both of the constants which determine the temperature change may be found for the gases studied; and the gases selected should be (1) series of equal molecular weights but varying complexity of molecular structure; (2) series of equal molecular weights but varying symmetry of molecular structure; (3) series of different molecular weights but like structure.

In later papers the writer hopes to present discussions of Reinganum's equation, as applied to published data; on capillary calibration and certain other points in the construction of viscosity apparatus; and on the viscosity of certain gases which have theoretical interest but to which as yet little attention has been paid.

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An excellent and complete résumé of the development and literature of the subject of viscosity is to be found in a thesis by F. M. Pedersen, submitted to the Faculty of Science, New York University, 1905.

A METHOD FOR THE DETERMINATION OF ELECTROLYTIC RESISTANCE AND CAPACITY, USING ALTERNATING CURRENTS.

BY A. H. TAYLOR.

IF, according to Varley,[1] the polarization capacity is constant for small E.M.F., it may be offset by an inductance in series with the electrolyte, as has been done by Wien.[2] Or the resistance and capacity of the electrolyte may be balanced by a similar combination of resistance and capacity in the symmetrical arm of a bridge, as accomplished by Gordon,[3] or perhaps in a better way, by Scott.[4]

For great accuracy the methods of Gordon and of Scott require a wide range in the compensating capacity, and in the method of Scott it is advisable to have a smoothly variable capacity. Hence it would seem that Wien's method, where the capacity of the electrolyte is determined in terms of a smoothly variable inductance, is preferable.

The objection to Wien's method is that it is not independent of frequency, even if the electrolytic capacity be independent of frequency, and the object of the present paper is to propose a method of attacking this problem which will allow the use of a commercial alternating current, *i. e.*, a current of many overtones, hence of several frequencies.

In regard to its behavior toward alternating currents, an electrolyte which has polarizing electrodes may be regarded as a resistance in series with a capacity, a second resistance shunting both the resistance and the capacity in some cases.

It is possible to balance this combination in a bridge by inserting a shunted inductance in the same arm with the shunted electrolyte.

[1] Varley, Proc. Roy. Soc., 1871.
[2] Wien, Annalen der Phys., 1896.
[3] Gordon, Annalen der Phys., 1897.
[4] Scott, Annalen der Phys., 1899.

The figure gives the arrangement of the circuit, which consists of a bridge whose two proportional arms *a* and *b* are identical, and whose fourth arm consists of a variable non-inductive resistance *R*.

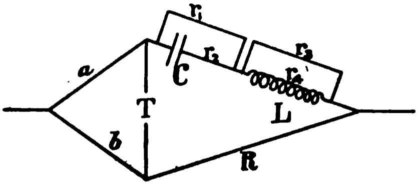

Fig. 1.

The third arm consists of a capacity *C* in series with a resistance r_2 and an inductance *L* of resistance r_4. As in the similar circuit devised by Trowbridge[1] for the differential transformer, the capacity and inductance must each be shunted in order to obtain a perfect telephonic balance, independent of the frequency.

The resistance r_3 shunts the inductance, while the resistance r_1 shunts the capacity $+ r_2$.

Let $c =$ the capacitance, $1/\omega C$, and $x =$ the inductive reactance, ωL. The impedance of the third arm may be divided into two parts, Z_1 and Z_2, where

$$\frac{1}{Z_1} = \frac{1}{r_2 + jc} + \frac{1}{r_1}; \quad \frac{1}{Z_2} = \frac{1}{r_3} + \frac{1}{r_4 - jx}.$$

$$Z_2 = \frac{r_1(r_2 + jc)}{r_1 + r_2 + jc}; \quad Z_2 = \frac{r_3(r_4 - jx}{r_3 + r_4 - jx}.$$

The bridge will be balanced when

$$Z_1 + Z_2 = R.$$

Substituting for Z_1 and Z_2, and multiplying by the conjugate complexes of the denominators,

$$R = \frac{r_1(r_2 + jc)(r_1 + r_2 - jc)}{(r_1 + r_2)^2 + c^2} + \frac{r_3(r_4 - jx)(r_3 + r_4 + jx)}{(r_3 + r_4)^2 + x^2}.$$

Separating into reals and imaginaries, and putting the imaginary parts equal to zero, since *R* has no imaginary component, we have

$$(1) \qquad R = \frac{r_1^2 r_2 + r_1 r_2^2 + r_1 c^2}{(r_1 + r_2)^2 + c^2} + \frac{r_3^2 r_4 + r_3 r_4^2 + r_3 x^2}{(r_3 + r_4)^2 + x^2}$$

and

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[1] Trowbridge, PHYS. REV., 1905.

$$(2) \qquad \frac{r_1^2 c}{(r_1+r_2)^2+c^2} = \frac{r_3^2 x}{(r_3+r_4)^2+x^2}.$$

Let $r_1 + r_2 = a$, and $r_3 + r_4 = b$. Then eq. (2) becomes

$$(b^2 + x^2) r_1^2 c = a^2 r_3^2 x + c^2 r_3^2 x,$$

or

$$(b^2 + x^2) r_1^2 = \frac{a^2 r_3^2 x}{c} + c x r_3^2.$$

Since $x = \omega L$, and $c = 1/\omega C$, we have

$$b^2 r_1^2 + \omega^2 L^2 r_1^2 = a^2 r_3^2 \omega^2 LC + \frac{L r_3^2}{C}.$$

This can be put in the form

$$(3) \qquad C = \frac{L r_3^2 + \omega^2 (a^2 r_3^2 LC - L^2 r_1^2)}{b^2 r_1^2}.$$

If the balance is to be independent of the frequency, the term in ω^2 must disappear; that is, $a^2 r_3^2 LC = L^2 r_1^2$, or

$$(4) \qquad C = \frac{L r_1^2}{a^2 r_3^2}.$$

If the term in ω^2 does disappear in (3), it becomes

$$(3)' \qquad C = \frac{L r_3^2}{b^2 r_1^2}.$$

Combining this result with (4) we have,

$$\frac{L r_3^2}{b^2 r_1^2} = \frac{L r_1^2}{a^2 r_3^2}; \quad \text{or} \quad \frac{r_1^2}{r_3^2} = \frac{a}{b} = \frac{r_1 + r_2}{r_3 + r_4} \qquad \text{Eq. (B).}$$

Equation (3)′ thus may be thrown into the form

$$C = \frac{L}{ab} \qquad \text{Eq. (A),}$$

a form which is very easy to use in calculation.

Going back to equation (1) we may put it in the form

$$R(a^2 + c^2)(b^2 + x^2) = (b^2 + x^2)(r_1 r_2 a + r_1 c^2) + (r_3 r_4 b + r_3 x^2)\, a^2 + c^2)$$

Solving for c_1^2.

$$(5) \qquad c^2 = \frac{(b^2 + x^2)(r_1 r_3 a - Ra^2) + a^2 (r_3 r_4 b + r_3 x^2)}{R(b^2 + x^2) - r_1 (b^2 + x^2) - r_3 r_4 b + r_3 x^2)}.$$

Solving equation (2) for c^2 we have,

$$(6) \qquad c^2 = \frac{(b^2 + x^2)r_1{}^2c - a^2r_3{}^2x}{r_3{}^2x}.$$

Eliminating c^2 from (5) and (6) we have,

$$\frac{(b^2 + x^2)r_1{}^2c - a^2r_3{}^2x}{r_3{}^2x} = \frac{(b^2 + x^2)(r_1r_2a - Ra^2) + a^2(r_3r_4b + r_3x^2)}{(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)}.$$

Solving the equation for c,

$$c = \frac{r_3{}^2x}{(b^2 + x^2)r_1{}^2}\left[\frac{(b^2+x^2)(r_1r_2a-Ra^2)+a^2(r_3r_4b+r_3x^2)}{(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)}\right] + \frac{a^2r_3{}^2x}{(b^2 + x^2)r_1{}^2}$$

$$= \frac{r_3{}^2x}{(b^2+ x^2)r_1{}^2}\left[\frac{b^2 + x^2)(r_1r_2a - Ra^2) + a^2(b^2 + x^2)(R - r_1)}{(R - r_1)(b^2 + x^2) - (r_3r_4b + r_2x^2)}\right]$$

$$= \frac{r_3{}^2x}{r_1{}^2}\,\frac{[r_1r_2a - Ra^2) = a^2(R - r_1)]}{[(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)]}$$

$$= \frac{r_3{}^2x}{r_1}\,\frac{(r_2a - a^2)}{[(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)]}.$$

Since $c = 1/\omega C$, and $a = r_1 + r_2$,

$$\omega C = \frac{x^2(R - r_1 - r_3) + b^2(R - r_1) - r_3r_4b}{-xr_3{}^2(r_1 + r_2)}$$

$$= \frac{x(R - r_1 - r_3) + \dfrac{b^2(R - r_1) - r_3r_4b}{x}}{-r_3{}^2(r_1 + r_2)}.$$

Finally,

$$C = \frac{L(R - r_1 - r_3) + \dfrac{b^2(R - r_1) - r_3r_4b}{\omega^2L}}{-r_3{}^2(r_1 + r_2)}.$$

If the balance is to be independent of the frequency, $b^2(R - r_1) = r_3r_4b$. Since b cannot equal zero, the only other relationship that will satisfy this condition is

$$b(R - r_1) = r_3r_4 \quad \text{or} \quad R = r_1 + \frac{r_3r_4}{r_3 + r_4} \quad \text{Eq. } (C).$$

Now equation (c) is simply the condition for steady current

$$(2)\qquad \frac{r_1^2c}{(r_1+r_2)^2+c^2}=\frac{r_3^2x}{(r_3+r_4)^2+x^2}.$$

Let $r_1+r_2=a$, and $r_3+r_4=b$. Then eq. (2) becomes

$$(b^2+x^2)r_1^2c=a^2r_3^2x+c^2r_3^2x,$$

or

$$(b^2+x^2)r_1^2=\frac{a^2r_3^2x}{c}+cxr_3^2.$$

Since $x=\omega L$, and $c=1/\omega C$, we have

$$b^2r_1^2+\omega^2L^2r_1^2=a^2r_3^2\omega^2LC+\frac{Lr_3^2}{C}.$$

This can be put in the form

$$(3)\qquad C=\frac{Lr_3^2+\omega^2(a^2r_3^2LC-L^2r_1^2)}{b^2r_1^2}.$$

If the balance is to be independent of the frequency, the term in ω^2 must disappear; that is, $a^2r_3^2LC=L^2r_1^2$, or

$$(4)\qquad C=\frac{Lr_1^2}{a^2r_3^2}.$$

If the term in ω^2 does disappear in (3), it becomes

$$(3)'\qquad C=\frac{Lr_3^2}{b^2r_1^2}.$$

Combining this result with (4) we have,

$$\frac{Lr_3^2}{b^2r_1^2}=\frac{Lr_1^2}{a^2r_3^2};\quad\text{or}\quad\frac{r_1^2}{r_3^2}=\frac{a}{b}=\frac{r_1+r_2}{r_3+r_4}\qquad\text{Eq. (B).}$$

Equation (3)′ thus may be thrown into the form

$$C=\frac{L}{ab}\qquad\text{Eq. (A),}$$

a form which is very easy to use in calculation.

Going back to equation (1) we may put it in the form

$$R(a^2+c^2)(b^2+x^2)=(b^2+x^2)(r_1r_2a+r_1c^2)+(r_3r_4b+r_3x^2)a^2+c^2)$$

Solving for c_1^2.

$$(5)\qquad c^2=\frac{(b^2+x^2)(r_1r_3a-Ra^2)+a^2(r_3r_4b+r_3x^2)}{R(b^2+x^2)-r_1(b^2+x^2)-r_3r_4b+r_3x^2)}.$$

Solving equation (2) for c^2 we have,

$$c^2 = \frac{(b^2 + x^2)r_1^2c - a^2r_3^2x}{r_3^2x}. \tag{6}$$

Eliminating c^2 from (5) and (6) we have,

$$\frac{(b^2 + x^2)r_1^2c - a^2r_3^2x}{r_3^2x} = \frac{(b^2 + x^2)(r_1r_2a - Ra^2) + a^2(r_3r_4b + r_3x^2)}{(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)}.$$

Solving the equation for c,

$$c = \frac{r_3^2x}{(b^2 + x^2)r_1^2}\left[\frac{(b^2+x^2)(r_1r_2a - Ra^2) + a^2(r_3r_4b + r_3x^2)}{(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)}\right] + \frac{a^2r_3^2x}{(b^2 + x^2)r_1^2}$$

$$= \frac{r_3^2x}{(b^2 + x^2)r_1^2}\left[\frac{b^2 + x^2)(r_1r_2a - Ra^2) + a^2(b^2 + x^2)(R - r_1)}{(R - r_1)(b^2 + x^2) - (r_3r_4b + r_2x^2)}\right]$$

$$= \frac{r_3^2x}{r_1^2}\frac{[r_1r_2a - Ra^2) = a^2(R - r_1)]}{[(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)]}$$

$$= \frac{r_3^2x}{r_1}\frac{(r_2a - a^2)}{[(R - r_1)(b^2 + x^2) - (r_3r_4b + r_3x^2)]}.$$

Since $c = 1/\omega C$, and $a = r_1 + r_2$,

$$\omega C = \frac{x^2(R - r_1 - r_3) + b^2(R - r_1) - r_3r_4b}{-xr_3^2(r_1 + r_2)}$$

$$= \frac{x(R - r_1 - r_3) + \dfrac{b^2(R - r_1) - r_3r_4b}{x}}{-r_3^2(r_1 + r_2)}.$$

Finally,

$$C = \frac{L(R - r_1 - r_3) + \dfrac{b^2(R - r_1) - r_3r_4b}{\omega^2L}}{-r_3^2(r_1 + r_2)}.$$

If the balance is to be independent of the frequency, $b^2(R - r_1) = r_3r_4b$. Since b cannot equal zero, the only other relationship that will satisfy this condition is

$$b(R - r_1) = r_3r_4 \quad \text{or} \quad R = r_1 + \frac{r_3r_4}{r_3 + r_4} \quad \text{Eq. } (C).$$

Now equation (c) is simply the condition for steady current balances, and might have been anticipated, since a balance independent of ω must hold for $\omega = 0$.

To summarize, the perfect balance condition means that

$$R = r_1 + \frac{r_3 r_4}{r_3 + r_4} \tag{C}$$

$$\frac{r_1^2}{r_3^2} = \frac{r_1 + r_2}{r_3 + r_4} \tag{B}$$

$$C = \frac{L}{ab} = \frac{L}{(r_1 + r_2)(r_3 + r_4)} \tag{A}$$

If C is an ordinary condenser, and L a variable inductance, it is easy to obtain a perfect telephonic balance, such that these three equations are satisfied. In the case of the electrolyte, substituted for $C + r_2$, perfect balance is attained only for low current densities.

Equation (C) allows us to calculate r_1. If the calculated value is smaller than r_1, as observed, it will indicate a leaky condenser action.

Equation (B) allows us to calculate r_2, the *effective* electrolytic resistance.

Equation (A) is used for computing the capacity. The equation (3)[1], for capacity, is identical with Trowbridge's[1] equation (12) although this circuit contains the resistance r_2 which is not present in his circuit. The explanation is that his equation (12) is an approximation which is exceedingly close if R — r_3 is small compared to r_1. It is, however, impossible to realize this experimentally if very large capacities are to be compared with inductances which are under say, 40 millihenrys.

The order of precision is necessarily practically the same as for his method, when used for similar purposes.

The advantage is that capacities as large as 100 microfarads may be compared with relatively small inductances. It is applicable to the case of electrolytic capacity and resistance, and might possibly be used to advantage for the determination of condensers which have large absorption, since this is equivalent to r_2, the resistance in series with the condenser.

UNIVERSITY OF WISCONSIN,
November 29, 1906.

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[1] Trowbridge, loc. cit.

FLUORESCENCE ABSORPTION IN RESORUFIN.[1]

BY FRANCES G. WICK.

THE relation between absorption of light by a fluorescent substance and emission during fluorescence is difficult to understand. Waves of one length are absorbed while those of another length are emitted, in apparent contradiction to Kirchhoff's law. At least a step toward an explanation of this difficulty has been made in the experiments of Burke[2] and in the more recent work of Nichols and Merritt,[3] who found that a change takes place in the absorbing power during fluorescence. Previous observations of absorption had been made when the body was not under excitation. The work of Burke upon uranium glass, and that of Nichols and Merritt upon fluorescein in water, eosin in alcohol, and resazurin in alcohol, showed that during fluorescence a substance acquires the temporary power of absorbing the same wave-lengths which it emits. M. Camichel,[4] who has since made measurements of a similar kind, failed to find any change in absorbing power during fluorescence, throwing doubt upon the existence of the phenomenon.

The subject thus brought into question seemed of sufficient interest to justify further study. At the suggestion of Professors Nichols and Merritt the following series of tests for fluorescence absorption was made upon Weselski's diazo-rezorufin. Some experimental work[5] just completed, in which a study had been made of the fluorescence and ordinary absorbing power of resorufin, suggested this as a desirable substance for further investigation.

A preliminary set of experiments showed that resorufin acquires a measurable increase in absorbing power during fluorescence. This phenomenon was then studied in its relation to:

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[1] A paper presented at the Ithaca meeting of the American Physical Society, June, 1906.

[2] Burke, John, Philosophical Transactions, Vol. 191A, p. 87, 1898.

[3] Nichols and Merritt, PHYSICAL REVIEW, Vol. XIX., p. 397, 1904.

[4] Camichel, M., Comptes Rendus, 140, p. 139.

[5] Wick, Frances G., PHYSICAL REVIEW, Vol. 24, p. 356, 1907.

1. Intensity of transmitted light.
2. Intensity of fluorescent light.
3. Thickness of absorbing layer.
4. Wave-length.
5. Concentration.

Method and Apparatus.

The method used was essentially the same as that employed by Nichols and Merritt, the instrument used being the Lummer-Brodhun spectrophotometer. The fluorescent solution, contained in a rectangular glass cell *R* (Fig. 1), was placed in front of slit *S*.

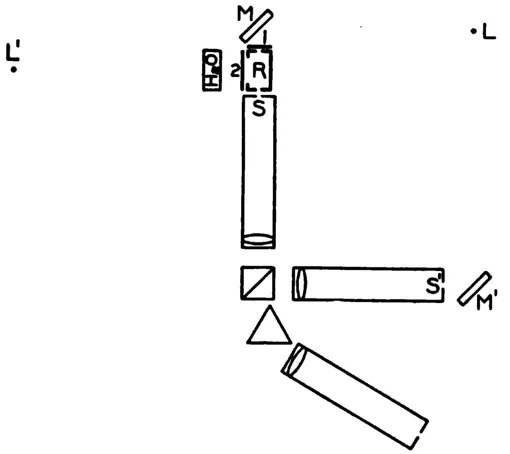

Fig. 1.

A bank of four acetylene flames *L'* was used as a source of fluorescence excitation. The source for transmission was an acetylene flame *L* from which light was diffusely reflected by a block of magnesium carbonate *M*. The intensity of transmission was adjusted by altering the angle of *M*. Screens 1 and 2 were arranged like shutters so that light from *M* or *L'* could be entirely cut off. For observations in which the intensity of fluorescence was varied, and those in which the thickness of the solution was changed at a constant wave-length, a glass cell containing a dilute solution of the fluorescent substance was placed between the flame *L* and the carbonate *M*. This acted as a screen to prevent fluorescence excitation by the transmitted light.

To determine the increase in absorption due to fluorescence three readings of the micrometer screw attached to S' were necessary.

1. *Transmission* (T). — Intensity of light transmitted from M, shutter 1 being open and 2 closed.

2. *Fluorescence* (F). — Intensity of the fluorescent light, shutter 2 being open and 1 closed.

3. *Transmission and Fluorescence combined* (C). — Intensity of light observed with 1 and 2 both open.

If the absorbing power were not changed during fluorescence the sum of the first two readings ($T + F$), would equal the third (C). This, however, did not prove to be the case. From actual observations

$$T + F > C.$$

An increase in absorption takes place which is measured by the expression

$$A_F = T + F - C.$$

This increase, A_F, is called by Nichols and Merritt "fluorescence absorption." The values of T, F, C and A_F were obtained in terms of the width of slit S' as read from the drum of its micrometer screw. Different sets of observations are not comparable since the comparison source was so adjusted as to produce the best possible match in color and intensity.

Difficulties and Sources of Error.

In all spectrophotometric work, especially determinations such as this, in which a small final result is obtained from the difference of two relatively large observations, the probability of error is great. This work was done in a dark room and special precautions were taken in the adjustment of screens and shutters to eliminate stray light. All possible care was taken in the arrangement of every part of the apparatus.

The most serious difficulty, however, was one of color match, and this could not be avoided by any mechanical adjustment. In making a set of observations extending through the fluorescence band the width of the collimator slit S (Fig. 1) had to be kept constant. A perfect color match was possible only when the intensities of the two sources of light was such as to make slit widths *S* and *S'* (Fig. 1) equal. But as different wave-lengths were observed, keeping the comparison source *L* constant, the intensities of *T*, *F* and *C*, varied in such a way as to require the width of *S'* to be made in some cases greater and in others less than that of *S*.

Since the proper adjustment is difficult to make with accuracy it is essential that the observer's eyes be kept sensitive and that they be allowed to become perfectly restored after exposure to light before any observations are made. Care was taken in this work to have the periods of observations short, and results were rejected as inaccurate when the individual settings for any one measurement began to vary widely. An average of from three to six settings of the micrometer screw was taken for each of the values obtained.

FLUORESCENCE ABSORPTION AND INTENSITY OF TRANSMITTED LIGHT.

The first series of measurements was made to determine the relation between fluorescence absorption and the intensity of transmitted light. Tables I. and II. give the results obtained by a variation in the intensity of transmitted light, all other conditions being kept constant. Fluorescence absorption (A_F), it will be observed, is nearly constant for all values of transmission (T), in a given set of measurements.

According to Nichols and Merritt, as the intensity of the transmitted light increases from zero, fluorescence absorption must also increase to a certain maximum value, at which saturation is reached. At this point the intensity of transmitted light is so small as to make accuracy of measurement impossible. The tables here given serve as additional evidence of the truth of the conclusions drawn by Nichols and Merritt, that fluorescence absorption does not take place in accordance with the ordinary laws of absorption. An increase in the intensity of the incident light is not accompanied by a corresponding increase in absorption.

FLUORESCENCE ABSORPTION AND INTENSITY OF FLUORESCENCE.

In the next set of observations the intensity of the fluorescent light was varied by changing the distance of *L'* from the face of the cell (*R*) (Fig. 1).

TABLE I.

Dependence of Fluorescence Absorption upon the Intensity of the Incident Light. I., II. and III. indicate the Individual Observations from which the Average was Computed. The Measurements were made upon Concentration 1/32, *at Wave-length* .646 μ.

T				*F*				*C*				*T + F*	A_p
I.	II.	III.	Av.	I.	II.	III.	Av.	I.	II.	III.	Av.	Av.	Av.
25.2	24.8	24	24.7	12	11.4	11.3	11.57	34.3	34.5	33	33.93	36.27	2.34
36.4	36.5	35	35.97	11	11	11	11	44.5	44.7	44	44.4	46.97	2.57
56.2	60	59	58.4	11.3	10.8	11	11.03	65.4	66.5	66.2	66.03	69.43	3.4
68.3	67	67	67.43	10.6	10	11	10.53	75.5	75	76	75.5	77.96	2.46
74	73.8	74.8	74.2	10.6	11	11.2	10.93	82	82.8	82.4	82.4	85.13	2.71

TABLE II.

Dependence of Fluorescence Absorption upon the Intensity of the Incident Light. Concentration 1/128. *Wave-length* .646 μ.

T	*F*	*C*	A_p
13	10.4	21.73	1.67
31.5	10.1	39.56	2.04
68.23	9.93	75.2	2.96
89.9	10.43	97.8	2.53
98.27	10.3	105.375	3.195

Concentration 1/64. *Wave-length* .646 μ.

4.5	18.9	21.7	1.7
24.27	18.7	39.8	3.17
29.8	18.4	45.48	2.72

Concentration 1/128. *Wave-length* .646 μ.

13.17	10.5	21.43	2.24
32.85	10.5	39.4	3.95
57.47	10.87	62.5	5.84
68.65	10.5	76.96	2.19
84.3	10.6	90.7	4.2
114.1	11	117	8.1

The relation between fluorescence and fluorescence absorption was thus determined. The results indicate that for small values an increase in fluorescence causes an increase in fluorescence absorption. At a certain point saturation is reached, further increase in the intensity of fluorescence producing no change in fluorescence absorption. This is shown in Table III., graphically represented in Fig. 2. In the last two sets of observations given, there are no observations

TABLE III.

Dependence of Fluorescence Absorption upon the Intensity of the Fluorescent Light.
Concentration 1/64. *Wave-length* .646 μ.

T	F	C	A_F
30.2	3.4	33.2	.4
30.95	6.2	35.2	1.95
31.45	8.2	38.2	1.45
30.95	13.45	41.45	2.95
31.7	17.2	46.37	2.53
31.2	23.5	51.2	3.5
32.8	27.95	56.8	3.95
32.46	33.7	61	5.16
31.95	46.63	72.2	6.38
31.6	60.6	87.4	4.8
31.4	66.5	90.9	7
31.8	94.83	120.3	6.33
31.6	83.07	109.26	5.41
32.3	140.7	168.12	4.88
Concentration 1/64. *Wave-length* .646 μ.			
34.9	7.37	41.1	1.17
33.82	9.37	40.25	2.94
32.9	12.9	42.9	2.9
34.9	16.27	46	5.17
35.7	20.7	51.5	4.9
35.33	22.53	53.37	4.49
34.63	23.7	52	6.33
Concentration 1/128. *Wave-length* .646 μ.			
38.9	12.92	40.71	11.11
37.6	13.6	43.8	7.4
39.18	15.4	44.7	9.88
38.65	19	44.9	12.75
38.83	26	55.98	8.85
40.1	35	61.65	13.45
Concentration 1/128. *Wave-length* .646 μ.			
45.88	10.05	44.125	11.805
44.9	13.37	46.17	12.1
44.78	19.6	47.66	16.72
46.3	22.95	54.9	14.35

below the point at which saturation is reached. This same saturation effect was previously observed by Nichols and Merritt.

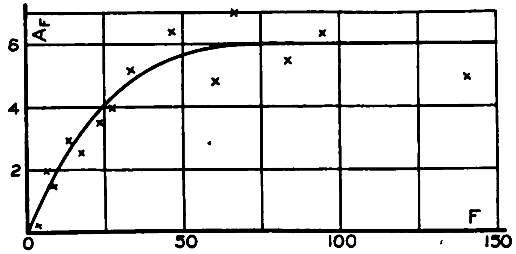

Fig. 2. Fluorescence absorption as a function of intensity of fluorescence.

FLUORESCENCE ABSORPTION WITH VARIATION IN THE THICKNESS OF THE ABSORBING LAYER.

In the previous experiments the section of the liquid excited to fluorescence through which the transmitted light passed was kept constant, *i. e.*, 5.45 cm. To find what effect a change in thickness has upon fluorescence absorption, the width of the section of the cell, *R*, exposed to the exciting light *L′* (Fig. 1) was varied, A section of the solution 1 mm. wide at the end of the cell next to the collimator slit *S* was first excited to fluorescence. The width of this section was gradually increased until it covered the whole face of the cell, readings being taken for each increase in width.

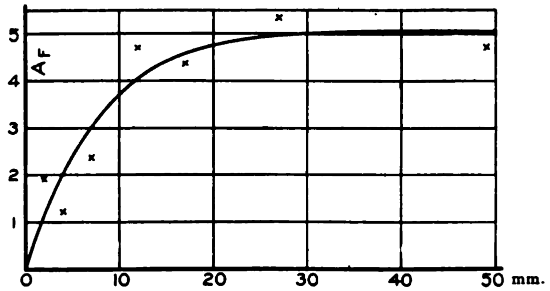

Fig. 3. Fluorescence absorption as a function of the thickness of the absorbing layer.

Fig. 3 shows the result. Fluorescence absorption increases with an increase in thickness up to a certain point, for this particular concentration about 1.25 cm. Further increase in thickness produces no change in fluorescence absorption. The complete set of curves from the first observations given in Table IV. is shown in Fig. 4.

As a check upon this work the same series of measurements was made beginning with the 1 mm. slit at the opposite end of the

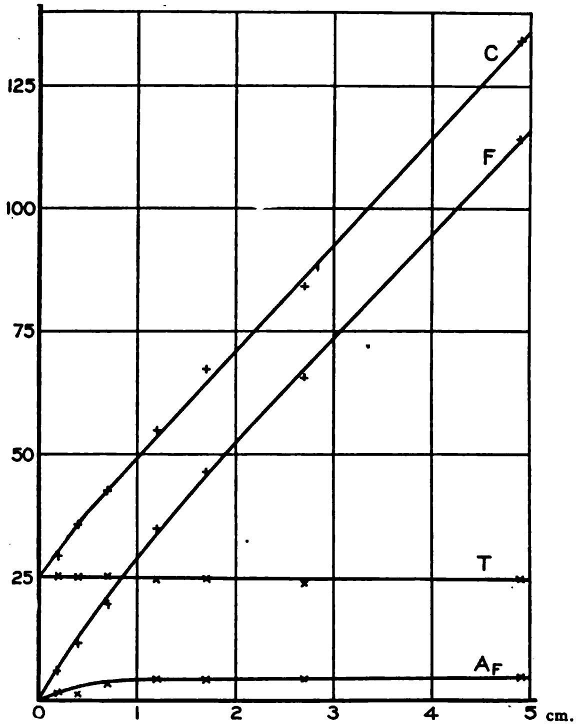

Fig. 4. Curves showing the effect of change in the thickness of the absorbing layer, the width of band being increased from the end of the cell next to the collimator tube toward the end next to the source of transmitted light.

F = observed fluorescence.
T = transmission.
C = fluorescence and transmission combined.
$A_F = T + F - C$ = fluorescence absorption.

cell, *i. e.*, the end next to M (Fig. 1), and gradually increasing the width of the opening toward the collimator tube until the whole face of the cell was excited. The results, given in the third set of observations in Table IV., and Fig. 5, were similar to those ob-

Table IV.

Dependence of Fluorescence Absorption upon the Thickness of the Absorbing Layer.
Concentration 1/64. *Wave-length* .646 μ.

Variation in thickness beginning at end of cell next to the collimator tube.

Thickness.	T	F	C	A_F
2 mm.	25.5	6.1	29.69	1.91
4	25.21	11.85	35.83	1.23
7	25.43	19.58	42.65	2.36
12	24.6	35	54.91	4.69
17	25.05	46.57	67.25	4.38
27	23.9	65.7	84.26	5.34
49	24.7	114.2	134.2	4.7

Concentration 1/64. *Wave-length* .646 μ.

1 mm.	28.9	3.95	31.94	.91
3	28.9	8.4	34.1	3.2
5	28.1	12.2	37	3.3
7	28.1	16	40.2	3.9
12	27.84	24.3	47.3	4.84
17	28.1	34.8	57.3	5.6
27	27.5	46.88	68.74	5.64
37	28.1	62.8	84.5	6.4

Concentration 1/64. *Wave-length* .646 μ.
Variation in thickness beginning at end of cell away from the collimator tube.

1 mm.	27.5	2.6	29.4	.7
3	27.6	6	32.1	1.5
5	27.63	8	33.6	2.03
7	26.58	12.8	35.3	4.08
12	26.9	20.37	42.7	4.57
17	26.2	28.45	49.6	5.05
22	26.2	37.3	54.45	9.05
32	26.2	46.9	67.2	5.9
42	26.2	59.7	80.7	5.2
54	26.2	71.5	91.8	5.9

tained from measurements in the other direction. This might be expected since the solution used was very dilute and at the wave-length studied ordinary absorption was insignificant.

In Fig. 5, the curve marked A_F shows the same saturation effect observed in Figs. 3 and 4. Curve *F*, Figs. 4 and 5, gives the observed fluorescence and curve *C* the fluorescence and transmis-
n combined. Curve *F* bends to about the point at which satu-
sio

ration is reached in A_F then becomes a straight line. This is explained by the fact, that, at first, with increased thickness there is an increase in the fluorescence absorption. This decreases the intensity of the observed fluorescent light. After saturation has been reached the increase in the observed fluorescence is proportional to the increase in thickness, fluorescence absorption entering

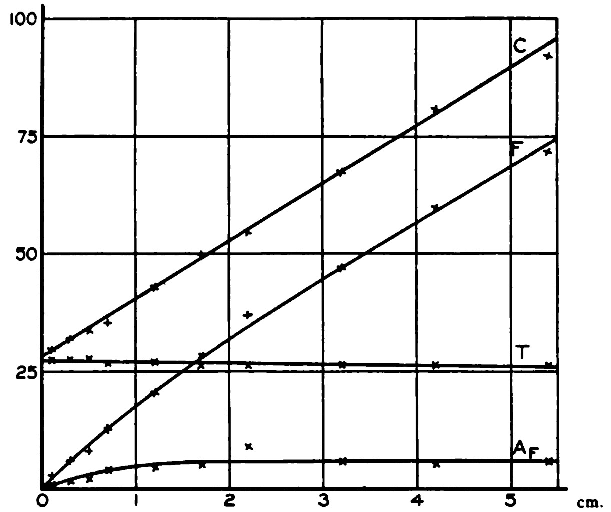

Fig. 5. Curves showing the effect of change in thickness of the absorbing layer, the thickness being increased from the end of the cell next to the source of transmission toward the collimator tube.

as a constant factor. Curve *C* is approximately a straight line. Other sets of measurements were made with similar results. In every case a saturation point was reached beyond which an increase in thickness produced no change in fluorescence absorption.

It thus appears that the energy absorbed from the transmitted light increases with increasing thickness up to a certain limit, beyond which fluorescence absorption is not changed by increasing the thickness of the fluorescent solution. A saturation point is reached similar to that observed in changing the intensity of fluorescence.

RELATION OF FLUORESCENCE ABSORPTION TO WAVE-LENGTH.

The relation between fluorescence absorption and wave-length was determined by shifting the telescope into different positions in the fluorescence band and making readings of T, F, and A_F at different wave-lengths.

TABLE V.

Dependence of Fluorescence Absorption upon Wave-length. Concentration 1/16.

Wave-length.	T	F	C	A_F
.542 μ	.7	1.7	1.5	.9
.562	.38	2.3	2.55	.13
.589	7.76	15.75	22.9	.61
.614	31.97	26.31	54.7	3.58
.646	39.88	13.8	52.7	.98
.69	36.55	7.82	43.35	1.02
Concentration 1/2.				
.589 μ	.55	4.325	4.9	.025
.614	11.3	20.63	29.575	2.355
.646	31.75	27.95	51.975	7.725
.69	40.4	15.9	52.375	3.92

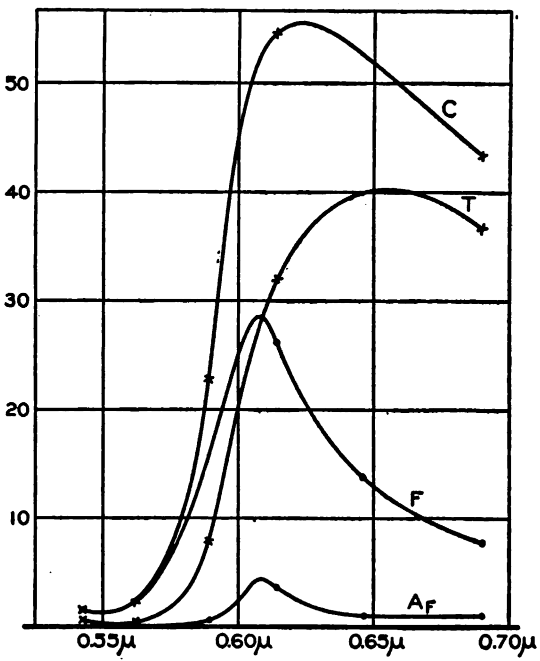

Fig. 6. Fluorescence absorption as a function of wave-length.

Table V., represented graphically in Fig. 6, gives the average of three sets of observations made upon concentration one sixteenth, a screen of one sixty-fourth resorufin being so adjusted between *L* and *M* (Fig. 1) as to cut off those rays which might produce fluorescence and thus introduce error. The fluorescence absorption curve, A_F, follows the fluorescence curve, *F*, the maxima of the two corresponding in wave-length. Table V. gives also the average of three sets of observations made upon concentration one half.

Other measurements given in Table VI. were made earlier without the interposition of the resorufin screen. These indicate greater errors of observation but in general it will be noticed that the greatest fluorescence absorption takes place in the region of greatest fluorescence.

TABLE VI.

Dependence of Fluorescence Absorption upon Wave-length. Concentration 1/128.

Wave-length.	T	F	C	A_F
.524 μ	11.7	2.5	11.8	2.3
.542	11.56	2.7	11.73	2.43
.562	9.86	4.55	13.35	1.06
.589	14.5	16.6	29.75	1.35
.614	23.65	17.9	37.25	4.3
.646	32.9	10.4	39.35	3.95
		Concentration 1/64.		
.524 μ	10.25	1.95	10.3	1.9
.542	7.73	1.8	7.5	2.03
.562	6.47	5.07	9.27	2.27
.589	8.6	35.9	42.4	2.1
.614	21.73	47.7	65.23	4.2
.646	30.83	26.9	53.97	3.76
		Concentration 1/128.		
.542 μ	11.2	1.2	11.45	.95
.562	7	4.2	10.2	1
.589	14.45	29.8	40.7	3.55
.614	34.4	39.7	64.9	9.2
.646	46.6	22.5	55.4	13.7
.69	48.2	19.45	59.8	7.85

CHANGE IN CONCENTRATION.

The relation between fluorescence and fluorescence absorption is further brought out by a comparison of the results obtained from different concentrations. The positions of maximum fluorescence and fluorescence absorption correspond in each case.

Fig. 7 gives results from two concentrations. Curves F and A_F represent fluorescence and fluorescence absorption for concentration one sixteenth, the corresponding maximum being .61 μ. Curves F' and A_F' represent fluorescence and fluorescence absorption of concentration one half. The maximum of fluorescence, F', has shifted to about .635 μ—an effect due to increased absorption on the side of the band next to the violet. The maximum fluorescence absorption, A_F', has also shifted to about the same point.

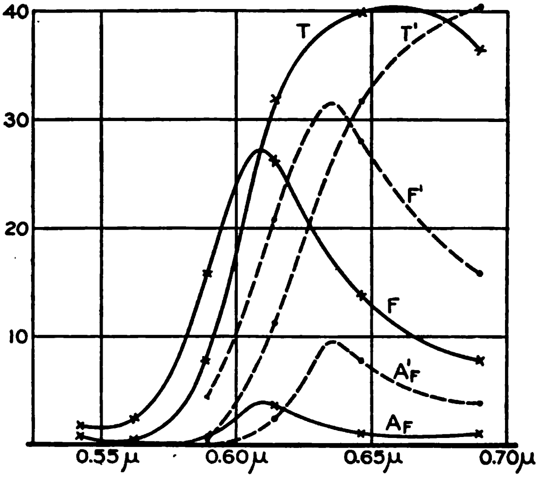

Fig. 7. Fluorescence absorption as a function of wave-length in different concentrations.

Curves T, F and A_F correspond to concentration $\frac{1}{16}$.
Curves T', F', and A_F' correspond to concentration $\frac{1}{2}$.

CONCLUSION.

The experiments described in this paper serve as a confirmation of the work done by Professors Nichols and Merritt for whose encouraging interest the author wishes to express her grateful appreciation.

The fact that during fluorescence a substance acquires a temporary power of absorbing those rays emitted during excitation seems reasonable; but certain phenomena observed in connection with this increase baffle explanation. It seems natural to expect that with an increase in the intensity of transmitted light an increase in fluorescence absorption will take place according to the known laws of ordinary absorption, but such is not the case. The increased absorption seems independent of the intensity of transmitted light.

The saturation effect observed with increase in the thickness of the fluorescent solution is equally strange. After a certain thickness has been reached further increase in thickness has no effect upon fluorescence absorption.

Another surprising fact brought out is that the fluorescence absorption band corresponds in position with the *observed* fluorescence band. It might be expected that a phenomenon which seems to be an accompaniment of fluorescence should show a maximum effect not at the observed maximum of fluorescence, after absorption has taken place, but at the actual maximum fluorescence point found by correcting for absorption.

In this paper no attempt has been made at explanations, the object being simply to present the observational facts with the object of discovering whether or not any peculiar absorption phenomena accompany fluorescence in resorufin.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

REFLECTION AND TRANSMISSION OF ELECTRIC WAVES BY RESONATOR GRATINGS.

REMARKS ON A PAPER UNDER THIS TITLE, BY F. C. BLAKE AND C. R. FOUNTAIN.

BY CLEMENS SCHAEFER.

IN a research,[1] which has just been published, Messrs. Blake and Fountain have investigated the reflection and transmission of electric waves by systems of resonators.

The method which they use is similar to that which Garbasso,[2] Aschkinass and I[3] have employed; we will speak later of the changes they have introduced. Their results confirm in part those previously obtained by the above-mentioned authors. Along with other work, I[4] published some of their results in an article under the title, "Concerning the Selective Properties of Resonator Gratings," in the Annalen der Physik, in January, 1905. This article has apparently not come to the notice of Messrs. Blake and Fountain. At least I can find no evidence that it has, although in the article by Pätzold, cited by them in their appendix, my work is frequently mentioned.

This is my reason for giving the work of Messrs. Blake and Fountain somewhat closer consideration, and especially since their method, in those points where it differs from that of Aschkinass and myself, is by no means an improvement, but to my mind at least is even poorer.

As Garbasso first showed, the resonator gratings here used possess selective properties for electric waves; *i. e.*, they reflect to a considerable extent only those waves whose period of vibration does not differ greatly from their own. Two methods present themselves for the study of the relation between reflection, with its corresponding transmission, and the wave-length:

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[1] F. C. Blake and C. R. Fountain, PHYS. REV., XXIII., p. 257, 1906.

[2] A. Garbasso, Atti Acc. di Torino, XXVIII., pp. 470 and 816, 1893. Journal de Physique, XXII., p. 259, 1893.

[3] E. Aschkinass and C. Schaefer, Ann. d. Phys., V., p. 489, 1901.

[4] C. Schaefer, Ann. d. Phys., XVI., p. 106, 1905.

I. We may use a *single resonator grating*, which remains unchanged during the experiment. In this case we must use a variable exciter and receiver to give us the necessary wave-lengths within the limits over which we wish to study the extent of reflections or transmission. This method is exactly that which is used in the corresponding measurements in optics. This method seems to me for certain reasons to have the advantage over the one to follow.

II. We work with a *single wave-length, i. e.*, with an unchanging exciter and receiver, varying instead the resonator gratings, *i. e.*, the length of the resonators. With this method, too, which in a certain sense is only a reversal of the previous one, we may obtain unimpeachable results. By this method, for example, the investigations of Garbasso, and Aschkinass-Schaefer were carried out. Messrs. Blake and Fountain have also used this second method, but in a way that is not entirely free from objection.

I will now try to make this clear. The characteristic point in the method used is the variation of the resonator grating. Let us see how this was done in this investigation. According to the table, page 265, the authors start out, for example, in their curves T and R of Fig. 8, with a grating, the structure of which can be seen from the accompanying figure.

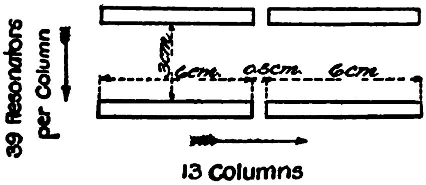

Fig. 1.

This grating they change by the gradual decrease of the resonators, *all other dimensions of the apparatus remaining unaltered.*

There are objections to this method, although at first sight it is apparently correct. In order to make this clear, consider the following: The power of a resonator grating to reflect, and consequently too, to transmit, depends not only on the length of the resonators but also on their number and their arrangement. So gratings made of resonators of different lengths are in general not at all comparable. That is, the question which rises in using the second method is this: How must resonator gratings, consisting of resonators of different lengths, be arranged, in order to be comparable with another? The answer to this question is easy, if we for a moment consider the relations holding when we employ the first method, which, as already stated, is entirely analogous to the method used in the corresponding optical measurements. Here the grating stays unchanged during the experiment, while the wave-length is varied. If we pass from a wave-length λ_1 to a second λ_2, the corresponding linear dimensions of the grating have changed in the proportion of λ_2 / λ_1, using as a unit the new wave-length λ_2. And here we see the answer to our question: In using the second method it is not sufficient to change the length of the resonators alone, but all corresponding dimensions of the grating must be changed in the same ratio. Only thus can we obtain comparable relations, as in the case of those optical experiments where we meet selective reflections, and in this way we get exactly these relatively simple relations. The error in the method of Blake and Fountain is that they did not notice this fact, although Aschkinass and I laid especial emphasis upon it.

This explanation makes it at once clear how much simpler the first method is, because this particular difficulty does not enter into it at all. Hence the curves of Messrs. Blake and Fountain are quite complicated, and cannot well be compared with those of the previous investigators, and it is not possible to apply the ordinary dispersion formulæ to them directly as would be the case with the curves of Aschkinass and myself. In short, Blake and Fountain obtained, on account of the construction of their gratings, complicated relations instead of relatively simple ones.

I have mentioned already that many of the results obtained by Messrs. Blake and Fountain are only substantiations of the work of previous authors. I will now make this clear in a special case. Blake and Fountain discuss this problem: How does the length of the resonators corresponding to maximal resonance change, when the elements of the grating are brought closer together? They find, that a diminution of the distance between the resonators in a direction at right angles to the electric force requires a greater resonator length to produce maximal resonance. According to their results, within the limits investigated, a diminution of the distance in a direction parallel to the vector of electric force makes no difference at all. From the standpoint of the first method, this means that in the first case the point of maximal resonance is changed so that it occurs at a smaller wave-length, while in the second case it is unaffected.

I discussed this same question in the article previously mentioned, and studied the phenomena by the first method, which seems certainly better on account of its simplicity. The results I obtained are these: (1) A diminution of the distance in a direction at right angles to the electric vector changes the maximal resonance to a point of smaller wave-length. This is the same result as obtained by Blake and Fountain. (2) A decrease of the distance in a direction parallel to the electric force changes the maximal resonance to a point of greater wave-length. This result is exactly opposite to that of Blake and Fountain; however this contradiction is easily explained, since I varied the distances within wider limits than they. Further it must not be forgotten that, strictly speaking, the curves of Blake and Fountain are not at all comparable with mine. The fact that they did not find the change in the point of maximal resonance in the second case, is of itself a proof of the unsuitability of their method. Moreover, according to the explanation which Blake and Fountain put forward for the change in the first case, one would also expect a change in the second. Consider this explanation for a moment. They say, page 268: "that the resonance length should decrease with increasing distance between the strips was to be expected; for since at any instant the currents in the resonators are all in the same direction, the effect of induction is reduced by decreasing the distance between them, hence in order to respond to a given wave the capacity of the resonators must be increased, that is, they must be lengthened."

I cannot understand this explanation because of its inexactness o statement. What is meant by "the effect of induction"? The only factor concerning the induction, which can be changed through a change in the distance between the resonators, is the coefficient of mutual induction. As I previously showed,[1] however, this change would, in the first case, have an effect exactly opposite to the observed; while in the second case, on the other hand, the theoretical changes would correspond with those observed by myself. These points make it clear that the explanation in the first case, by means of the coefficient of mutual induction, is not complete. We must rather take into account that a change in the distance between the resonators also changes in general their capacity. The change in capacity, however, causes in both cases a change in the point of maximum resonance corresponding to the experimental results. That is, the explanation of Messrs. Blake and Fountain is certainly, if not absolutely wrong, at least incomplete.

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[1] C. Schaefer, l. c.

I pass now to the consideration of a phenomenon which Messrs. Blake and Fountain discovered, and to which they gave the name of "Extra transmission." They met this phenomenon in the case of resonator gratings glued to glass plates. Essentially it is this, that such a resonator grating glued to glass, under certain conditions, *transmits more*, and consequently *reflects* less, than the glass plate alone. The authors explain this remarkable phenomenon by saying that the glass plate and resonators may be a medium with a variable index of refraction, and that, according to the optical dispersion formulæ, under certain conditions the index of refraction may become smaller than that of the glass itself. Although, as I have already shown, the optical dispersion formulæ are not here applicable entirely without restriction, still there is no objection to be raised to this explanation, *provided the phenomenon in question is a fact.* I must say, however, that the description of Blake's and Fountain's experiments has not convinced me of the reality of the phenomenon. My reasons are: first, that none of the earlier experimenters found this phenomenon, although, according to Blake and Fountain, it is quite marked. In particular, Aschkinass and I did not find this to happen. Hence in this latter investigation we must assume quite large errors of observation. Now, such large errors are *surely* here out of the question, as is easily seen from the excellent correspondence of our measurements of the dielectric constant with those of the other investigators. Moreover, I have repeated the measurements of Blake and Fountain in question, reproducing their arrangement of apparatus. The only change I made was to restrict the wave-flow by making it pass through two diaphragms, with openings of 24 × 32 cm. The resonator grating used was of the same size. It only remains to be said that the area of the resonator grating was *smaller* than the mouth of the concave mirror used. With this apparatus I was likewise unable to find any "extra transmission." More than that, the grating always transmitted less energy than the glass plate alone. If the diaphragms were removed, the transmittivity of the resonator grating *increased;* if the rays were made *divergent*, by moving the exciter a bit from the focus of the concave mirror, the transmittivity *increased still more.* Now this result is an *approximation* of the phenomenon of "extra transmission"; but I did not succeed in verifying the observations of Blake and Fountain.

The question is, must we accept the explanation of Blake and Fountain, or can we give another. To answer this, it must be kept in mind, that these authors did *not use diaphragms* to restrict the wave flow; moreover the area of their resonator grating was larger than that of their parabolic mirror. Now, if we assume that the original electric waves were not exactly parallel, but slightly *divergent*, it would actually be possible to observe a phenomenon like "extra-transmission." For the individual resonators of the grating would be set in vibration by the electric wave striking them, and thus would become themselves centers of spherical waves going out in all directions. Now, since in the experiments of Blake and Fountain the grating had a larger area than the concave mirror, in the focus of which the receiver is placed, "*side waves*" from the resonators on the border of the grating could reach the receiver when the grating was interposed, while the interposition of the glass plate alone let only direct waves reach the receiver. Under certain conditions these "side waves" may become so marked, that putting the grating in the path of the wave would actually give an *increase* of the amount of energy reaching the receiver. As a matter of fact, M. Paetzold observed this very thing in the case of *divergent* radiation without interposition of diaphragms with gratings, whose area was larger than the mouth of the mirrors he used. This is the explanation of the results of my repetition of the experiments of Blake and Fountain. As soon as the diaphragms are taken away, we find an increase of transmittivity, which is, and ought to be, under proper experimental conditions, *constant.* The increase above the normal is still greater if we intentionally make the paths of the radiation divergent.

I do not yet say, that the "extra transmission" of Blake and Fountain is likewise due to poor experimental arrangements. But so long as the above objections to the experimental arrangement of the authors are not refuted, there is no reason for the assumption of a *new phenomenon.*

BRESLAU, PHYSICAL LABORATORY OF THE UNIVERSITY,
January, 1907.

THE EFFECT OF A TRANSVERSE MAGNETIC FIELD ON METALLIC RESISTANCE.

BY E. P. ADAMS.

THE effect of a transverse magnetic field in changing the resistance of a conductor has been investigated by many observers; Patterson,[1] and very recently Grunmach[2] have made very careful measurements of this effect in a number of metals. Considering the size of the effect, the agreement between the results of these two observers is, on the whole, very good. In all the metals which have been tried, with the exception of the ferromagnetic metals, iron, nickel, and cobalt, a transverse magnetic field is found to produce an increase in the resistance. Grunmach found that in the case of the ferromagnetic metals, weak magnetic fields produced an increase in the resistance. On increasing the magnetic force the effect became smaller, until for a certain value of the magnetic force it disappeared; for greater values the resistance was diminished. With one specimen of iron, and with nickel, the effect appeared to be a decrease of the resistance from the start.

A theory of the effect of a transverse magnetic field in increasing the resistance of conductors has been given by J. J. Thomson.[3] He assumes that the electric current is carried by free corpuscles, which, when no external forces act, are in motion in all directions, just as the molecules of a gas inside a closed vessel. The corpuscles are, in fact, assumed to behave just as the molecules of a gas, with this exception; the collisions which take place are not between corpuscles and corpuscles, but between corpuscles and atoms of the metal. When an electric force is applied, a drift of the corpuscles in one direction is produced, which is the electric current. In this explanation, in order to get an effect of the right sign, it is necessary to assume that the collisions between corpuscles and atoms are greatly influenced by the electric charges carried by the former.

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[1] Philosophical Magazine, 3, p. 643, 1902.

[2] Annalen der Physik, 22, p. 141, 1907.

[3] Rapports presentes au Congres International de Physique, III., p. 138, 1900. Philosophical Magazine, 3, p. 353, 1902.

The assumption of collisions like those between hard elastic spheres cannot therefore be made. This theory can be made to apply to the ferromagnetic metals only by assuming that in them the collisions are of an altogether different character from the collisions in other metals. Lorentz,[1] on the other hand, expressly assuming collisions uninfluenced by the electric charges of the corpuscles, has successfully applied this theory to the thermoelectric properties of metals; it thus seems as if the assumption of collisions between corpuscles and atoms like those between hard elastic spheres ought also to explain the change in resistance of conductors in a magnetic field.

In the present paper it will be shown that, keeping to the assumption of collisions of this nature between corpuscles and atoms, and introducing the probable effect of the magnetic field in changing the molecular configuration of the metal, it is possible to explain the sign of the effect in the ordinary para- and diamagnetic metals, and in the ferromagnetic metals, as well. The theory of the effect as given by J. J. Thomson will first be developed, and afterwards this theory will be modified by introducing the effect of the magnetic field on the molecular configuration of the metal.

We assume that the corpuscles are all of one kind and move uniformly in all directions with the velocity U. In the paper of Lorentz, referred to above, the theory of electric and thermal conductivity has been given where the distribution of velocities among the corpuscles is assumed to be that given by Maxwell's law for the distribution of velocities among the molecules of a gas. But for the present purpose, since we can hope to get an idea of the order of magnitude only of the effect, this complication is unnecessary.

In a magnetic field the velocity of the corpuscles moving in a plane perpendicular to the magnetic force will be altered. Taking the xy plane as the plane of motion, the current being in the x-axis, and the magnetic force in the z-axis, the equations of motion of a single corpuscle are:

$$\left.\begin{aligned} m\frac{\partial^2 x}{\partial t^2} &= eX - He\frac{\partial y}{\partial t} \\ m\frac{\partial^2 y}{\partial t^2} &= He\frac{\partial x}{\partial t} \end{aligned}\right\} \qquad (1)$$

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[1] Archives Neerlandaises des Sciences Exactes et Naturelles, 10, p. 336, 1905.

m being the mass, e the charge, and H the intensity of the uniform magnetic field.

From this we get:

$$m\frac{\partial y}{\partial t} = Hex$$

since when $x = 0$,

$$\frac{\partial y}{\partial t} = 0.$$

As the effect produced by the magnetic field is known to be small it will be permissible to take for x in this equation the value it would have if no magnetic field were applied; *i. e.*:

$$x = \frac{1}{2}\frac{e}{m}Xt^2 + Ut$$

$$\frac{\partial y}{\partial t} = \frac{He}{m}\left(\frac{1}{2}\frac{e}{m}Xt^2 + Ut\right).$$

Substituting this in the first of (1):

$$\frac{\partial^2 x}{\partial t^2} = \frac{e}{m}X - \frac{H^2e^2}{m^2}\left(\frac{1}{2}\frac{e}{m}Xt^2 + Ut\right)$$

and hence:

$$\frac{\partial x}{\partial t} = \frac{e}{m}Xt - \frac{H^2e^2}{m^2}\left(\frac{1}{6}\frac{e}{m}Xt^3 + \frac{1}{2}Ut^2\right) + U.$$

Calling $\bar{u}$ the mean velocity of the corpuscle due to the electric and magnetic field between two collisions, and T the time of free path, we get:

$$\bar{u} = \frac{1}{T}\int_0^T \frac{\partial x}{\partial t}\,dt = \frac{1}{2}\frac{e}{m}XT - \frac{H^2e^2}{m^2}\left[\frac{1}{24}\frac{e}{m}XT^3 + \frac{1}{6}UT^2\right] + U. \quad (2)$$

If there are n corpuscles per unit volume, the current in the direction of the x-axis will be:

$$I = ne\bar{u} = \frac{1}{2}\frac{e^2}{m}nXT - \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4}n\frac{e}{m}XT^3 + nUT^2\right]. \quad (3)$$

The last term of (2) drops out because there will be as many corpuscles for which U is positive as negative. We cannot, however, say that the second term in the brackets in (2) drops out, since, owing to the factor T^2, this will be different for corpuscles whose initial velocities are in opposite directions. A corpuscle for which U is negative will have a greater value of T, as a result of the electric force, than one for which U is positive. This term can be easily evaluated if we consider the collisions between corpuscles and atoms to be like those between hard elastic spheres. For half of the corpuscles U will be negative; so this term can be written:

$$\frac{1}{2}nUT^2 - \frac{1}{2}nUT'^2 = nUT\Delta T,$$

T being the time between collisions when no electric force is applied. If L is the free path of a corpuscle:

$$\Delta T = T - T' = L\left\{\frac{1}{U + \frac{1}{2}\frac{e}{m}XT} - \frac{1}{U - \frac{1}{2}\frac{e}{m}XT}\right\}$$

so that:

$$nUT^2 = -\frac{e}{m}nXT^3$$

and the electric current is given by the expression:

$$I = \frac{1}{2}\frac{e^2}{m}nXT + \frac{1}{8}\frac{H^2e^3}{m^2}\frac{e}{m}nXT^3.$$

The first term on the right is the current when no magnetic force is applied. The effect of the magnetic force is thus to increase the current, or decrease the resistance, which is contrary to observed facts.

J. J. Thomson, however, considers that the collisions between corpuscles and atoms are strongly influenced by the electric charges carried by the corpuscles. Without going into any calculation it is easy to see that if this influence is sufficienly pronounced, the difference, ΔT, between the time of the free path in the direction of the current and the opposite direction, will be very small; hence the second term in the brackets of (3) may be neglected in comparison with the first term, and we get:

$$I = \frac{1}{2}\frac{e^2}{m}nXT - \frac{1}{24}H^2\frac{e^4}{m^3}nXT^3.$$

This indicates an increase in resistance due to the magnetic field.

Now if we suppose that a magnetic field produces a rearrangement of the molecules within a metal, it seems very reasonable to assume that the free path of a corpuscle will be altered. Without saying anything definite as to the nature of this change, let us assume that a magnetic field, H, produces a change, δT, in the time between two collisions of a corpuscle. δT will then, in general, be some function of H. With no magnetic field the current will be:

$$I = \frac{1}{2}\frac{e^2}{m}nXT. \tag{4}$$

In a transverse magnetic field the current will be, by (3):

$$I' = \frac{1}{2}\frac{e^2}{m}nXT' - \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4}n\frac{e}{m}XT'^3 + nUT'^2\right] \tag{5}$$

where T is changed to T' the free time in the magnetic field. The effect of the magnetic field will therefore be:

$$I - I' = \frac{1}{2}\frac{e^2}{m}nX\delta T + \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4}n\frac{e}{m}XT'^3 + nUT'^2\right]. \tag{6}$$

As we suppose the change in T produced by the magnetic field to be small, we can put:

$$T'^3 = T^3 - 3T^2\delta T,$$

$$T'^2 = T^2 - 2T\delta T.$$

Then by bringing in the assumption that the collisions between the corpuscles and atoms are like those between hard elastic spheres, we find, by the method previously employed:

$$I - I' = \frac{1}{2}\frac{e^2}{m}nX\delta T + \frac{1}{6}H^2\frac{e^3}{m^2}\left[-\frac{3}{4}n\frac{e}{m}XT^3 + \frac{1}{4}n\frac{e}{m}XT^2\delta T\right]. \tag{7}$$

The second term in the brackets will be small compared to the first, and we thus have:

$$I - I' = \frac{1}{2}\frac{e^2}{m}nX\left[\delta T - \frac{1}{4}H^2\frac{e^2}{m^2}T^3\right]. \tag{8}$$

If R is the resistance of the conductor with no magnetic field, and R' the resistance in the magnetic field, we have:

$$I - I' = \left(\frac{1}{R} - \frac{1}{R'}\right)X$$

or

$$\frac{\delta R}{R} = \frac{\delta T}{T} - \frac{1}{4}H^2\frac{e^2}{m^2}T^2. \tag{9}$$

In order that the magnetic field may increase the resistance, it is necessary, first, that δT be positive; this means that a change in molecular configuration arising from the magnetic field produces a diminution in the time between two collisions of a corpuscle; and, second, that $\frac{\delta T}{T}$ be greater than $\frac{1}{4}H^2\frac{e^2}{m^2}T^2$. As to the first condition, our lack of knowledge regarding molecular structure makes it difficult to decide whether it is satisfied or not. But there seems to be no evidence, at any rate, that it is not satisfied. From the second condition we can determine the order of magnitude of the change in the free time which is necessary in order to result in an increase of resistance in a magnetic field. But it is first necessary to get an estimate of the value of T.

The change in the free time of a corpuscle we have supposed due to a change in the configuration of the molecules brought about by the magnetic field. For the ordinary para- and diamagnetic metals, for which the magnetic susceptibility appears to be constant, independent of the strength of the magnetic field, this change in configuration must be supposed to increase with the magnetic field. So that if these metals show an increase in resistance for certain values of the magnetic field we would expect an increase in resistance in all magnetic fields, however great. This is the result obtained by experiment.

With the ferromagnetic metals the case is altogether different. These metals show the phenomenon of magnetic saturation. Up to a certain limit we may suppose that an increase in the magnetic force produces an increased change in the molecular configuration. After this limit has been reached, on further increasing the magnetic force no more change in the molecular configuration is produced, and consequently, no further change in the free time of a corpuscle. After a certain limit, therefore, the first term on the right of (9) remains constant, while the second term increases with H^2. It appears very probable, then, that for a certain value of H the second term on the right in (9) may become equal to the first term, and for greater values of H become larger than the first term. This would therefore result in a decrease of resistance due to the magnetic field; this is the result obtained by Grunmach experimentally.

For large values of the magnetic force, in the case of iron, for example, we can neglect the first term on the right in (9) in comparison with the second term, and obtain thus:

$$\frac{\delta R}{R} = -\tfrac{1}{4}H^2\frac{e^2}{m^2}T^2.$$

From Grunmach's results, we have, for iron:

$$H = 2 \times 10^4, \quad -\frac{\delta R}{R} = 10^{-3}, \quad \frac{e}{m} = 10^7,$$

and hence:

$$T = 3 \times 10^{-13}.$$

This gives the order of magnitude of the free time of a corpuscle in iron, and it will probably be not very different in any other metal. We can now find the order of the magnitude of the change in the free time of a corpuscle which is necessary to get an increase of resistance in a magnetic field. For this, we have found the condition to be:

$$\frac{\delta T}{T} > \tfrac{1}{4}H^2\frac{e^2}{m^2}T^2,$$

taking

$$H = 10^3, \quad \frac{e}{m} = 10^7, \quad T = 3 \times 10^{-13},$$

we get:

$$\frac{\delta T}{T} > 2.5 \times 10^{-6}.$$

It does not seem at all improbable that a magnetic field of the strength assumed should produce a change in T of this order of magnitude.

Having found the order of magnitude of T, we can easily calculate the number of corpuscles per unit volume which are involved in the electric current. We find, by (4)

$$\frac{1}{R}=\frac{1}{2}\frac{e^2}{m}nT.$$

For iron, R is approximately 10^4; $e=10^{-20}$ in electromagnetic units. Therefore:

$$n=7\times 10^{21}.$$

Taking $U=10^7$ as a probable value of the velocity of the corpuscles inside a metal, we get for the free path:

$$L=3\times 10^{-6}.$$

The effect of molecular configuration on the resistance of a conductor is well illustrated in the case of a distinctly crystalline metal like bismuth. In bismuth it is known that the resistance is different in different directions; this may well be explained by the differences in the free paths of corpuscles in different directions.

PRINCETON, N. J.

A HYDRAULIC ANALOGY OF RADIATING BODIES FOR ILLUSTRATING THE LUMINOSITY OF THE WELSBACH MANTLE.

By R. W. Wood.

THE device about to be described enables us to illustrate to a class the behavior of different types of radiating bodies when introduced into a flame, and will be found especially useful in explaining the remarkable luminosity of the incandescent mantles used in modern gas-lighting. It is of course not intended to explain the mechanics of radiation, but merely to enable us to describe certain phenomena in terms of easily grasped notions.

Students are told that the more powerfully a body absorbs, the more powerfully will it emit when heated, this relation holding for every individual wave-length. Black bodies then give out the most light when heated. The fact that a white block of lime is far more luminous than a carbon rod when heated in the oxy-hydrogen flame is not usually cited in support of this law, while the fact that the most luminous body of all, the Welsbach mantle, is also quite white is equally unsatisfactory as an illustration, for white bodies are in reality transparent, that is they are made up of masses of small transparent particles, and transparent bodies ought not to emit at all. It is of course necessary to define just what we mean by transparency in this case, and it may be well to consider first a somewhat analogous case. The absorption which is accompanied by high emissivity is true absorption, and not selective reflection,—which is sometimes confused with absorption. A highly reflecting polished metal surface is a poor radiator, but by properly constructing its surface we may give it the power to absorb and emit. A bundle of polished steel needles with their points all turned towards the source of light reflects scarcely any light at all, the rays undergoing multiple reflections between the conical ends of the needles. Such a bundle of needles should emit much more powerfully than a polished steel surface, and it is easy to see just why it should do so. Each needle, seen end on, sends not only emitted light to the eye, but reflects rays coming from its neighbors. The surface formed by the points of the needles can be regarded as an absorbing surface, which absorbs in virtue of its structure: it is analogous to the hollow "black bodies" with which we are now familiar.

The point which I wish to emphasize is that such a surface, which absorbs not at all in virtue of its molecular nature, is also a powerful radiator, the mechanism by which its radiating power has been increased, being as indicated above. Suppose now we take a perfectly transparent body, which like a perfect reflector has no emitting power. A bead of microcosmic salt (sodium pyro-phosphate) heated in a blast lamp is a good example. Though the platinum wire which supports it glows with vivid incandescence, the bead remains perfectly dark. A glass bead however emits a good deal of light, doubtless from the fact that its transparency is much less at high temperatures, a very common behavior of transparent substances. The microcosmic salt on cooling becomes traversed by hundreds of cleavage planes, which give it a milky appearance. On reheating it it emits light strongly until it finally fuses into a transparent drop, when it instantly becomes dark again. The reason for this behavior is not quite as apparent as in the case of the needles. In fact I am not quite sure that I understand it at all. Quartz behaves in the same way. A drop of clear fused quartz, heated in the blast, emits little or no light, but if it contains spots made up of an emulsion of quartz and air, these spots emit strongly. In other words an opacity resulting from a pulverization of the transparent medium seems to be accompanied with a strong emitting power. Apparently we cannot apply the same reasoning as in the case of the needles, and it looks rather as if the radiation was largely a surface effect. If this is so, it is obvious that an increase of the surface, by enclosures of air, will increase the radiating power. It is my intention to make some measurements of the intensity of the light radiated from the ends of long and short cylinders of red-hot glass.

The hydraulic analogy of radiating bodies which we will now consider occurred to me during a lecture on radiation, and proved quite useful in explaining the different behavior of various types of radiators.

The radiator is represented by a tall hollow cylinder, open at the top and closed at the bottom, provided with a number of outflow pipes of different sizes as shown in Fig. 1. Water flows into the cylinder at a certain definite rate from a horizontal pipe or flume, the height of which (T_2) above the base of the cylinder represents the temperature of the flame. Obviously the level of the water in the cylinder will rise until the rate at which the water flows out exactly equals the rate at which it flows in. This height (T_1) is the temperature which the radiator acquires in the flame. The jets of water which issue from the tubes represent radiation of different wave-lengths, the small jets representing the short waves. Their velocity corresponds to *intensity* of radiation. We will first suppose our hydraulic radiator to represent a black body, say a lump of carbon. In this case all of the pipes at the bottom are wide open and we have the maximum outflow of all wave-lengths for any given temperature, *i. e.*, for any given height of the fluid within the cylinder. (If we take the cylinder empty, and plunge it into water, jets will squirt into it through the pipes, that is it is a perfect absorber for all wave-lengths.) With all of the pipes open however the level of the water within the cylinder will not rise to any great height, owing to the limited rate at which water flows in from the horizontal pipe. This means that the lump of carbon in the flame does not rise to a very high temperature because it radiates energy at a high rate. At the low temperature there is comparatively little visible light in the radiation, for the shorter waves only appear in quantity at high temperatures. We can imitate this condition in our hydraulic model if we choose by putting valves on the inside of the tubes, those on the small tubes opening only at high pressures.

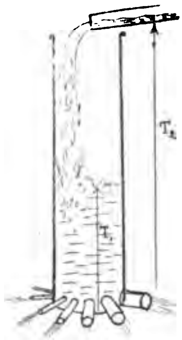

Fig. 1.

To make our model imitate the bead of microcosmic salt we plug up all of the pipes. The cylinder now represents a transparent body. If immersed in water it absorbs nothing through the pipes, and no matter how high the level of the water rises in it when water is poured in there is no emission of fluid, in other words no radiation.

The body rises in temperature until the temperature is equal to that of the flame, but there is no radiation. Take next the case of the lime in the oxyhydrogen flame. It is a partially transparent substance, and we can imitate it by plugging the tubes with glass beads or cotton. Owing to the lesser rate at which the water now flows out through the tubes, the level rises much higher than when the tubes are all open, and owing to the greater pressure (temperature) we have liquid jets through the small tubes (short wave-length radiation). The inferiority in the emissivity is more than made up for by the higher temperature which the body can acquire. We are now ready for the Welsbach mantle. It has been conclusively shown by Rubens, that the peculiar brilliancy of the thorium mantles caused by a small trace of cerium, is due to the fact that the cerium makes the thorium selectively absorbing for the short waves at high temperatures. If we wave a bunsen flame over a mantle in a brilliantly lighted room, it will be seen to turn yellow at a temperature a little below a red heat. In other words it becomes a strong absorber for the short waves. It is however transparent for the long waves, consequently it does not emit energy at anything like the rate at which a black body does, and in consequence can rise to a high temperature in the flame, exactly as a pure thorium mantle does. Its band of absorption in the blue region enables it to pour out visible radiations nearly as powerfully as those which a black body at the same temperature would emit, hence its enormous brilliancy. Our hydraulic model, with all of its tubes plugged with cotton represents the mantle of pure thoria, while to transform it into the Welsbach mantle we have only to pull out the porous plugs from some of the smaller tubes. In this condition, owing to the impeded flow in the large tubes, the water will rise in the cylinder to a great height, and we get very powerful jets from the small tubes which we have opened, much more powerful than in either of the previous cases considered. Of course with all of the tubes open we could get equally intense small jets if we poured the water in at the top at a sufficient rate. There is a limit to this rate however, for it is obvious that the rate at which the water is poured in at the top, corresponds to the rate at which the flame can pour energy into the radiating body, a circumstance which depends on the conductivity of the body for heat and other things.

It is, of course, not necessary to make the hydraulic apparatus, for its action is so easily understood that a diagram answers every purpose. Its utility lies in the fact that it fixes in the mind of the student the behavior of different types of radiators, when plunged into a flame.

It could be made perhaps to illustrate the displacement of the point of maximum energy in the spectrum, which accompanies a rise in temperature, but it is doubtful whether any such complications would prove beneficial. It seems best on the whole not to try to illustrate too much with it, as its relation to a radiating body is at best rather far-fetched.

Johns Hopkins University.

ON THE ESTABLISHMENT OF THE HIGH TEMPERATURE SCALE.[1]

By C. W. Waidner and G. K. Burgess.

IN the interval 400° to 1200° C. the temperature scale is defined by the freezing points of certain pure metals, and in practice is usually reproduced with thermocouples. This scale, which is based primarily on the gas thermometer measurements of Holborn and Day, is the same as that used at the Physikalisch-Technische Reichsanstalt and the National Physical Laboratory.

Reproducibility of Scale. — Experiments undertaken to determine how exactly the scale may be reproduced by the use of metals from various sources, sold as chemically pure, show that Sn, Zn, Cu, and Au may be obtained in this country of sufficient purity to reproduce the above scale to 0°.5 or better. Sb showed differences of 10° or more, and Al about 3°.

Optical Scale. — The scale has been extended above 1200° by optical pyrometers whose calibration is based on the Wien-Planck equation. The realization of this scale necessitated the construction of an experimental black-body in which the temperature distribution could be exactly controlled. The uniformity of temperature attained was about 1° at 1300° throughout two-thirds of the radiating enclosure 30 cm. long.

The high temperature scale was then fixed by the calibration of five pyrometer lamps in terms of the radiation from this nearly ideal black-body whose temperature was measured with thermocouples. By the use of calibrated absorbing glasses and sector discs this scale has been extended to the highest attainable temperatures. The order of agreement of the lamps is well within 2° at 1500°.

Various types of electrically heated radiators were studied to determine the departure of their radiation from ideal black-body radiation.

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[1] Communication from the National Bureau of Standards.

Radiation from Platinum and Palladium. — Platinum is useful as a radiating source in many investigations at high temperatures. Further work has been done on the monochromatic radiation from Pt and also from Pd, both in air and hydrogen. The measurement of the temperature of the radiating surface and the changes which the surface undergoes have added greatly to the difficulties of the problem. Three independent methods of temperature measurement were used, one based on the indication of a thermocouple within a platinum spiral, another based on the melting of minute particles of metal on the platinum, and a third on the indications of a thermocouple heated in a colorless gas flame to the same brightness as the radiating platinum. The results are sumarized in the following table, where t° is the true temperature and s° the corresponding black-body temperature for red ($\lambda = 0.66\mu$) and green ($\lambda = 0.55\mu$), as determined in air.

$s^\circ =$	700°	800°	900°	1000°	1100°	1200°	1300°	(1400°)	(1500°)
(red) $t^\circ - s^\circ =$	46°	58°	72°	90°	110°	132°	155°	(178°)	(200°)
(green) $t^\circ - s^\circ =$	—	—	53°	68°	86°	103°	121°	139°	(155°)

The difficulty of determining the temperature of a radiating surface limits the accuracy of these results to about 10°.

Platinum and palladium agree in their radiating properties to within the limits of precision attainable to 1250°.

Determination of Fixed Points. — For the temperature scale above 1200° C. it is of great importance to determine accurately a number of fixed points. The melting points of palladium and platinum, which are especially suitable for this purpose, have been made the subject of numerous researches, but are still uncertain by some 40°.

Determinations of these constants were made by observing with an optical pyrometer the temperatures of melting within an iridium tube furnace heated with an alternating current. The temperatures, as given in the following table, are based on the extrapolation of Wien's equation, $J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$, assuming $c_2 = 14500$.

Melting Points of Pd and Pt in Iridium Furnace.

Metal.	Wave Length.	Absorption Coefficient.	Number of Observations.	Melting Point.	Average Deviation of an Observation.
Pd	0.666 μ	Glasses, $k=200$	8	1546°C	2°.5
	.666	$k=1$	7	1553°	2°.0
	.666	Glasses, $k=200$	7	1545°	2°.5
Pt	.668	Sector Disc $k=35.4$	10	1753°	2°.0
	.668	Glasses, $k=199$	16	1754°	3°.5
	.668	Glasses, $k=199$	7	1752°	5°.0
	.547	Sector Disc $k=35.4$	6	1748°	2°.0
	.462	Sector Disc $k=35.4$	4	1749°	3°.0

At least three pyrometer lamps were used in each series. The second value found for *Pd* is an independent extrapolation of the calibration equations of the lamps and therefore not based on Wien's equation.

Simultaneous measurements were made with thermocouples of the following types: Pt, 90 Pt–10 Rh; Pt, 90 Pt–10 Ir; 90 Pt–10 Rh, 80 Pt–20 Rh; Ir, 90 Ir–10 Ru. The results obtained differ by as much as 40° at the Pt point for the same couple according to the extrapolation formulæ used. The usual formulæ

$$(a) \qquad E = -a + bt + ct^2$$

$$(b) \qquad E = mt^n$$

gave, for the 10 per cent. Rh and Ir couples, the following values when calibrated at the freezing points of the metals indicated. Convincing evidence is at hand that formula (*a*) gives low values of temperature when extrapolated beyond 1200° C.

Melting Points with Thermocouples.

Metal.	Equation (*a*).	Equation (*b*).	
	Zn, Sb, Cu.	Zn, Cu.	Sb, Cu.
Palladium.	1530°	1544°	1542°
Platinum.	1706	1732	1730

The melting points of Pd and Pt were also determined by observing the black-body temperatures of ribbons of these metals at the instant of melting.

Melting Points Based on Platinum Radiation.

Metal.	Wave Length.	S° by Wien's Equation.	S° by Lamp Extrapolation.	Melting Point.
Pd	.652 μ	1389°		1563°
	.662		1386°	
	.547		1425	
	.462		1450	
Pt	.666	1534		1750
	.666		1544	
	.547	1578		
	.547		1593	
	.462		1627	

The melting point of Pd is higher by this method because the assumption is made that Pd and Pt have the same emissivity. Observations indicate that the values of $t^\circ - s^\circ$ increase less rapidly at high temperatures for Pd.

The present investigation[1] gives for the melting points of palladium and platinum, **1546°** and **1753°**, respectively, based on the extrapolations of the Wein-Planck equation, in which the constant c_2 is taken as 14500, and using as the melting point of gold, 1064°. If c_2 were taken as 14200,[2] these melting points would become about 1555° and 1760°. On account of the more satisfactory theoretical basis of the Stefan-Boltzmann law, which also gives extrapolated temperatures on the thermodynamic scale, it is planned to attack the problem from this standpoint.

WASHINGTON, D. C.,
February 19, 1907.

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[1] A more complete account of this investigation is given in Bulletin, Bureau of Standards, vol. 3, 1907.

[2] Holborn and Valentiner, Ann. d. Physik, 22, p. 1, 1907.

THE VAPOR PRESSURE OF SOLID CARBONIC ACID.

By H. du Bois.

IN 1899 I published with A. P. Wills some preliminary measurements relative to the thermometric and cryogenic applications of solid carbonic acid.[1] The object was to establish some approximate data with reference to practical applications of solid carbonic acid in the laboratory, and in no wise to attempt an exact final determination of the temperature-pressure function. On account of the return of Professor Wills to America, the proposed more accurate determinations referred to in the last part of the original article were not made.

Recently there have appeared two articles relative to this subject, one by J. Zeleny and R. H. Smith, and one by J. and A. Zeleny.[2] The $P\theta$-curve found lies about one degree higher than ours, and this is displaced toward the axis of pressure; instead of the value 5.5 cm. Hg. per degree for $dP/d\theta$ at atmospheric pressure found by us, the first of the above mentioned articles gives the value 6.35, and the latter the value 5.87, *i. e.*, 1 cm. Hg. corresponds to a temperature change of .17°.

In accordance with the above introductory remarks it is quite possible that such a difference might exist; also it is well known that the graphical determination of the slope of a curve with only a few given points is much more uncertain than the inverse graphical integration.

In a final determination of the constants it seems to me that the employment of pure carbonic acid is absolutely necessary, but the physicists mentioned above used the material as it is to be bought, as did likewise we ourselves. Our material was obtained through the burning of coke, and 99.5 per cent. at least was carbonic acid although there might have been traces of air, water and grease.

In America, as I am informed by reliable authorities, liquid carbonic acid is very frequently obtained from brewery gases and is not in general so pure as that obtained here, since it frequently unavoidably contains odoriferous material and the like, associated with the brewing process. It is scarcely possible to judge without further information whether, or in how far, such circumstances are adequate to account for the differences referred to above.

Berlin, November 5, 1906.

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[1] H. du Bois und A. P. Wills, Verh. D. Phys. Ges., I., 168, 1899.

[2] J. Zeleny and R. H. Smith, Phys. Zeitschr., 7, 667, 1906; J. und A. Zeleny, l. c., p. 716; Phys. Rev., 23, 308, 1906.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

NOTE ON THE HISSING METALLIC ARC.[1]

BY W. G. CADY.

THE arc between iron electrodes in pure nitrogen was investigated in order to determine whether the presence of oxygen plays the same part in causing the hissing stage of the iron arc as is the case with the carbon arc. It was found that the iron arc in nitrogen resembles the hissing rather than the quiet arc. No quiet state, properly speaking, could be obtained, as the arc went out whenever the current was decreased to the point where the positive electrode ceased to be vaporized. This suggests that the quiet state of the metallic arc may be defined as that in which the material of the anode is not heated to the boiling point. All observations on the iron arc in air, including rate of consumption of electrodes, temperature of oxid globules as determined by a thermo-element, and appearance of the spectrum of the arc between iron and copper, tend to confirm this view.

THE TEMPERATURE OF THE MERCURY ARC.[2]

BY CHAS. T. KNIPP.

THE purpose of this investigation was to determine the temperature of the mercury arc as recorded by Cu-Pt thermo-couples. The form of the tube was that used in the Hewitt lamp. The electrodes were replaced by mercury columns supported by atmospheric pressure. A third electrode (mercury column) was inserted midway between the terminals of the lamp. The tube was exhausted by a power Geryk pump which was kept running continuously while taking the observations. The temperature measurements were made by inserting the couples, each protected by a thin walled glass tube, up through the mercury columns and adjusting them to any desired height. The end couples were placed 2 cm. above the surfaces of the mercury. It was found that the galvanometer deflections increased steadily as the distance above the surface of the mercury increased. The form of the tube prevented making this distance greater than 2 cm. as it was desired to keep the three junctions on the axis of the tube. The length of the tube was 20 cm., and its diameter 3.8 cm.

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[1] This paper, presented at the meeting of March 2, 1907, is supplementary to one on 'The Hissing Point of the Metallic Arc" read at the meeting of the Physical Society, in New York, December 29, 1906. See PHYS. REV., p. 381, April, 1907.

[2] Abstract of a paper presented at a meeting of the Physical Society held December 27-29, 1906.

Adjusting the current to 4 amperes — which value caused the lamp to give out approximately the same illumination as in the commercial lamp — and a potential difference of 20.7 volts, the temperature as recorded by the junctions was 147° C. at the anode, 164° C. at the cathode, and 178° C. at the point midway between the anode and cathode. In general, for values of current ranging from 3 to 9 ampers, the cathode was found to be hotter than the anode, while the midway point was hotter than either. Again, the individual readings obtained at the cathode fluctuated much more than those obtained at the anode, while there was scarcely any variation from a smooth curve either for increasing or decreasing currents at the midway point.

ON THE CURRENT CARRIED BY CANAL RAYS IN A DISCHARGE TUBE.[1]

BY C. C. PERRY.

THE method employed in this work was that of allowing canal rays to penetrate into an earthed Faraday cylinder, one end of which served as kathode in the discharge tube. The rays were allowed to fall on an insulated brass electrode, which was connected to earth through a D'Arsonval galvanometer. The deflections of the galvanometer could then be used as a measure of that part of the total discharge current which was carried into the Faraday cylinder by the canal rays. The galvanometer circuit was so arranged by means of suitable mercury cup switches, as to first allow the "canal ray current" to be measured, and then the total current carried by the discharge. The ratio of the two, canal ray current to total current, was taken (for different gas pressures) as a measure of the relative numbers of canal ray particles present in discharges at these pressures, provided the discharge potential be kept constant.

Several such series of measurements were made with gas fillings of air, oxygen and hydrogen. When these measurements are plotted, with the ratios of canal ray current to total current as ordinates, and the gas pressures as abscissae, the curves in every instance exhibit a maximum ordinate for pressures which in every case observed were in excess of the pressures at which canal rays show their maximum fluorescent effects. (.035 to .04 mm. of Hg.). This pressure for the maximum production of canal rays as measured by the galvanometer, occurs always at pressures not far removed from those at which the first kathode ray fluorescence becomes visible, and appears to be different for different gas fillings.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

Several sources of error were investigated. Among these, the most important seemed to be, 1st, as to whether phenomena could be attributed to the diffusion of ions from in front of the kathode, and, 2d, as to whether the phenomenon of the production of secondary rays (soft kathode rays) as described by J. J. Thomson[1] and Austin[2] could account for the maximum in the curves.

The first of these was shown to be improbable by the introduction of a sufficient back e.m.f. in the circuit from the insulated electrode to earth, to preclude any diffusion phenomena. The second was tested by applying a magnetic field parallel to the surface of the insulated electrode, and of sufficient strength to turn completely back any secondary rays while not strong enough to disturb the main part of the discharge. In this way the observations of Thomson and Austin were confirmed, the effect observed was a diminution of the current from the electrode to earth when the field was on. This did not however change the general form of the curve, for if two sets of readings were taken, at the various gas pressures, one with the field on and one with it off, the corrected curve still showed a maximum slightly displaced from its former position. These secondary rays appear to begin at a perfectly definite gas pressure and hence a perfectly definite kinetic energy for canal rays in each gas tried, if the discharge potential is kept constant. The magnitude of the secondary effect increases with increasing mean free path.

The following conclusions have been deduced: 1°. Canal rays are present in a discharge tube in the greatest numbers at gas pressures so great that they do not acquire energy enough to be detected by fluorescence effects.

2°. The velocity at which this maximum occurs, appears to be connected in some way with the beginning of kathode rays in the discharge, and also to vary with the gas filling.

Further work is in progress to test if possible, the increase of kathode rays as the canal rays disappear, and also to measure the velocity of the canal rays at their point of maximum number.

PHYSICAL LABORATORY, SHEFFIELD SCIENTIFIC SCHOOL,
YALE UNIVERSITY, February 18, 1907.

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[1] J. J. Thomson, Proc. Camb. Phil. Soc., Vol. 13, part 4, p. 212, 1906.

[2] Austin, PHYS. REV., Vol. 22, pp. 312–319, 1906.

DEWITT BRISTOL BRACE

Volume XXIV. *June, 1907.* *Number 6*

THE

PHYSICAL REVIEW.

ON THE MAGNETIC PROPERTIES OF HEUSLER'S ALLOYS.

BY J. C. MCLENNAN.

SINCE the discovery by Heusler some three years ago that it was possible to prepare alloys of copper, manganese, and aluminium which possessed magnetic properties similar to and comparable in amount with those of cast iron, considerable attention has been paid by a number of investigators[1] to this interesting phenomenon.

Among others Mr. H. F. Dawes, M.A., and Miss L. B. Johnson, M.A., demonstrators in the Physical Laboratory at Toronto, and Messrs. H. A. McTaggart and J. K. Robertson, senior students in the same laboratory have from time to time during the past year, as opportunity offered, studied the properties of some of these alloys which they prepared, and the following paper contains a few notes on the results of their work, which is still being continued by two of them.

In order to prepare samples of the alloys a supply of manganese, copper, and aluminium, sold as chemically pure and free from iron was obtained from Messrs. Eimer and Amend of New York. The metals were melted in an ordinary coke furnace, such as is used in brass foundries, and poured into moulds just as in making brass castings. A graphite crucible was used for the purpose, and in the process the copper was first heated until it reached a molten state, after which the manganese was added to it. When this in turn was melted the aluminium was added, and the molten mass then stirred for a short time with a porcelain rod, after which the crucible was taken from the furnace and the melted metal poured into the moulds.

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[1] Fr. Heusler, W. Starck and E. Haupt, Verh. der Phys. Ges., 5, p. 219, 1903. Heusler, Starck and Haupt über der ferromagnetischen Aigenschaften von Legierung unmagnetischer Metalle, Marburg, 1904. Gumlick, Ann der Physik, 16, p. 535, 1905. Austin, Verh. der Deut. Phys. Gesell., 6, p. 211, 1904. Fleming and Hadfield, Proc. Roy. Soc., 76, p. 271, 1905. E. Take, Verh. der Deut. Phys. Gesell., 7, p. 133, 1905, Ann. der Physik, Vierte Folge, Band 20, p. 249. Hill, PHYS. REV., 21, p. 335, 1905; 23, p. 498, 1906. Guthe and Austin, Bulletin of the Bureau of Standards, Washington, Vol. 2, No. 2, page 297, Aug., 1906.

For a preliminary set of the alloys four castings were made. As Heusler had found that the most highly magnetic alloy contained manganese and aluminium in the proportion of their atomic weights, one casting was made with as nearly as possible this proportion of the two metals. A second was made with a larger proportion of aluminium and two others with smaller proportions of this constituent.

The percentages of the metals used in each of the four castings and the proportions of aluminium to manganese by weight and by atomic ratios are shown in Table I.

TABLE I.

Number of Alloy.	Composition of Alloys — Percentages.			Ratio of Manganese to Aluminium by Weight.	Atomic Ratio of Aluminium to Manganese.
	Aluminium.	Manganese.	Copper.		
I.	8.0	32.1	59.8	.250	.51
II.	9.7	25.6	64.6	.378	.77
III.	14.3	28.6	57.1	.498	1.01
IV.	15.9	23.9	60.3	.683	1.392

In each casting sufficient material was used to make two cylindrical rods about 20 cm. long, and 8 mm. in diameter, as well as two rings about 7 cm. in diameter, and 2.5 cm. in height. It was thought desirable to make the castings in the form of rings as well as rods, in order to investigate more fully the permeability of each specimen.

In the whole of the operation the greatest care was taken to keep the alloy free from iron in order that there might be none of this metal present to vitiate the results or to cast doubt upon their validity.

The investigations described below include one by Mr. Dawes on the phenomenon of magnetostriction exhibited by one set of the rods, one by Messrs. McTaggart and Robertson on the same phenomenon with the second set of rods, and one by Miss Johnson on the permeability of the alloys cast in the form of rings.

A. MAGNETOSTRICTION. (EXPERIMENTS BY MR. DAWES.)

I. *Apparatus.* — The apparatus used in measuring the elongation was, with some changes, modelled after that used by Rhoads[1] in his investigations on magnetostriction in iron. The apparatus is shown in section in the accompanying diagram, Fig. I. A long hollow solenoid *L* produced the magnetic field in which the rod to be tested was placed. The lower end of the rod *F* was screwed into the fixed brass stand *H*, and the → upper end carried a brass rod *E*. A cylindrical water jacket *K* placed between the coil and the rod served to keep the latter at a constant temperature. The change of length of the rod was magnified by means of an optical lever. A brass block *D* carrying a brass pivot indicated by the inverted arrow was clamped to the rod *E*, and so moved up or down as the rod *F* was elongated or shortened. A brass tube *G* which was attached to *H* at the base of *F* supported a horizontal projection *A*. *A* carried two brass pivots indicated by the second arrow. A lever *B* rested horizontally between these sets of pivots as indicated. At *C* a mirror was mounted so that it could rotate about an axis perpendicular to the plane of the paper and attached to *A*. This mirror was supported in a vertical plane by an attachment resting on the end of *B*. The brass points rested on agate beds and the axis was jewelled. The image of a scale as seen in *C* was

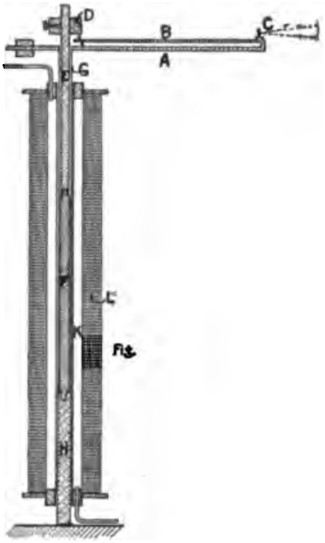

Fig. 1.

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[1] Rhoads, PHYS. REV., Vol. VII., No. 2, p. 65, 1898.

observed by means of a telescope. In order to trace the method by which the change of length was measured, suppose that the rod was slightly elongated relative to the tube *G*. The brass point carried by *D*, would then move upwards, and in consequence the *C*-end of the lever *B* would be allowed to drop. This would cause the mirror to be tilted backwards, and hence cause a change in the scale reading as observed by the telescope. It will be seen that if the change of length was very small, it would be proportional to the change of scale reading, and the proportionality factor could be found by measuring the ratio of the arms of the lever, the distance between the axis of rotation of the mirror and its point of support, and the distance of the scale from the mirror.

In the present experiment a modification was made in the apparatus as used by Rhoads. Formerly a plane mirror was used at *C* and the scale was placed at a distance of about 4 meters from it. This mirror was replaced in these experiments by a concave mirror of one meter radius of curvature, and the scale was placed at a distance from it of one half meter, the focal length of the mirror. By this means the image of the scale was magnified sixteen times its former size and the sensibility, *i. e.*, the change of reading for a given change of length was double that obtained with the plane mirror. A glass scale was used in the telescope instead of a cross hair, and the number of divisions on this scale over which a line of the image moved was taken as the reading. The sensibility of the arrangement was such that a change of reading of one division resulted from a change in length of the specimen of 10^{-5} mm.

II. *Measurements.* — In making measurements the change of scale reading was found for a series of applied fields ranging from about 30 to about 900 C.G.S. units. The strength of the applied field was calculated from the value of the current, and the known dimensions of the coil and the change of length per centimeter was deduced from the scale readings. For each series of readings curves were plotted, the strengths of field, *H*, being taken as abscissæ, and the changes per unit length multiplied by 10^7 as ordinates.

In the first few series of measurements, the readings were taken by applying each field for an instant and then withdrawing it.

By this procedure, an average result was obtained for the effect investigated, but the method was not suitable for bringing out any hysteresis effect which might be present. It was therefore discarded, and the method of increasing the field step by step and then reducing it in the same way was adopted.

It was found that a cycle of readings could be taken in this way in about two minutes.

When the current was kept passing through the coil it might be expected that the heating would effect the change of length. In order to test for errors from this cause, a brass rod was inserted in the apparatus in place of one of the specimens investigated, and a cycle of fields similar to that used in working with the alloys applied to it. As brass is non-magnetic and consequently does not exhibit the phenomenon of magnetostriction, the development of heat by the current and its diffusion into the walls of the water jacket would be followed by the elongation of *G*, Fig. 1, which would produce an apparent shortening of *D*. This test showed that during the time of a cycle of observations no error from heating was introduced.

Elongation curves for each of the four specimens are shown in Figs. 2–5, the corresponding sets of values being given in Table II. Curves 1 and 3 *A* were taken by the first method, the others by the method of cycles.

Table II.

Alloy No. 1.

H	$\frac{dl}{l}\cdot 10^7$	H	$\frac{dl}{l}\cdot 10^7$
33	0	220	.3
50	0	900	.5
130	.1		

Alloy No. 2.

33	0	495	– .5
95	.15	375	– .75
225	.25	335	–1.0
335	.25	286	–1.0
375	.25	225	–1.25
491	–.25	95	–1.5
	–.5		
945	–.25		

Alloy No. 3. Curve A.

22	.25	315	3.8
44	.5	352	4
67	.6	385	4.2
90	.8	435	4.5
124	1.0	456	5.5
192	2.0	980	9
255	2.3		
276	2.5		

Alloy No. 3. Curve B.

33	.5	470	32
56	1.4	385	31
79	4.4	351	30.4
100	8	310	30
143	15	260	29
215	20	215	27.5
260	23.2	143	22.8
310	26	100	19
351	27	79	14
385	27.4	56	9.5
470	27.5	33	5.8
910	33	0	3

Alloy No. 4. Curve A.

33	.5	912	3.2
56	1	475	3
85	1.5	375	3
110	2	357	2.5
143	2.4	320	2.3
220	2.5	275	1.5
275	2.5	221	1
320	2.5	143	.5
385	2.5	110	0
475	2.6	85	−2.5

Alloy No. 4. Curve B.

33	2	490	11.5
56	5.5	400	11.5
90	7.25	332	11.5
111	8.5	221	11
170	9.5	170	11
221	11	110	11
332	11.5	90	10.5
480	11.5	56	10
940	23	33	16
		0	3

III. *Discussion.* — A careful examination of these and the other curves found in the series of measurements showed that three factors entered into the change of length of the bar.

1. When a change was made in the field to which the bar was subjected an almost instantaneous change was made in the length. This change consisted in a lengthening as the field was increased, and a shortening as it was decreased.

2. As the field was decreased there was a certain lag or hysteresis effect so that the length did not have the same value for a given field as the field was approached from above or below.

3. In some cases when the specimen was left in a field for some time there was a continuous shortening. This effect when it existed was very marked in strong fields so that the time for which the specimen was left in these fields had a comparatively great influence on the length. This effect was also observed by Austin in some experiments made by him with these alloys. It does not appear, however, to have been found by him in some measurements he has recently made, in collaboration with Guthe, with other samples of the alloys.

The form of the curve arising from a cycle of readings on a given specimen would depend, therefore, on the relative importance of these three factors. In the early stages of an increasing field there would be no effect from (2) and that from (3) would be small so that the initial portions of the elongation curve would correspond to the change of length arising from the first source. In each of the curves shown, this part of the curve exhibits the same character. The curve rises slowly in the very weak fields and then in fields extending from about 50 to 200 units quite rapidly. In still higher fields the rise becomes smaller and tends to a maximum value.

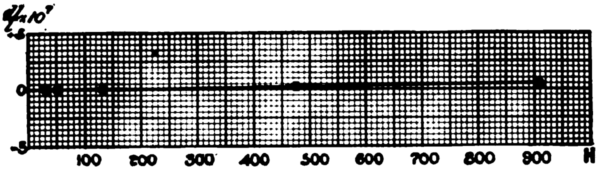

Fig. 2.

In the case of alloy No. 2, Fig. 3, it will be seen that while the actual changes were small compared with those in alloys No. 3 and No. 4, the third factor was of more importance than the first, with

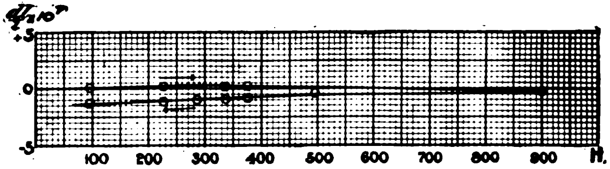

Fig. 3.

the result that in the higher fields the bar diminished a little below its original length. This was also observed in the case of alloy No. 1, although it is not shown in the curve as the experimenter failed to keep a record of his measurements.

In curves 3, *B* and 4, *B*, Figs. 4 and 5, this cause of change of length was not sufficiently great to make the return half of the curve cut the forward half, *i. e.*, to overcome the hysteresis effect. But if the strong fields had been left on a little longer this would probably

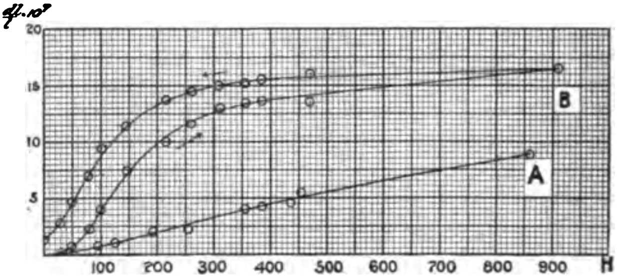

Fig. 4.

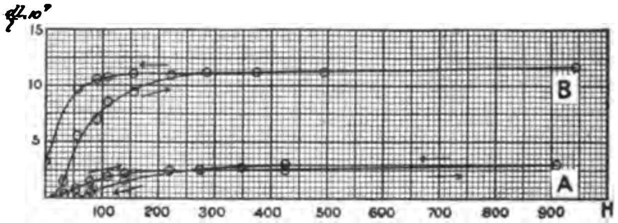

Fig. 5.

have been the case and there would have been a final resultant contraction. An example of this effect is shown in 4, *A*, Fig. 5, where the return portion of the curve crosses the forward initial portion and consequently masks the presence of the hysteresis effect shown in 4, *B*.

In Figs 4 and 5, curves *A* represent the elongation obtained with two of the alloys when magnetized for the first time, and curves *B*, in the same figures represent the elongations obtained with the same rods after they had been magnetized and demagnetized several times, and then allowed to lie undisturbed for about thirty hours.

This increase in the elongation has not hitherto been observed, and it may possibly be connected with the increase of permeability observed by Starck and Haupt,[1] Gumlich,[2] Hill[3] and others with these alloys when they were heated for some time to a moderate temperature, such as a 110° C. after being cast. It may possibly, too, point to changes in the constitution of the alloy analogous to those marking the recovery of iron from overstrain observed by Muir,[4] on heating strained iron bars for short periods in boiling water.

Considering the curves as a whole it will be seen that the elongation increased with the percentage of aluminium, present in the alloy, the maximum elongation, curve *B*, Fig. 4 being obtained with the alloy in which the manganese and aluminium were present in the proportions of their atomic weights.

The maximum elongation in this case, it will be seen, was 16 ten-millionths or about one third the maximum for iron.

The effect of higher percentages of aluminium is shown in Fig. 5, curve *B*, where the maximum elongation was 11.75 ten-millionths.

Chemical analyses of the alloys have not as yet been made, but if the constituents of the different samples of the alloys can be taken to have the same relative proportion as the metals from which they were cast, these results would go to show that the maximum elongation was obtained with alloys containing the same proportions of manganese and aluminium as were found by Heusler to give the highest permeability.

[1] Deutsch. Phys. Gesell. Verh., 5, 12, pp. 224-232, June, 1903.

[2] Electrotech. Zeit., XXVI., 9, Mar., 1905.

[3] Hill, Phys. Rev., Vol. XXIII., No. 6, p. 498, 1906.

[4] Phil. Trans. Roy. Soc., London, Vol. 193, pp. 1-46, Series A.

B. MAGNETOSTRICTION. (EXPERIMENTS BY MESSRS. H. A. MCTAGGART AND J. K. ROBERTSON.)

In the experiments made with the first set of rods a very marked difference was observed in the elongations obtained with alloys Nos. 3 and 4, when magnetized for the first time, from what was obtained with them after they had been magnetized and demagnetized several times. This result pointed to the existence of an unstable equilibrium among the molecules of newly cast specimens of the alloys, and it forms a corroboration of the opinions of other experimenters who have arrived at the same conclusions from their observations on the changes in permeability of the alloys produced by variations in temperature.

TABLE III.

Alloy No. 2.

H	$\frac{dl}{l} \cdot 10^7$	H	$\frac{dl}{l} \cdot 10^7$
235	0.8	300	−0.8
370	1.7	218	−1.7
531	2.5	155	−2.5
675	2.5	0	−3.3
440	0		

Alloy No. 3.

55	1.6	580	13.3
96	3.3	376	12.5
130	4.2	255	10.8
198	6.6	202	10
255	8.3	155	8.3
272	9.2	118	6.6
376	10.8	75	4.2
474	12.5	0	−1.6

Alloy No 4.

32	4.2	255	15.8
62	8.3	180	15
96	10.8	125	14.2
145	12.5	84	12.5
202	14.2	55	10
250	15	40	8.3
355	15.8	10	4.2
474	16.7	0	0
595	17.1		

It seemed desirable to investigate this point more closely, and therefore a set of measurements was made by the above mentioned senior students on the elongations of the second set of rods cast at the same time as those used by Mr. Dawes. Before proceeding with this however, a set of readings was taken with the rods used by the latter to see whether any change had taken place in them in the six months interval during which they had been laid aside. The results of their measurements are given in Table III. and represented graphically by the curves in Figs. 6–8.

With alloy No. 1 no elongation whatever could be observed by them. With alloy No. 2, Fig. 6, the general form of the curve they obtained is similar to that given in Fig. 3, by Mr. Dawes, although the maximum elongation they obtained in increasing fields and the maximum shortening in decreasing fields was greater than that observed by him with this specimen.

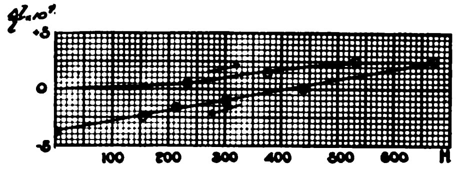

Fig. 6.

Their results with alloy No. 3, Fig. 7 show the presence of the three effects pointed out in the first set of measurements, the hysteresis and the shortening being clearly marked. The maximum elongation obtained by them with this specimen corresponded to a field of 600 units and is slightly less than that obtained in the first tests for the same field.

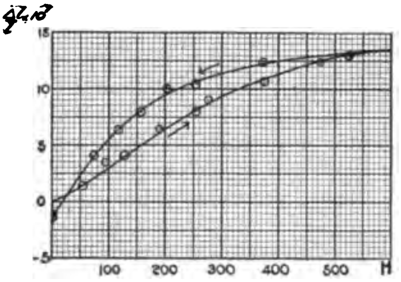

Fig. 7.

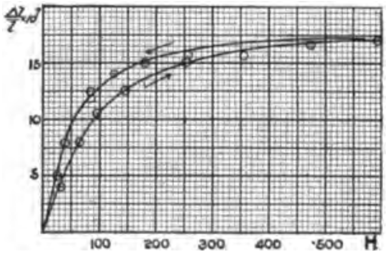

Fig. 8.

With alloy No. 4, Fig. 8, the elongation was about 50 per cent. greater than that shown in Fig. 5. In both curves there exists a well marked lag, but in the later measurements, the rod returned to its original length whereas in the earlier ones it showed an elongation when the field was removed.

It is evident from these measurements that considerable modifications must have taken place in the structure of the alloy in the interval which elapsed between the two tests, and that the lapse of time is a factor as well as changes in temperature and magnetization in producing alterations in the molecular groupings in the alloy.

TABLE IV.

Alloy No. 2A_1.

H	$\frac{dl}{l} \cdot 10^7$	H	$\frac{dl}{l} \cdot 10^7$
0	3.3	445	2.9
174	3.3	330	2.9
425	3.75	284	2.5
745	4.1	180	1.7
670	3.75	104	0.8
635	3.3		

Alloy No. 2A_2.

118	0.8		
280	1.7		
705	2.1		
425	1.7		
155	0.8		
0	0		

Alloy No. 3A_1.

42	2.5	910	27.5
86	8.3	600	25
138	12.5	405	23.3
180	15	320	21.7
215	16.7	280	20.8
312	20	215	19.2
390	21.7	121	15
460	22.5	75	8.3
545	24.2	42	5.8
705	25	0	4.2
890	26.7		

TABLE IV.—*Continued.*

Alloy No. $3A_2$.

50	2.5	705	20.8
75	4.2	880	22.5
104	6.7	635	20.8
125	8.3	405	19.2
155	10	270	16.7
185	12.5	175	12.5
238	14.2	104	8.3
320	16.7	60	4.2
398	18.3	0	0
495	19.2		

Alloy No. $3B_1$.

53	1.7	740	23.3
80	4.2	592	22.5
110	5.8	425	20.8
138	8.3	370	20
192	12.5	326	19.2
225	14.2	284	17.5
277	16.7	255	16.7
348	18.3	208	15
372	19.2	174	12.5
458	20.8	118	8.3
565	21.7	68	4.2
610	22.5	0	1.7

Alloy No. $3B_2$.

55	1.7	545	21.7
85	4.2	390	20.8
134	8.3	348	20
187	12.5	327	18.3
275	16.7	255	16.7
390	19.2	187	12.5
480	20.8	125	8.3
665	22.5	68	4.2
760	23.3	0	0

Alloy No. $4A_1$.

112	3.3	565	7.5
208	4.2	452	7.5
270	5	320	7.5
362	5.8	215	7.5
390	6.7	103	6.7
478	6.7	0	5
705	7.5		

TABLE IV.—*Continued.*

Alloy No. $4A_2$.

50	1.7	362	2.9
110	2.5	255	2.5
362	3.3	118	2.5
552	3.4	70	2.1
610	3.4	42	1.25
590	2.9	0	0

The measurements made with the duplicate set of rods are given in Table IV., and curves representing them are given in Figs. 9, 10, 11, 12, 13, and 14. For convenience in description the rods used in this investigation are designated as No. 1, *A*, No. 2, *A*, No. 3, *A*, and No. 4, *A*, and where subscript figures are used, for example in No. 3, A_1, and in No. 3, A_2 they represent the first and second tests respectively made with the specimen. Alloy No. 3, *B* is a rod of another casting made over a year ago from the same metals and with the same proportions as the pair No. 3 and No. 3, *A*.

With No. 1, *A* no elongation was observed which accorded exactly with the behavior of the twin rod No. 1.

With No. 2, *A*, Fig. 9, the results obtained were very similar to those obtained with rod No. 2. Curve No. 2, A_1 represents the elongations obtained with the first magnetization. As the field was gradually increased to 750 units the rod steadily lengthened, and as the field was decreased the rod shortened, and finally when the field was removed entirely it remained slightly shorter than its original length. It was then repeatedly magnetized and demagnetized and afterwards put through the usual cycle of fields. The curve No. 2, A_2 represents its behavior after this treatment. The

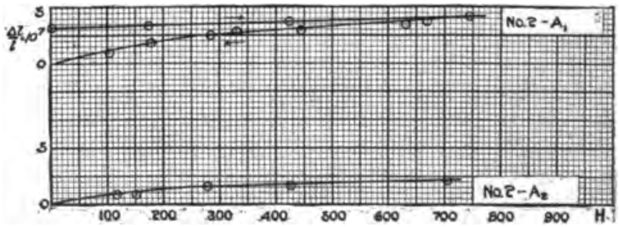

Fig. 9.

rod as will be seen had assumed a steady state, and the upward and downward curves coincided exactly. No hysteresis action or shortening under high fields was observed.

With alloy No. 3, *A* the measurements made with the first magnetization are given in Fig. 10. Here it will be seen a lag was obtained in the return portion of the cycle, the rod ending with a slightly greater length than it had originally.

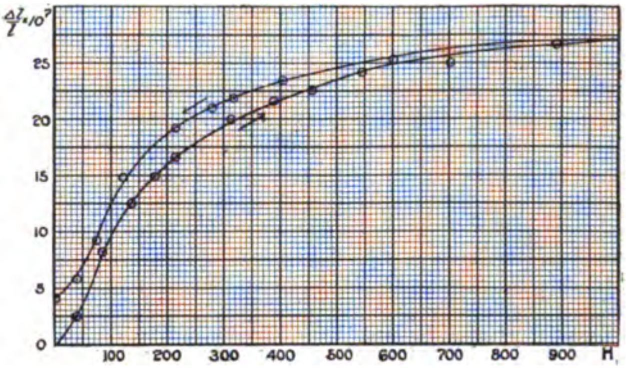

Fig. 10.

Curve No. 3, A_2, Fig. 11, shows its behavior after being repeatedly magnetized. Here again a lag was observed in the return portion of the cycle, but the shortening referred to under heading No. 3 page 455 had entirely disappeared, so that the initial and final lengths of the rod were the same.

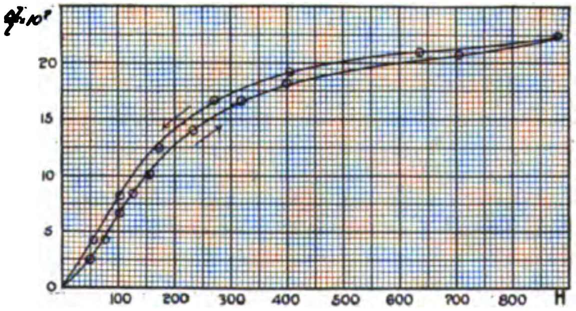

Fig. 11.

The first and second tests with alloy No. 3, *B* are shown in Figs. 12 and 13. Here again it will be seen, precisely the same behavior was obtained as with No. 3, *A*. As with the latter a lag was again observed in the return portion of the cycle and a final lengthening of the specimen occurred. In the second tests, Fig 13, made after repeated magnetizations, this change of length disappeared, and the rod after being put through its cycle returned to its original length.

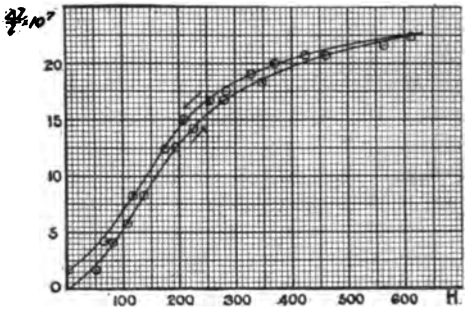

Fig. 12.

With alloy No. 4, *A*, the behavior was exactly the opposite of that obtained with No. 2, *A*. In the initial test, No. 4, A_1, Fig. 14, the rod steadily lengthened as the field was increased to 700 units and afterwards as the field was decreased a lag occurred and the rod ended somewhat longer than it was at the commencement of the cycle. After it had been repeatedly magnetized and demagnetized it was again put through a cycle, and curve No. 4, A_2 was obtained. Here again the values in the downward portion of the cycle coincided exactly with those obtained in the forward portion and no lag and no shortening was observed.

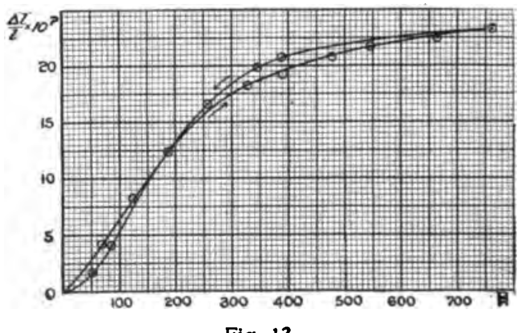

Fig. 13.

In the measurements with these rods, the greatest elongation was obtained with samples No. 3, *A* and No. 3, *B*. With them the maximum elongation observed was about 22.5 ten-millionths which is somewhat greater than that obtained with the specimen No. 3 where the maximum elongation obtained was 16 ten-millionths.

The maximum elongation obtained with rod No. 4, *A* was somewhat less than that obtained by Mr. Dawes with rod No. 4. With the former specimen the maximum elongation was about 3 ten-millionths while that obtained with the latter was over three times as great. This discrepancy was very likely due to a difference in the composition of the two rods which should be brought out by a chemical analysis of the specimens. It is interesting to note that with both sets of rods the maximum elongation was obtained when the composition of the alloy was approximately the same as that of the specimens which were found by Heusler to give the highest permeability.

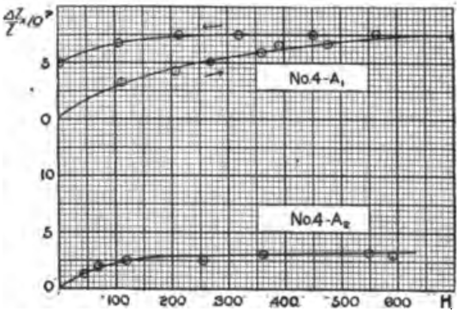

Fig. 14.

Summarizing the results of the two measurements it would appear that the structure of the alloys when freshly cast is very unstable, and that it undergoes a rapid change under repeated magnetizations and a slower change under the lapse of time.

In their experiments on magnetostriction Guthe, and Austin state that no after-effect either in expansion or in contraction was observed even with the strongest fields, whereas Austin in his first communication refers very definitely to the observation of such an effect in his measurements. The explanation probably is that in the first investigations the rods were in the same condition as those with which the initial tests described in this paper were made; while in the later investigation the rods tested (which were obtained from Dr. Heusler) had very likely been subjected to such conditions prior to the test as to bring them into the stable condition.

III. PERMEABILITY. (EXPERIMENTS BY MISS L. B. JOHNSON.)

During the period covered by the experiments on magnetostriction, a series of measurements was made by Miss L. B. Johnson on the permeability of the four alloys. Two rings had been made at each casting, in addition to the two rods mentioned above, and with these the experiments on permeability were carried out. The magnetization was measured by the ballistic method, by reversing the magnetizing force, and the arrangement of apparatus was similar to that used by Ewing and Klaasson.[1]

TABLE V.

Designation of Ring.	Height.	Thickness.	Mean Diam.	Cross Section.	Turns in Primary.	Turns in Secondary.
No. I.	1.56 cm.	.881 cm.	6.59 cm.	1.374 sq.cm.	155	100
No. II.	1.47 "	.855 "	6.615 "	1.256 " "	153	100
No. III*a*.	2.44 "	.623 "	6.98 "	1.516 " "	150	50
No. III*b*.	2.37 "	.68 "	6.87 "	1.61 " "	115	50
No. IV*a*.	2.39 "	.703 "	6.86 "	1.684 " "	157	50
No. IV*b*.	1.447 "	.838 "	6.64 "	1.212 " "	136	46

TABLE VI.

Alloy No. I.

H	B	μ	H	B	μ
1.39	1.39	1.0	14.40	18.7	1.3
2.82	4.01	1.4	28.22	37.8	1.3
3.93	5.35	1.3	37.63	50.8	1.3
7.52	10.7	1.4	86.45	78.9	1.3

TABLE VII.

Alloy No. II.

H	B	μ	H	B	μ
.814	1.464	1.7	17.5	27.8	1.5
1.66	2.92	1.7	42.5	68.8	1.6
4.99	8.78	1.7	69.3	106.8	1.5
8.55	13.17	1.5			

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[1] Phil. Trans., Vol. CLXXXIV., A, p. 987, 1893.

TABLE VIII.

Alloy No. IIIa.

H	B	μ	H	B	μ
.68	6.442	9.3	17.1	201	11.6
1.8	17.1	9.8	28.3	332.8	11.7
4.4	46.1	10.3	36.9	364	9.8
7	71.2	10.1	42.9	413.8	9.6
11.9	129.8	10.8	59.3	483.1	8.1

The dimensions of the rings which have been designated as I., II., III*a*, III*b*, IV*a*, IV*b* for the purpose of identifying them with the rods which were denoted by the same numbers, are given in Table V. The table also contains the number of turns in the primary and secondary coils of each ring.

TABLE IX.

Alloy No. IIIb.

H	B	μ	H	B	μ
.99	4.596	4.6	11.24	70.17	6.2
1.45	9.138	6.3	23.4	160.8	6.8
2.04	11.42	5.5	30.81	221.5	7.1
3.08	18.27	5.9	50.75	352.1	6.9
6.01	34.26	5.7			

The first set of measurements was made with the rings, just as they were cast, and the results are given in Tables VI., VII, VIII., IX., X., XI. The permeability curves are given in Fig. 15.

TABLE X.

Alloy No. IVa.

H	B	μ	H	B	μ
.2745	13.1	48.1	8.692	565.7	65.1
.5032	30.5	58	16.47	933.9	56.7
1.464	96.1	65.6	34.31	1487.2	43.3
2.379	157.2	66.1	45.29	1724.08	38
4.575	314.5	68.7	69.54	2072.3	29.8

TABLE XI.

Alloy No. IVb.

H	*B*	*μ*	*H*	*B*	*μ*
.0379	3.108	82	16.45	1366.93	83.0
.147	13.67	92.9	27.14	2113.44	77.8
.452	49.72	110	34.55	2257.76	65.4
1.192	136.75	114.7	41.77	2424.24	58
2.467	279.72	113.3	51.00	2548.4	49.8
3.331	360.52	108.2	55.93	2586.4	44.4
5.264	535.81	101.7	80.61	2735.04	33.8
7.81	739.70	94.7	92.13	2735.04	24.6

From these it will be seen that Alloys IV*a* and IV*b* showed the highest permeability. Alloys I. and II. were but feebly magnetic, and Alloys III*a* and III*b* only slightly so.

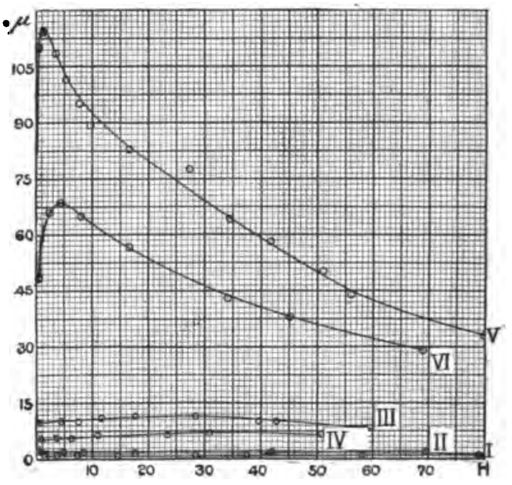

Fig. 15.

It had been expected that the permeabilities of the rings would give an indication of the relative permeabilities of the corresponding rods used in the measurements on magnetostriction. The very considerable difference in the permeability of the two rings No. IV*a* and No. IV*b*, which were made at the same casting, showed, however, that even different portions of the same casting did not exhibit the same magnetic properties, and that before any definite comparisons could be made, it was necessary to make a chemical analysis of each of the specimens tested. This difference in magnetic properties was also very marked with alloy No. III. The rods of this casting gave the greatest elongation, while the rings, as the figures show, possessed but a small permeability. Guthe and Austin[1] have pointed out that the greatest elongation was obtained by them with rods which possessed the greatest permeability and as some experiments which are now being made on the permeability of the rods No. III. show that they possess a very much higher permeability than that exhibited by the rings, this conclusion is very probably correct.

TABLE XII.

Alloy No. IIIa.

H	*B*	*μ*	*H*	*B*	*μ*
.73	8.59	11.9	17.1	309.2	17.9
1.28	15.03	11.7	27.5	440.1	15.9
1.97	25.77	13	36.1	541.1	14.9
6.4	94.4	14.7	46.4	674.3	14.5
6.8	105.2	15.3	64.5	751.6	11.6
8.5	138.2	16.2	91.1	843.9	9.2
11.5	196.5	17			

Alloy No. IVa.

H	*B*	*μ*	*H*	*B*	*μ*
.164	9.83	59.9	8.88	758.02	85.3
.814	65.33	80.5	16.4	1187.42	72.4
1.41	121.02	85.8	34.31	1857.5	54.1
2.33	203.15	87.1	43	2090.1	48.8
4.3	415.05	96.7	71.32	2402.6	33.6
6.03	538.26	89.2			

Alloy No. IVb.

H	*B*	*μ*	*H*	*B*	*μ*
.7621	10.567	13.9	24.38	528.22	21.6
1.258	28.172	16.3	31.24	654.99	20.9
3.35	56.34	16.4	44.2	795.85	18
6.09	116.2	19	58.68	993.06	16.9
9.9	211.29	21.3	86.11	1253.6	14.55
15.24	352.15	23.1			

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[1] Loc. cit.

In addition to the initial tests just described, some measurements were also made on the changes in the permeability produced by continued heating.

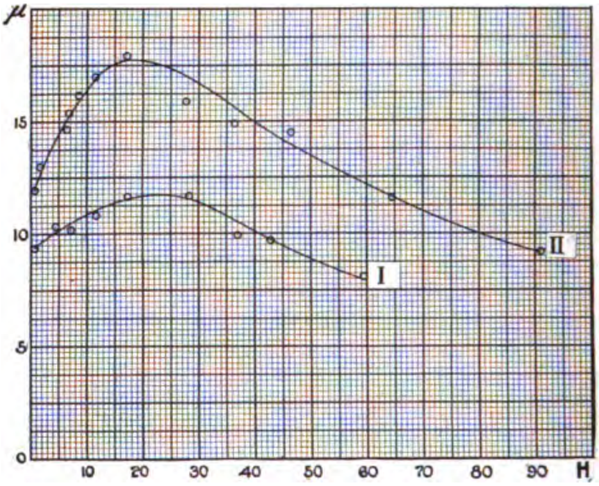

Fig. 16.

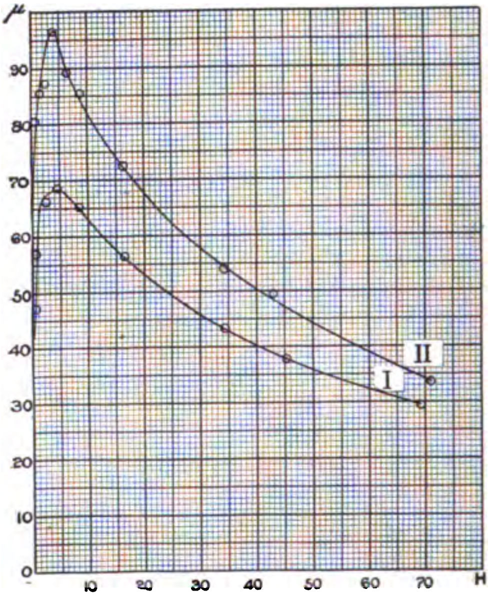

Fig. 17.

Examples of these measurements with alloys No. III*a* and No. IV*a* and No. IV*b*, are given in Table XII.

The values of the permeability of alloy No. III. were obtained when the specimen had become cool after being heated to 107° C. and kept at this temperature for 40 hours. A permeability curve representing the results is given in Fig. 16, and a comparison with the curve made in the initial test which is plotted in the same figure shows that although the magnetic properties of the specimens were not very marked the heating nevertheless produced a very considerable increase in the permeability.

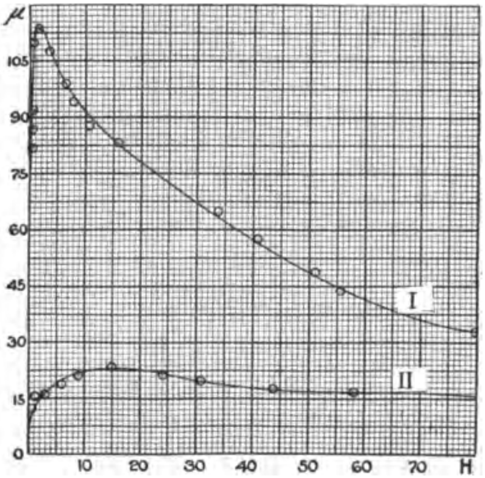

Fig. 18.

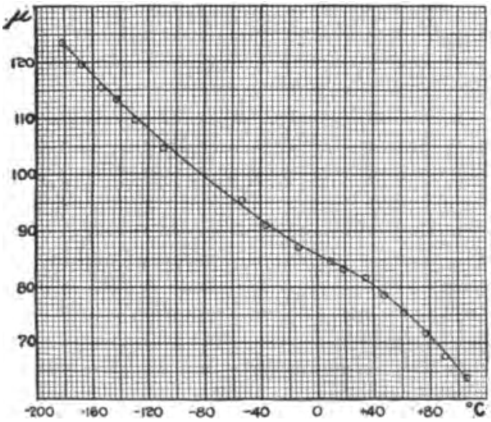

Fig. 19.

This increase in the permeability is also shown by the numbers for alloy No. IV*a*. Curve I., Fig. 17, shows the permeability of the alloy in different fields when freshly cast, and curve II. its permeability in the same fields after it had been repeatedly heated up to 100° C. and cooled to the temperature of liquid air.

The results obtained with alloy No. IV*b* show the effect of raising the alloy to a red heat and then allowing it to cool slowly. This specimen in the initial tests showed the highest permeability of all the rings, but after being treated as described a very considerable decrease in the permeability ensued. Curves I. and II. represent respectively the initial and final tests with this specimen.

In the interval between which the results with alloy IV*a* recorded in Fig. 17 were obtained, this specimen was examined under a constant magnetizing force of 3.934 C.G.S. units for temperatures ranging from — 182° C. to 105° C. The temperatures were estimated from variations in the resistances of a platinum wire wound round the ring between the primary and secondary coils. The results of these measurements are given in Table XIII. and a curve representing them is given in Fig. 19.

TABLE XIII.

Alloy No. IVa. $H = 3.934$ *C.G.S. units.*

Temperature.	B	μ	Temperature.	B	μ
−182	487	123.7	0	338.5	86
−169	471.8	119.9	8.2	334.2	84.9
−154	454.3	115.4	19.6	327.6	83.2
−143.5	445.6	113.2	35.3	321.1	81.6
−131	432.5	109.9	48	308.8	78.5
−110.5	410.6	104.3	63.1	297	75.5
−53	375.7	95.4	69	288.3	73.2
−46	364.8	92.7	76.4	281.8	71.6
−36	358.2	91	91.6	266.5	67.7
−13.5	342.9	87.1	105	521.21	63.8

The permeability, it will be seen, showed a steady increase as the temperature was lowered.

The results of these various tests, it will be seen are in accordance with the observations of other investigators, who have found that the permeability of these alloys can be considerably increased by continued heating at moderate temperatures.

The results obtained with alloy No. IV*a* lends support to the view expressed by Hill, that when these alloys are raised to a temperature higher than the transformation point, and then cooled, they possess a permeability which is largely determined by the temper ature from which the cooling took place. This alloy when cooled from the melting point as curve I., Fig. 18, shows, possessed a permeability very much higher than that which it had when cooled from a red heat.

This suggests a parallel between the behavior of these alloys and those studied by Heycock and Neville.[1] In their investigations it was found that the structure of an alloy at any temperature could be ascertained very closely by suddenly chilling it from that temperature. It is possible that these alloys under varying conditions of temperature, may pass through different phases, some of which may be magnetic, and others not. From the general results of the present investigations it is evident that the magnetic properties of the different alloys are intimately associated with their molecular structure, and since the structure of an alloy at any temperature can be ascertained by rapid cooling from that temperature, it is possible that the magnetic properties of the alloy may also be investigated by the same procedure. The results obtained by Hill and other investigators would seem to point in that direction.

Physical Laboratory,
University of Toronto, December 20, 1906.

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[1] Rapports Congrès Int. de Phys., Paris, 1900, p. 131.

SOME NEWLY OBSERVED MANIFESTATIONS OF FORCES IN THE INTERIOR OF AN ELECTRIC CONDUCTOR.[1]

BY EDWIN F. NORTHRUP.

SOME months ago, my friend, Carl Hering, described to me a surprising and apparently new phenomenon which he had observed. He found, in passing a relatively large alternating current through a non-electrolytic, liquid conductor contained in a trough, that the liquid contracted in cross-section and flowed up hill lengthwise of the trough, climbing up upon the electrodes. With a further increase of current, he found that this contraction of cross-section became so great at one point that a deep depression was formed in the liquid with steeply-inclined sides like the letter V. This depression extended in the case of a liquid metal as deep as six inches. With a current of constant value, the condition was a stable one, but the liquid on the inclined surfaces showed great agitation. With a still greater increase of current, the depression reached the bottom of the trough, thereby rupturing the circuit; this, of course, resulted in the liquid flowing together again, and again breaking, a violent interrupter being thus formed. He thus concluded that there is a limit to the current which it is possible to pass through a liquid conductor and that this limit may be reached before the volitalization limit. He suggested that the blowing of fuse wires and the action of the Caldwell interrupter may, perhaps, be aided by this phenomenon. Mr. Hering suggested the idea that this contraction was probably due to the elastic action of the lines of magnetic force which encircle the conductor, which lines, he said, acted on the conductor like stretched rubber bands, tending to compress it, especially at its weakest point. As the action of the forces on the conductor is to squeeze or pinch it, he jocosely called it the "pinch phenomenon."

The phenomenon described appeared to the writer to have a considerable theoretical interest. The theory and the experiments de-

scribed in the following are the results of the writer's analysis of the problem. In the development of the theory which follows, he has been much assisted by the discussions which he had with Mr. Hering who offered many useful suggestions.

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[1] A paper presented at the New York meeting of the Physical Society, March 2, 1907.

Depression Obtained with a Small Current.

To show the phenomenon observed by Mr. Hering, the writer devised several laboratory experiments in which mercury was used as the liquid conductor. In a thick sheet of hard rubber, about five inches square, two square holes or depressions were cut. These depressions were about three quarters of an inch deep and about one and three quarter inches on a side. Their two adjacent sides were parallel and about one half inch apart. A channel one quarter inch wide and three quarter inch deep was cut so as to connect the depressions. The depressions and channel were nearly filled with mercury. Electrodes of brass placed in the depressions enabled a storage battery current to be passed in the mercury from one depression to the other through the narrow channel.

It was found that about 800 amperes caused the mercury in the channel to depress lengthwise of the channel in the form of a V, about one half inch below the general level of the mercury. A small increase in the current caused the angle of the depression to reach the bottom of the channel and rupture the circuit. The liquid would again flow together and again rupture thus forming a slow and irregular interrupter.

The following experiment was then devised by which the forces at work could be shown in a more striking manner and with much smaller currents.

The apparatus shown in Fig. 1 was constructed. This is a box made of wood. The box is divided as shown into two rectangular compartments which are connected by a deep channel. One side of the channel was formed of a sheet of transparent mica, so the height of any liquid in the box could be observed from the side. In each end of the box were fastened brass electrodes.

The interior of this box, thus arranged, was filled to a depth of two inches with a liquid alloy of potassium and sodium. The remaining space was filled with kerosene oil. The liquid alloy

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scribed in the following are the results of the writer's analysis of the problem. In the development of the theory which follows, he has been much assisted by the discussions which he had with Mr. Hering who offered many useful suggestions.

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[1] A paper presented at the New York meeting of the Physical Society, March 2, 1907.

Depression Obtained with a Small Current.

To show the phenomenon observed by Mr. Hering, the writer devised several laboratory experiments in which mercury was used as the liquid conductor. In a thick sheet of hard rubber, about five inches square, two square holes or depressions were cut. These depressions were about three quarters of an inch deep and about one and three quarter inches on a side. Their two adjacent sides were parallel and about one half inch apart. A channel one quarter inch wide and three quarter inch deep was cut so as to connect the depressions. The depressions and channel were nearly filled with mercury. Electrodes of brass placed in the depressions enabled a storage battery current to be passed in the mercury from one depression to the other through the narrow channel.

It was found that about 800 amperes caused the mercury in the channel to depress lengthwise of the channel in the form of a V, about one half inch below the general level of the mercury. A small increase in the current caused the angle of the depression to reach the bottom of the channel and rupture the circuit. The liquid would again flow together and again rupture thus forming a slow and irregular interrupter.

The following experiment was then devised by which the forces at work could be shown in a more striking manner and with much smaller currents.

The apparatus shown in Fig. 1 was constructed. This is a box made of wood. The box is divided as shown into two rectangular compartments which are connected by a deep channel. One side of the channel was formed of a sheet of transparent mica, so the height of any liquid in the box could be observed from the side. In each end of the box were fastened brass electrodes.

The interior of this box, thus arranged, was filled to a depth of two inches with a liquid alloy of potassium and sodium. The remaining space was filled with kerosene oil. The liquid alloy

SOME NEWLY OBSERVED MANIFESTATIONS OF FORCES IN THE INTERIOR OF AN ELECTRIC CONDUCTOR.[1]

By Edwin F. Northrup.

SOME months ago, my friend, Carl Hering, described to me a surprising and apparently new phenomenon which he had observed. He found, in passing a relatively large alternating current through a non-electrolytic, liquid conductor contained in a trough, that the liquid contracted in cross-section and flowed up hill lengthwise of the trough, climbing up upon the electrodes. With a further increase of current, he found that this contraction of cross-section became so great at one point that a deep depression was formed in the liquid with steeply-inclined sides like the letter V. This depression extended in the case of a liquid metal as deep as six inches. With a current of constant value, the condition was a stable one, but the liquid on the inclined surfaces showed great agitation. With a still greater increase of current, the depression reached the bottom of the trough, thereby rupturing the circuit; this, of course, resulted in the liquid flowing together again, and again breaking, a violent interrupter being thus formed. He thus concluded that there is a limit to the current which it is possible to pass through a liquid conductor and that this limit may be reached before the volitalization limit. He suggested that the blowing of fuse wires and the action of the Caldwell interrupter may, perhaps, be aided by this phenomenon. Mr. Hering suggested the idea that this contraction was probably due to the elastic action of the lines of magnetic force which encircle the conductor, which lines, he said, acted on the conductor like stretched rubber bands, tending to compress it, especially at its weakest point. As the action of the forces on the conductor is to squeeze or pinch it, he jocosely called it the "pinch phenomenon."

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This was proved experimentally as follows: In the apparatus shown in Fig. 4, a fine manganin wire of high resistance was placed in the liquid column. One end of the wire was fastened to the upper copper disk near its circumference and the other end to the corresponding point of the lower copper disk. The wire was slightly longer than the distance between the disks and hence hung loose between them. It was arranged to have it hang near the circumference of the liquid column. When current passed through the liquid column a current in the same direction and of greater current density also passed through the wire. On making the circuit, the loose wire was instantly urged from the circumference and formed a bow which bent toward the axis of the liquid column.

In place, now, of an actual wire in the inside of the liquid column, we may picture this liquid column as being made up of a great many filaments parallel to the column. Each filament will be a conductor which carries a current proportional to the total current and to its own cross-section. All the imaginary conducting filaments, except the one coinciding with the axis, will be in a field of magnetic force, the intensity of the field in which any filament lies being proportional to its distance from the axis. All the filaments, except the one on the axis, will experience a force which, if free to act, will urge them toward the axis of the conducting liquid column.

From this conception, it is evident that, if the conducting liquid column have elastic walls, it will contract so as to diminish in cross-section and in consequence increase in length. If the column cannot increase in length, it will tend to contract. In other words, there will be a pressure at the center greater than the pressure near the surface. This pressure is hydrostatic. Its existence and magnitude are shown by the following experiment.

EXPERIMENT TO SHOW PRESSURE IN THE INTERIOR OF A CONDUCTOR.

An apparatus was constructed which is shown in section in Fig. 6. *T* is the tube of fiber, and *t* another tube of fiber having an internal diameter of 2.54 cm; *B* and *D* are brass plugs tightly fitted in the ends of the inside tube. Heavy copper terminals enable these plugs to be joined to a battery of large current capacity. At the center of the tube *t* was placed a fiber ring to reduce the cross-section of the tube at one place. The hole in the fiber ring had a diameter *2R* or a radius of .635 cm.

Through the sides of the tube and fiber ring several small holes, *h*, were drilled radially to the axis. The bottom of the annular space between the tubes *T* and *t* was closed by a fiber piece *P*. The two tubes were then filled with mercury to the level shown in the cut. According to the conception given above, when current is passing through the mercury column in the tube *t*, a hydrostatic pressure should exist in its interior, which is greatest along its axis. Hence, as the mobile conductor has no other escape than the hole *H* provided in the plug *D*, it should rise in this hole from the pressure at the axis, more being at the same time drawn in at the holes, *h*. Sufficient pressure should raise the mercury in the hole *H* until it overflows the upper surface of the plug *D*, and falls back into the well from whence it came. Thus, a continuous stream of mercury should flow by the path, through the holes *h*, up the tube *t* and the hole *H*, into the well and thence again to the holes *h*.

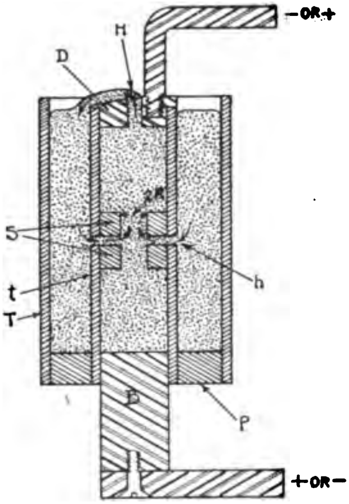


Fig. 6.

In trying this experiment, these results were amply realized. With about 1,800 amperes, the mercury flowed in a continuous and rapid stream, as described.

To test the magnitude of the pressure, a glass tube was tightly fitted in the hole *H*, and the mercury column was found to stand, with 1,800 amperes, six tenths of an inch in the tube above the general level of the mercury in the well.

So large a current soon heated the mercury and it was not found practical to keep it on more than a minute or two at a time. It is scarcely necessary to remark that the flow and the rise of the mercury is independent of the direction of the current.

It was thought that iron might be used to increase the induction and so get the mercury to flow continuously with less current. The previous experiment was modified as follows.

A disk of soft iron was inserted in the narrow portion of the tube t in the manner shown in Fig. 7, a being a vertical section and b a horizontal section of the middle portion of the tube t and its fittings. A slot was cut in the iron disk parallel to its axis. This slot was 1.6 mm. wide and extended in radically to the center of the disk. The iron disk was located in the narrow portion of the tube t so that the end of the slot registered with one of the holes, h, Fig. 6. It is evident that the intensity of the magnetic field in the space of this narrow slot should be greatly increased by the presence of the iron. Also that the conducting mercury filaments in the slot should be urged with increased force toward the axis with a resulting greater hydrostatic pressure at the axis.

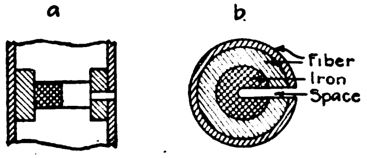


Fig. 7.

In trying the experiment, it was found that the force previously obtained was about doubled and that with 1,800 amperes the mercury would rise in the glass tube about one and a quarter inches. With 900 amperes and with the tube removed, the mercury would flow continuously, the heating being now reduced so that the current could be kept on for many minutes.

Law of the Pressure in the Interior of a Conductor.

Let Fig. 8 represent the cross-section of a cylindrical conductor. Conceive this cross-section to be made up of a large number of annular spaces as 1, 2, 3, etc. Let the radial depth of each of these annular spaces be called dr. Let R be the radius of the cylindrical conductor, and r the radial distance from the axis to any point within the circular cross-section. It should further be supposed that the conductor is of very great length, having its axis perpendicular to the plane of the paper, and far removed from any return circuit. By the theory and experiments cited above, all portions of the conductor, as the annular sections, 1, 2, 3, etc., are under the influence of a force, when the conductor is carrying a current, which tends to urge them radially toward the axis. This force, as measured in dynes, will have at any point distant r from the axis, a definite value per unit of area. The total force or pressure which acts upon the surface of any imaginary cylinder of radius r and length l, will be, evidently, the force per unit of area multiplied by the total area of a length l, of this imaginary cylinder.

It is first required, to find the force, g, per unit of area, at any point within the conductor distant r from the axis, and second, the force or pressure P on the surface of an imaginary cylinder of radius r, and axial length l.

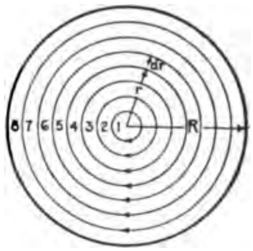


Fig. 8.

This problem may be solved by conceiving the action of the magnetic field on all portions of the conductor carrying current, or by considering only the mutual attractions of all the elements of the conductor.

By the first method, we may proceed as follows:

At a point within the conductor distant r from the axis the intensity of the magnetic field is by equation (2).

$$T_r = \frac{2Ir}{R^2},$$

where I is the total current flowing in the conductor. The lines of force of this field are circles, having as their common axis, the axis of the conductor.

Consider any single annular space as 5. It will carry current di, which is in the same ratio to the total current as the area of the annular space is to the total area of the cross-section of the conductor. If we call da the area of the annular space which has an inside boundary of radius r, we have

$$da = \pi(r + dr)^2 - \pi r^2 = 2\pi r dr,$$

since the square of dr can be neglected. Then as πR^2 is the total area, we obtain

$$di = \frac{2Irdr}{R^2}.$$

This current, *di*, is disposed in a field of intensity

$$T_r = \frac{2Ir}{R^2}.$$

If the current I flows downward, the lines of force would act on a unit north pole to move it in a clockwise direction around the axis; and an element of the conductor carrying a downward current would tend to move radially toward the axis. If the direction of the current were reversed, the direction of the magnetic lines would be reversed but the current elements would still tend to move toward the axis. The force in dynes with which a length l of any conductor is acted upon to move at right angles to the lines of magnetic force is, in electro-magnetic measure, numerically equal to the product of the length of the conductor, to the strength of the current in the conductor and to the field intensity where the conductor is located.[1] Thus calling dF the force with which the element carrying the current di tends to move radially toward the axis, we have

$$dF = l di T_r = \frac{4I^2 l r^2 dr}{R^4}.$$

It should be noted that the force, dF, is distributed so as to act normally on a surface the area of which is $2\pi rl$. Hence, calling dg the force per unit of area, or the force intensity acting radially inward, we have

$$\frac{dF}{2\pi rl} = dg = \frac{2I^2 r dr}{\pi R^4}.$$

This is the intensity of the pressure at distance r from the axis, due to the current in a single annular space of radial depth, dr. It is necessary in order to obtain the total intensity, g, at the distance r from the axis, to take the sum of the force intensities due to the currents in all the annular spaces, each of radial depth dr, which lie in the space included between the radius R and the radius r. This sum is given by the integral

$$g = \frac{2I^2}{\pi R^4}\int_r^R r dr.$$

---

[1] Consult Maxwell, Vol. II., § 490.

Performing the integration, we have

$$g = \frac{I^2}{\pi R^4}(R^2 - r^2). \tag{5}$$

Equation (5) gives the inward radial pressure per unit of area on the material of the conductor itself at any distance r from the axis of the conductor.

If I_1 is the current per unit of cross-section, or the current density in the conductor, $I^2 = I_1^2\pi^2R^4$, and we obtain as another expression for the pressure

$$g = \pi I_1^2(R^2 - r^2). \tag{6}$$

per sq cm

The pressure at the center, where $r = 0$, is therefore equal to the area of the cross-section of the conductor multiplied by the square of the current density.

The total pressure on the surface of an imaginary cylinder of length, l, and radius, r is, evidently, the force per unit of area multiplied by the total area of the surface of the cylinder considered. Calling P this pressure, we have:

total

$$P = \frac{2lI^2r}{R^4}(R^2 - r^2) \tag{7}$$

or

$$P = 2\pi l I_1^2 r(R^2 - r^2). \tag{8}$$

These same expressions are easily obtained by conceiving the imaginary conducting shells to exert an attraction according to the law of mutual attraction between currents flowing parallel and in the same direction. The law of attraction between any two infinitely long parallel conductors distant r from each other and carrying currents i and i_1 is

$$F = 2ii_1\frac{l}{r}. \tag{9}$$

Here l is the portion considered of the length ot *one* of the conductors.[1]

In the case under consideration, we can take one of the conductors to be the cylinder of radius r, and the other conductor to be the annular area of inside radius r and outside radius $r + dr$.

---

[1] See Maxwell, Vol. II., § 495.

If I is the total current in the large conductor of radius R, the current carried by the conductor of radius r, is

$$i=\frac{r^2}{R^2}I,$$

uniform current density being supposed.

$2\pi rdr$ is, as previously found, the cross-section of the annular area and the current which it carries is therefore

$$di=\frac{2rdr}{R^2}I.$$

The currents i and di attract and produce a pressure over the surface of the cylinder of radius r. Calling dF this pressure, we have

$$dF=\frac{2idil}{r}=\frac{4lI^2r^2dr}{R^4}.$$

The pressure per unit of area is:

$$\frac{dF}{2\pi rl}=dg=\frac{2I^2rdr}{\pi R^4},$$

whence, in integrating, we obtain

$$g=\frac{2I^2}{\pi R^4}\int_r^R rdr=\frac{I^2}{\pi R^4}(R^2-r^2)$$

and

$$P=\frac{2lI^2r}{R^4}(R^2-r^2).$$

These last two expressions are seen to be identical with (5) and (7).

ATTRACTION IN THE INTERIOR OF A CONDUCTOR.

The attraction in the interior of a cylindrical conductor carrying I_1 units of current per unit of cross-section which would be exerted upon a unit length of a unit of current distant r from the axis, may be found as follows:

. In equation (9) let $l=1$, and we have

$$F=\frac{2ii_1}{r}.$$

The current carried by the conductor of radius r is

$$i = \frac{r^2}{R^2} I,$$

which combined with the above gives

$$F = \frac{2ri_1 I}{R^2}.$$

Calling I_1 the current per unit of cross-section of the conductor and taking i_1 equal to unity, we find

$$F_i = 2\pi I_1 r. \qquad (10)$$

Equation (10) expresses the attraction in the interior of a conductor carrying current. It is analogous to the gravitational attraction in the interior of a body of the same form as the conductor, due to its mass only.

Gravitational Analogy.

It is interesting to compare the forces produced by an electric current in a conductor of circular cross-section with the forces produced by gravity in the interior of an infinitely long cylinder of the same radius and section as the electric conductor.

For the sake of comparing methods, we may use Poisson's equation to find the expressions giving the forces in the cylinder produced by gravity.

It is shown in Thomson and Tait's Natural Philosophy, Vol. II., page 37, that for the case considered, this equation takes the form

$$rF = C - 4\pi M \int_r^R r\,dr = C - 2\pi M(R^2 - r^2).$$

Here F is the force of gravitational attraction which acts radially toward the axis of the cylinder; M is the mass per unit of volume, and C a constant. Since we know, by symmetry, that F is zero at the axis, where r is 0, we obtain

$$C = 2\pi M R^2.$$

Hence

$$F = 2\pi M r. \qquad (11)$$

Equation (11) is an expression of the same form as (10).

If we express F in dynes and r in centimeters, we must take for the unit of mass, that mass which at one centimeter from a like mass will cause an attraction between the two of one dyne. The value of this unit in grams is given in Everett's "C.G.S. System of Units," page 73, 1891 edition, as equal to 3,928. Hence, if m be a mass expressed in grams, we have

$$M = \frac{m}{3928},$$

or

$$F = \frac{2\pi mr}{3928} \text{ dynes.} \tag{12}$$

We shall now have for the force acting on the matter in an annular space of radius r, radial depth dr and length l,

$$dF_1 = \frac{2\pi mr}{3928} \cdot \frac{2\pi lmrdr}{3928} = \frac{4\pi^2 lm^2 r^2 dr}{3928^2}.$$

The intensity of this force, or the dynes pressure per unit of area at distance r from the axis is

$$\frac{dF_1}{2\pi rl} = dg_1 = \frac{2\pi m^2 rdr}{3928^2}.$$

And by integrating, we obtain

$$g_1 = \frac{2\pi m^2}{3928^2}\int_r^R rdr = \frac{\pi m^2}{3928^2}(R^2 - r^2). \tag{13}$$

Or, if we call G the number of grams of matter in a unit of length of the cylinder of radius R,

$$g_1 = \frac{G^2}{\pi R^4\, 3928^2}(R^2 - r^2). \tag{14}$$

Comparing equations (5) and (14), we note that

$$g = 1.513 \times 10^7 g_1 \frac{I^2}{G^2}, \tag{15}$$

which means that the hydrostatic pressure in the interior of a conductor at any distance r from the axis when carrying I electromagnetic units of current is 1.513×10^7 times the pressure in this conductor, having G grams of matter per unit length, due to gravitational attraction alone.

If we compare equations (10) and (12), we note that the attraction due to the electric current, which would act on a unit length of a unit current, is of the same nature and in the same direction as the gravitational attraction which would act on a gram of matter, but 3,928 times as great, the number of units of current per unit of cross-section of the conductor being equal to the number of grams of matter per unit of cross-section in a unit of length of the cylinder.

GRAPHS.

The leading features respecting the forces discussed above are summarized in the curves shown in Fig. 9. The curves in full line apply to the cylindrical conductor which is shown in cross-section in full line. The curves in broken line, and the curve 2 apply to the conductor shown in broken line, having one half the diameter of the first conductor. In both cases, the larger and smaller conductor are supposed to carry the *same* current. Curve 1 gives the intensity of the field in the interior of the conductor. Curve 2, the intensity of the field exterior to the conductor. Curve 5, expresses the pressure in dynes per square centimeter. This pressure is a maximum at the center and zero at the circumference. At one quarter of the distance from the center to the circumference, the pressure has diminished but 6¼ per cent. — a fact having importance in certain practical constructions.

When the conductor is diminished in diameter to one half, its area is one fourth and the current density is increased four times, hence it follows from equation (6) that the pressure at the center is increased four times. This is shown by curve 6. The intensity of the field in the inside of the conductor is doubled, curve 4. On the outside of the conductor, the intensity of the field is not altered beyond the radius of the larger conductor, curve 2; but in the space outside the smaller and inside the larger, there is a field the intensity of which is given by curve 3.

From what has been given, the following deductions are easily made.

Suppose a conductor to have originally a radius R. Imagine the conductor to contract in cross-section until its radius becomes aR, a being a number less than unity. Further, assume the conductor to always carry the *same* total current I and to preserve a circular cross-section. Let a portion of the length of the conductor be called l. Then the total number of lines of force in the interior of a length, l, of the conductor is given by the expression

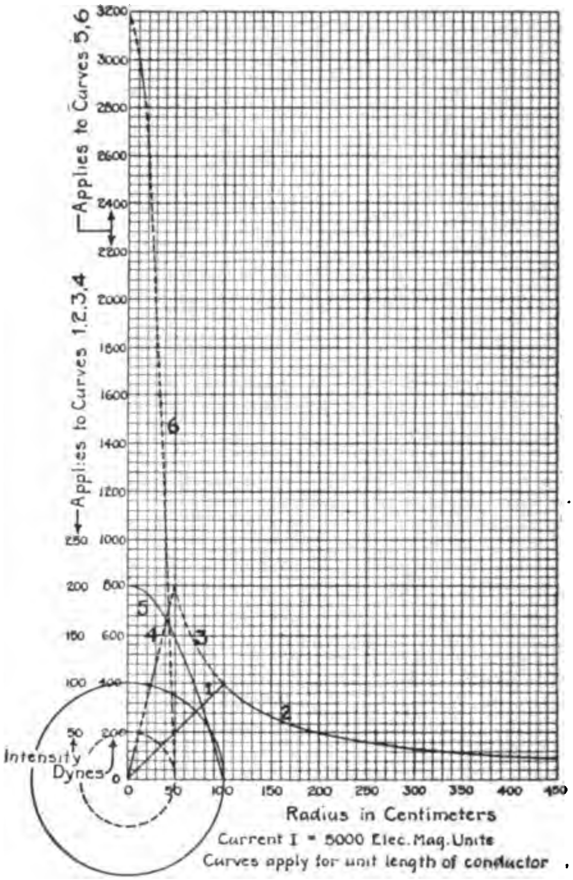


Fig. 9.

$$N = \frac{2Il}{(aR)^2}\int_0^{aR} r\,dr = lI. \tag{16}$$

Hence, we conclude that, however, the conductor may change in diameter, the total number of lines of force in its interior remains the same, provided the current is always the same, and that per unit length of the conductor the total number of lines of force in the conductor is numerically equal to the current in electromagnetic measure.

An inspection of curves 2 and 3 shows that when the conductor contracts from a radius R to a radius aR, the current remaining the same, lines of force are developed. These all lie in the medium around the conductor and their number per length l of the conductor is given by the expression

$$N_0 = 2Il \int_{aR}^{R} \frac{dr}{r} = 2Il \log_e \frac{1}{a}. \tag{17}$$

From this, it follows that the total number of lines exterior to the conductor is increased when the conductor diminishes in diameter, and as the number inside the conductor remains the same, we conclude that the coefficient of self-induction of a small wire is greater than that of a large wire.

It may be further concluded that, if the current be maintained constant, when the conductor diminishes in diameter, electromagnetic energy is stored up in the medium around the conductor.

It may be shown that the electromagnetic energy in the interior of a length l of the conductor is given by the expression

$$E = \frac{lI^2}{R^4} \int_0^R r^3 dr = \frac{lI^2}{4}. \tag{18}$$

Thus the energy stored in the conductor, as well as the number of lines of force, is independent of the diameter of the conductor. When the conductor contracts from radius R to radius aR, the energy added to the medium is stored in the space occupied by the conductor previous to its contraction less the space occupied by the conductor after its contraction. The energy so added per length l of the conductor is given by the expression

$$E_0 = I^2 l \int_{aR}^{R} \frac{dr}{r} = lI^2 \log_e \frac{1}{a}. \tag{19}$$

The above expressions (16), (17), (18) and (19) are given without proof, for they may be readily obtained from the facts stated above; and the energy relations given by Maxwell.

Adding Pressures in Series.

The writer soon perceived that to obtain forces of sufficient magnitude to be useful in practical applications means should be devised whereby the internal pressure at one point in a conductor might be added to that in another point and so on. This result may be attained as follows: A cylindrical conductor is constructed which consists of alternate strata of solid and liquid, as copper and mercury. This heterogeneous conductor is tightly encased in a tube of insulating or high resistance material. In each of the solid portions of the conductor small tubes of insulating material are placed in a manner to connect the axis of one liquid layer with the circumference of the next liquid layer. This forms a continuous passage which goes in steps from the axis to the circumference of each liquid portion. A tube from one end of this conductor connects with a

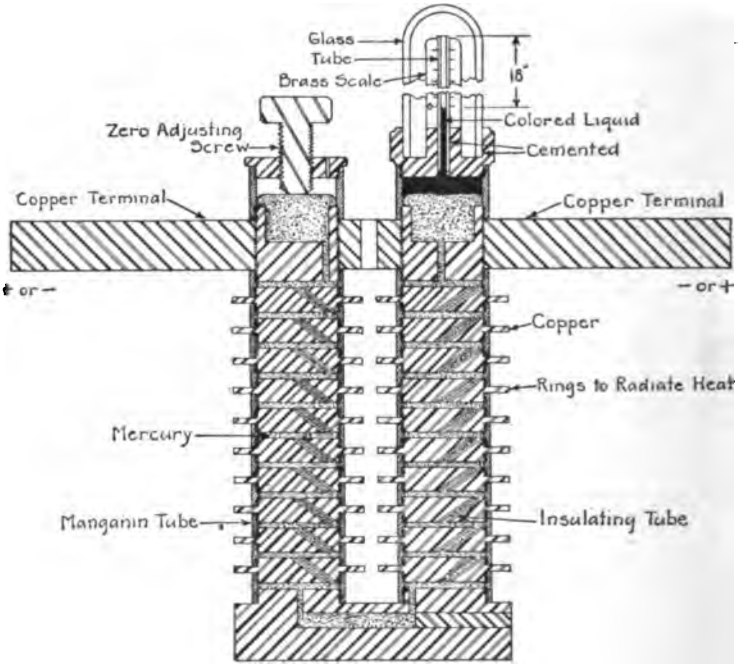
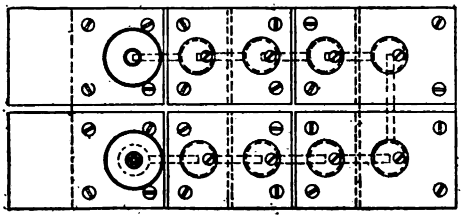
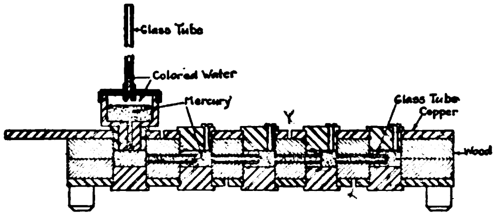
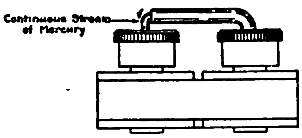


Fig. 10.







reservoir of the liquid and another tube of glass is raised from the other end of the conductor which connects with the end liquid layer and in which the liquid may rise. Heavy copper terminals lead the current, which may be direct or alternating, in, and out of, the compound conductor. When the electric current flows, the pressures produced at the axes of the liquid portions of the conductor add in series and the liquid may be thus raised to useful heights in the glass tube. Fig. 10 shows in section such a contrivance which it is evident might serve as a direct or alternating current ammeter of large capacity. A construction similar to that described was made and operated, and is a useful alternating current ammeter.

Fig. 11 shows another embodiment of the same principle. This apparatus was constructed and the calculated height of about 22 cm. to which the colored water would rise in the tube with 600 amperes was practically attained.

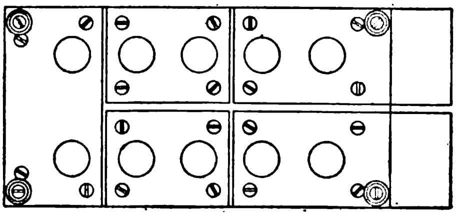


Fig. 11.

It should be noted that the hydrostatic pressure obtained is independent of the axial length of the liquid strata, while the heat developed by the current is directly as this length. With this consideration in mind, the axial lengths of the liquid portions of the apparatus shown in Fig. 11 were afterwards reduced to 1.6 millimeters each. The pressure was not lessened and the efficiency or cool running was greatly improved. The theoretical pressure or height of mercury that may be obtained in any particular case may be deduced easily from the curves (Fig. 9), or calculated from the expression

$$g_s = nAI_1^2 \tag{20}$$

which is derived from equation (6), when $r = 0$, $\pi R^2 = A$, the cross-section, $I_1 =$ the current density and $n =$ the number of times the pressures at the axis are added in series.

If C is the current density per square cm. expressed in amperes and g_p is the pressure in pounds per square cm. and A is the cross-section in square cm. the expression (20) takes the form

$$g_p = \frac{nAC^2}{44479100}. \tag{21}$$

In the apparatus shown in Fig. 11, $n = 10$, $A = 1.2$, $C^2 = 250{,}000$, which corresponds to nearly 600 amperes in the conductor. Whence we get $g_p = .067$ lb. per square cm. or .432 lb. per square inch. This pressure should give a rise of .88 inch of mercury or about 11.9 inches of water. The rise obtained with 600 amperes in the apparatus of Fig. 11 was about 9 inches, but this lessened height is due to the fact that the mercury itself had to raise some to produce the rise in the water. The point to note, however, is that with 600 amperes, the apparatus runs cool. Hence we can conclude that with a hetèrogeneous conductor of one half inch diameter and made up of 1,000 cells, or mercury sections of say, 1 mm. length each, we could obtain with 600 amperes the large pressure of 43.2 lbs. per square inch, and this without iron in the circuit. This pressure would sustain a column of mercury over seven feet high.

Some Final Remarks.

One is naturally led to inquire if the recognition of this force that exists in the interior of electric conductors and the establishment of its laws of action is likely to result in new and useful applications. It is quite certain that this force which depends only on the square of the current and linear dimensions may be used as an accurate measure of the current, either direct or alternating. As the forces can be enhanced by a proper disposition of iron in the heterogeneous circuit, motors without brushes or slip rings may be made operative. And remembering that power is the product of current and electromotive force, it is not impossible, even if large operative currents are required, that fair efficiencies may be obtained.

Further thought along an engineering line may bring about developments of a unique and useful character.

THE DETERMINATION OF POTENTIAL BY CARBON PENCILS.

BY C. D. CHILD.

AN investigation of the determination of the potential of hot gases by carbon pencils has shown that error may often be introduced when measurements are made by this means. Such errors appear to have been made by the present writer in the work on the magnetic deflection of the arc[1] and possibly by Stark, Retschinsky and Schoposchnikoff[2] in their work on the relative velocities of the ions of the carbon arc. The phenomena which give rise to these errors may be described as follows.

Potential Difference between hot Carbon and surrounding Bodies. It has already been shown by the present writer that when the temperature of carbon in air is gradually raised, it first discharges positive electricity but not negative, that secondly it discharges both positive and negative, but the positive in greater amount, and finally that it discharges negative more readily than positive In the similiar case with hot platinum wire it was shown by Elster and Geitel[3] that if there was no external E.M.F. between the hot wire and a neighboring insulated body, the wire was positive as compared with the other body, when it first became luminous and negative when it became white hot. It seemed probable that the same thing would occur with carbon.

To test this a carbon pencil, 1.6 mm. in diameter, was placed inside an insulated brass cylinder 2 cm. in diameter. The pencil was heated by passing a current through it. A high resistance shunt was placed about the pencil and the middle of this shunt was connected to one pair of the quadrants of an electrometer and the cylinder to the other pair. When the pencil first became luminous, it was negative as compared with the cylinder, and when white hot, it was positive. It varied approximately from minus four to plus three volts. Thus it may happen that in some cases the hotter of two carbons will be positive as compared with the cooler, while in others it will be negative. When one is red hot while the other is non-luminous, the hotter will be negative. When one is white hot while the other is either cool or red hot, the hotter will be positive.

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[1] PHYS. REV., 20, 100.

[2] Drude's Ann., 18, 230.

[3] Wied. Ann., 31, 109.

When potential is measured by hot carbon pencils, error may be introduced by overlooking either of two phenomena ; one, that at certain temperatures carbon discharges positive electricity the more easily, while at others the negative is more easily discharged ; the other, that there is an E.M.F. between carbons of different temperatures and that in some cases the hotter will be positive, while in others it will be negative.

Magnetic Deflection of the Arc. — Let us consider the bearing of these facts on the experiment of the magnetic deflection of the arc to which reference has been made. It was apparently shown in that article that when the arc was deflected to one side by a magnet, the side toward which it moved assumed a higher potential than the opposite one. In making the measurements the potential was determined by placing carbon pencils in the arc. It was evident that the pencil toward which the arc was deflected would become hotter than the other, and it was recognized at the time that this might be the cause of the phenomena. This was tested by blowing the arc to one side by a draught of air and the results thus obtained were opposite to those observed with deflection by a magnet. The conclusion seemed evident that the phenomena were not due merely to differences in temperature.

However, such a conclusion was probably incorrect. At that time it was found that when the arc was blown to one side by a draught of air and the potential difference between the two pencils measured by a voltmeter, errors were liable to be incurred because of the increased resistance between the two pencils when one was cool. An electrometer was, therefore, substituted for the voltmeter. This electrometer had a period of 14 sec. and in order to observe any effect it was necessary to keep the arc blown to one side for some time, and in fact, it was blown entirely to one side, in order that there might be no question as to which carbon was the hotter. Apparently it happened that when the arc was deflected by the magnet both pencils remained luminous and that the one which was very hot was positive as compared with the cooler one, but that when the arc was deflected by draughts of air, the side from which the arc was blown was not luminous and the other was luminous but not white hot, so that the hotter pencil in this case was negative.

In order to test this further the experiment with the arc blown to one side by a draught of air was repeated, care being taken not to deflect the arc more than a slight amount, and it was found that the effect was then the same as when the arc was deflected by a magnet. For this purpose it was found best to use a millivoltmeter with a needle having a short period. With this the arc could be deflected and the effect observed before the pencil had become non-luminous.

In measuring the potential differences with a voltmeter the effects produced are largely due to changes in conductivity at the surfaces of the pencils, since the resistance of the voltmeter is often small as compared with that between the pencils. This, however, would give the same apparent changes in E.M.F. as the real changes described above. If two pencils are placed in the arc and connected by a voltmeter, it is impossible to place them so that for an appreciable length of time there will be no reading of the voltmeter. If they are placed as in Fig. 1, where *a* is nearer to the positive carbon, *a* will have a higher potential and a current will flow through the voltmeter from *a* to *b*. *a* is here receiving positive ions from the arc and giving negative ions to it, *b* is receiving negative ions and giving off positive ones. If now the arc were to be deflected slightly to the right *a* will be hotter and better able to give off negative ions, while *b* may not become so cold as to change appreciably its discharging power. The current through the voltmeter will increase, as if *a* had acquired a higher potential.

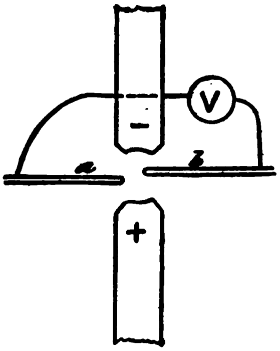

Fig.-1.

On the other hand, if the arc is deflected to the left, *a* will be less able to discharge negative ions and the voltmeter will read less than before, as if the potential of *a* had been lowered. So that whether we consider the effect which the temperature has on the potential of the pencils when we use an electrometer, or its effect on the conductivity near the pencil when a voltmeter is used, the apparent effect due to the change of temperature is the same.

Such an explanation as the preceding would account for the fact that the effect which had been observed with the magnetic deflection was less apparent with arcs between cored carbons, or with arcs into which salts were introduced, or with arcs in a vacuum. In each of these cases the arcs were not so hot and probably neither of the pencils reached the temperature where they gave off negative ions in abundance. In a vacuum the pencils may even be non-luminous while in the arc.

Measurement of Potential to Determine Relative Velocities of Positive and Negative Ions. — Passing now to the experiment of Stark, Retschinsky, and Schaposchnikoff, their method may be described as follows. If there is uniform ionization between the plates *A* and *B*, Fig. 2, and if a potential difference be maintained between them, the curve indicating the potential at various points will be similar to the curve *acb*. If the ionization is the same on both sides of the middle point *c*, and if the positive and negative ions have equal velocities, the drop in potential will be the same on the two sides. If, however, the negative ions have the greater velocity, other conditions being the same, the drop between *a* and *c* will be less than that between *c* and *b*. If the positive ions have the greater velocity, the drop between *a* and *c* will be the greater.

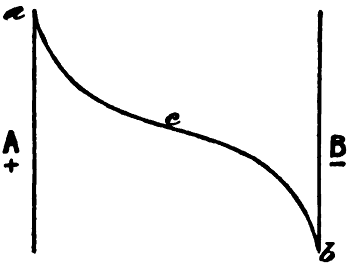

Fig. 2.

The writers to whom reference has been made attempted to find the relative velocities of the positive and negative ions by placing two small graphite pencils on opposite sides of the arc and between them a third pencil for testing the potential. All of these were near enough to the arc, so that they became self-luminous. The two opposite pencils were connected to an E.M.F. of about 7 volts. By this means they found that the drop in potential at the negative pencil was the greater and they concluded that the negative ions moved the more rapidly.

It is possible, however, that a different conclusion should be drawn from these facts. If, as before, there were uniform ionization between *A* and *B*, Fig. 2, and the two kinds of ionization had the same velocities, and if in addition there was ionization at the surface of *A*, then the drop in potential at *A* would be less than than that at *B*. That is, the positive ions coming from *A* would neutralize the electrostatic effect of the negative ones coming toward *A*. Or to state it in other words, the conductivity near *A* being increased, the potential gradient there would be diminished. A similar statement may be made concerning the effect of ionization at *B*.

In the experiment which has been described there was ionization not only in the arc, but also at the surface of the graphite pencils. It might easily be complicated by these phenomena. This would be the more serious in this case, since a large part of the fall of potential between the graphite pencils occurs near the pencils. This, at least, is the condition which we find at the electrodes placed in a flame, especially when the electrodes are much cooler than the gas between them as they are with the arc. Under these conditions a slight increase in the conductivity near the electrodes will have a large effect on the drop in potential, and the drop may be the smaller at the electrode which gives off the greater number of ions quite irrespective of the velocity of the ions.

It is possible to test this matter, for if the results are due to the ionization at the surface and not to relative velocities of the ions, we should expect the drop in potential at the positive pencil to be greater than that at the negative, when the pencils are placed near the center of the arc. It should, however, be borne in mind that no matter how hot the point of the pencil may be, there will be some region further from the center of the arc, which will be just hot enough to give positive ions, but not negative, so that to have a preponderance of negative ions given off, it is necessary that a comparatively large part of the pencil should be very hot.

Method for Testing this Explanation. — The apparatus was arranged as shown in Fig. 3. The arc occurs between *A* and *B*, which were solid carbons 12 mm. in diameter. The carbon pencils *a* and *b* were inserted in or near the arc, the two being approximately equally distant from the center of the arc. These were 3 mm. in diameter. A potential difference was maintained between them by a battery *C*. *ef* is a resistance shunted between *a* and *b* and *d* was the middle point of this shunt. *c* is an exploring pencil placed in the center of the arc, and *E* is an electrometer for determining the potential difference between *d* and *c*. *D* is a reversing key. If the greater drop in potential is at the positive pencil, *c* will have a lower potential than *d*, whichever way the current will be flowing through the arc or between *a* and *b*.

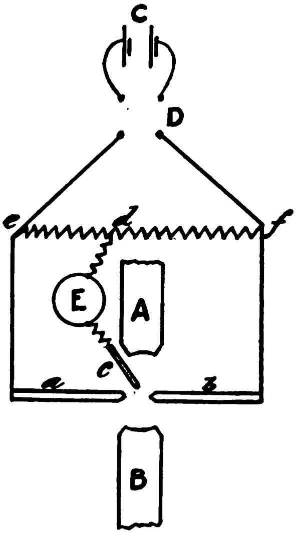

Fig. 3.

The experiment was first performed with a direct current through the arc of approximately 12 amperes, and a potential difference of 13 volts between *a* and *b*. The data taken are given in the following table, column 1 giving the distance between *a* and *b* in millimeters, column 2 the potential difference between *c* and *d* in volts, the sign indicating the potential of *c* as compared with *d*. The readings given in the table are the average of four readings, both the currents through the arc and through *a* and *b* being taken direct and reversed.

Distance Between Exploring Pencils, *a* and *b*.	Potential Difference Between *c* and *d*.
30	+ .4
25	+1.4
15	−1.9
10	−1.1
5	+1.0

In the first readings the exploring carbons were so far from the arc that they were non-luminous. In this case the result was not complicated by ionization at the surface of the exploring carbon. This would indicate that the positive ions had a greater velocity than the negative. This was consistent with what had previously been observed, when studying the discharge from the carbon arc to a surrounding cylinder.[1] It was at that time found that with a constant potential difference between the arc and the surrounding cylinder the current to the cylinder was only one half as great when the cylinder was positive as when it was negative. Here with a constant current we find the drop at the anode to be greater than that at the cathode. Both of these phenomena would appear to indicate that the positive ions drawn from the arc move more rapidly than the negative.

When the distance between the exploring carbons was 15 mm., they were both luminous. The drop at the cathode was then the greater. This is in agreement with the results obtained by Stark, Retschinsky and Schoposchnikoff.

When the exploring carbons were 5 mm. apart the drop at the anode was again the greater. If these results depend only on the relative velocity of the positive and negative ions, they would indicate that at the center of the arc, the velocity of the positive ions is greater than that of the negative. It can hardly be believed that the velocity of the positive ions outside the arc is greater than that of the negative, that it is less in the cooler parts of the arc, and that it is again greater in the center. It seems more probable that any effect produced by the relative velocities of the ions is masked by the ionization at the surface of the carbon and that the phenomena observed is due to the relative ease with which the different ions are given off at different temperatures.

The experiments were then varied, first by changing the potential difference between *a* and *b* to 7 volts, secondly by using an alternating current for the arc, and third by using an alternating potential difference between *a* and *b*. In all of these cases the results were of the same general character as those given in the table.

When cored carbons were used the average potential at *c* was at all times above that of *d*, being more, however, when the distance between *a* and *b* was in the neighborhood of 15 mm. This is probably due to the fact that with cored carbons the temperature of the arc and of the exploring carbons is not as high as with solid carbons.

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[1] PHYSICAL REVIEW, 12, 143.

It should be noticed that if this explanation is correct, we are not at any time measuring the true drop in the potential at the different pencils, for the middle exploring pencil does not take the true potential of the hot vapor in which it is placed. It shows rather that the effects are too complicated to admit of an explanation depending merely on the relative velocities of the ions.

One might think at first that the difficulty here shown could be obviated to a great extent by taking greater potential differences between the exploring carbons, so that any potential difference between the middle exploring pencil and the gas would be negligible as compared with other potential differences. This, however, is impracticable, because an arc would start between the exploring pencils, if a large potential were maintained between them.

This experiment does not in any way throw doubt on the valuable work which Stark, Retschinsky, and Schoposchnikoff performed on the relative velocities of the ions in the mercury arc, for in that case the exploring pencils did not become luminous, nor does it *prove* that the negative ions have not the greater velocity in the carbon arc. It does, however, throw doubt on the certainty of their method of proof and leave it desirable that some further experiment be made on the subject.

Measurement of the Potential of the Arc Vapor. — The measurements which have been made on the potential of the vapor of the arc and on the drop in potential at the anode and at the cathode must also be subject to some error, because of this E.M.F. between the vapor and the exploring carbon pencil. However, it does not seem probable that such error is much, if any, greater than the errors which are know to exist. As far as is known to the present writer no attempt has been made to determine whether such error exists and no investigation has been made of the effect which may be produced on the apparent potential of the arc vapor when the exploring pencil is inserted for different distances into the arc.

Swinton's Experiment on the Relative Velocities of the Positive and Negative Ions. — It is also a question whether the experiment performed by Swinton[1] proves anything concerning the relative velocity of the ions of the arc in air. He describes his experiment as follows: "One of the carbon electrodes was made hollow and was pierced axially by a small hole, and an insulated Faraday cylinder of brass, with its aperture in a line with and facing the aperture in the electrode, being fixed inside of the electrode. The insulated Faraday cylinder and the pierced electrode were connected together through a galvanometer, and it was found that when the opposite electrode was made negative, the pierced electrode being positive, a negative charge was imparted to the Faraday cylinder, while, if the opposite electrode was made positive and the pierced electrode was made negative, the Faraday cylinder acquired a positive charge."

He found that the negative charge was greater than the positive and from this concluded that the velocity of the negative ion was the greater. He would have found the same, however, if he had placed the exploring carbon in the arc, or any where else near the arc, instead of in the cylinder. Ions diffuse in all directions from the arc and since the potential difference between the vapor of the arc and the anode is greater than that between the vapor and the cathode, the potential difference between the exploring pencil and the anode would be greater than that between the pencil and the cathode, whenever it was placed in the same position with reference to the cathode as it had previously been placed with reference to the anode.

If penetrating anode and cathode rays were given off by the arc which could be studied, they would give indications concerning the velocities of the ions in the arc, but Swinton has himself shown[2] that there are at least no such rays which penetrate .00265 mm. of aluminum, and we have no evidence whatever that there are any which would penetrate even a millimeter of air. It seems, therefore, probable that the phenomena observed by Swinton was produced by the diffused ions, and that these indicated nothing concerning the relative velocities of the ions in the carbon arc at atmospheric pressure.

Summary. — We may summarize by saying that between hot carbon and surrounding bodies there is an E.M.F. which depends in amount and sign on the temperature of the carbon and that this E.M.F. is liable to introduce error in determining the potential of a hot gas.

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[1] Proc. Roy. Soc., A 76.

[2] Phil. Mag. (6), 11, 829.

Such an error was introduced by the present writer in an article on the arc in a magnetic field. In all probability the phenomena there described were due to such an E.M.F. It also seems probable that the phenomena observed by Stark, Retschinsky, and Schoposchnikoff in attempting to determine the relative velocity of the ions of the carbon arc in air were due to this cause rather than to differences in the velocities of the ions.

It also seems probable that the phenomena observed by Swinton were due to the diffusion of the ions in the arc and not to any anode or cathode rays. So that some definite determination of the relative velocity of ions in the carbon arc in air is a measurement yet to be made.

Colgate University,
February, 1907.

THE HIGH RESISTANCE CONTACT THERMO-ELECTRIC DETECTOR FOR ELECTRICAL WAVES.[1]

BY L. W. AUSTIN.

IF two metals standing far apart in the thermo-electric series be so brought in contact that their contact resistance is sufficiently high and if the surfaces of the metals are of such a nature that heat is not conducted away too rapidly from the point of contact, then the passage of electrical oscillations through the junction will produce direct electromotive forces which may be detected in a galvanometer or telephone in series with the thermo-element. The principle is extremely simple but in practice it is found that there are very few pairs of metals which give suitable contact conditions. If the contact is of too low resistance the heating is insufficient and if the contact is too loose as in the coherer the action is irregular, and the instrument less sensitive.

On account of its extreme position in the thermo-electric series tellurium seemed desirable for one of the metals and it was tried with a large number of others. Fair results were obtained with several of the more easily oxidized metals and also with carbon but with all these the action was irregular and adjustment difficult. Finally aluminium was fixed upon as being not only the most sensitive but by far the most reliable among the common metals. Silicon has also proved very satisfactory.

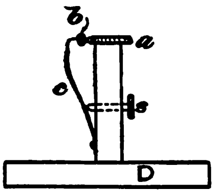

Fig. 1.

The instrument has been made in three forms. The first form is shown in Fig. 1. *D* is a hard rubber support at the top of the upright piece of which the pointed rod of aluminium *a* is mounted. *b* is a bead of tellurium melted onto the tip of a piece of brass wire the latter being soldered to the top of the spring *c*. The closeness of the contact between *a* and *b* is regulated by the screw *s*. A second form was also made in which the aluminium is in the form of a rod fitted in bearings and rotated slowly by means of a pulley. The tellurium is pressed against it by a spring as before. The constantly changing contact point seems to keep the conditions more constant and does away with any necessity of readjustment even when heavy atmospheric discharges have passed through it. The sensitiveness is about the same in the two types. As certain portions of the tellurium surface seem to be more sensitive than others it is desirable in both forms to have the point of contact adjustable.[1]

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[1] Briefly described in a letter to the Electrical World, Vol. XLVIII., p. 924, 1906. A similar detector using silicon and copper was independently devised by G. W. Pickard, Electrical World, Vol. XLVIII., p. 1003, 1906.

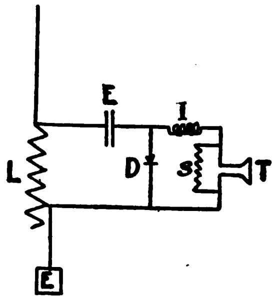

Fig. 2.

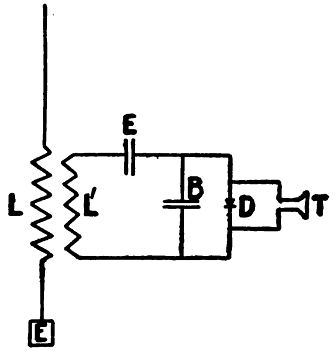

Fig. 3.

An exhaustive comparison of the thermo-electric receiver with the well-known electrolytic type, extending over several months and including some long distance work, showed that in sensitiveness it compared favorably with the latter, being slightly less sensitive on an average than the free Wollaston wire electrode type but more so than the type with the small platinum electrode sealed into glass. The comparisons were made by the shunted telephone method in which the loudness of the signals is measured by the smallness of the shunt *s* (Fig. 2) across the telephones necessary to produce silence. The telephones used were of the ordinary head type and had a resistance of about 1200 ohms.

In the figures *L* represents the inductance, *D* the detector, *E* a condenser of about 0.05 m.f. used to prevent the thermal currents flowing around through the inductance, *T* is the telephone and *I* (Fig. 2) is a choking coil preventing the oscillations flowing through the telephone circuit when the shunt *s* is closed, *B* (Fig. 3) is a tuning condenser.

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[1] A third form in which the contact metals are tellurium and silicon made up like form one, promises in some respects to be even more satisfactory than the others.

Experiments were also made using 120 cycle alternating currents, various alternating potential differences being applied to the detector from a fall of potential wire. The direct currents were measured by a D'Arsonval galvanometer having a sensitiveness of about 1.10^{-9} amp. per mm. of scale. The results are shown in the following table.

Volts. a.c.	Galvanometer Deflections. mm.
0.015	210
.010	105
.007	50
.005	25
.002	5
.001	1

The resistance of the detector under sensitive conditions varies between 1000 and 3000 ohms therefore the alternating current corresponding to a deflection of 1 mm. lay between 1.10^{-6} and 3.10^{-7} amp.

On account of its high resistance, in working with high frequency currents, the detector is not placed directly in the oscillating circuit but is shunted across the inductance as shown in Figs. 2 and 3.

WASHINGTON, D. C.,
January, 1907.

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[1] The contact surfaces should be cleaned frequently with petroleum.

UPON THE CHEMICAL NATURE OF THE RADIUM EMANATION.

By L. M. Dennis and L. F. Hawley.

RUTHERFORD, Ramsay and Soddy have described experiments made upon the emanations from thorium and from radium that led them to conclude that if the emanation in either case is due to the presence of a known gas, that gas must be a member of the argon group. Rutherford finally decided,[1] however, that the emanation cannot be due to the excited activity of a member of the argon group or of any other constituent of the atmosphere because when carbon dioxide is passed over thoria, the amount of emanation is the same as when air is substituted for the carbon dioxide.

Notwithstanding this statement by Rutherford it still seemed possible that the emanation might be due to the excitation of an atmospheric gas such as neon, for the reason that it is by no means certain that the carbon dioxide employed by Rutherford was wholly free from atmospheric gases. If he prepared carbon dioxide from marble and an acid, the gas would almost certainly contain appreciable amounts of air. For this reason it seemed desirable to repeat the experiment under such conditions as would exclude the possibility of the presence of a constituent of the atmospheric air, with the aim of ascertaining whether under these circumstances the amount of the emanation is the same as when air is present.

The radioactive material that was employed in the measurement was radium-barium chloride that had been extracted from gummite and uraninite in this laboratory and had been fractionally concentrated by repeated precipitation with hydrochloric acid gas. About 0.1 gram of this material, possessing an activity of 350 uranium units, was dissolved in water. This solution was boiled to free it from air and was cooled in a current of hydrogen.

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[1] Radioactivity, p. 226.

The apparatus employed was that shown in Fig. 1. The Kipp generator *T* was first filled with hydrogen and was then charged with pure zinc and with dilute sulphuric acid that had been boiled and cooled in an atmosphere of hydrogen. By means of the connections shown in the drawing an atmosphere of pure hydrogen was always maintained above the acid in the Kipp apparatus. An excess pressure of hydrogen in the upper bulb was automatically relieved by the escape of the gas through mercury in the beaker *H*. Diminished pressure of the gas above the acid was relieved by momentarily opening the stopcock *P*. With the aid of the stopcocks *R* and *S* it was possible to pass hydrogen either directly through the measuring tube *W* or first through the radium-barium chloride solution in the flask *L* and then through *W*. The measuring tube *W* was of glass, with platinum plates *A* connected to platinum wires fused through the walls of the tube. To diminish the effect of local electrostatic charges on the glass, the outside surface of *W* was covered with tin foil, and the inside surface was coated with silver[1] to within one quarter inch of the platinum wires. These inner and outer metallic layers were grounded by the wires *N*, *O* and *J*. The battery *F* consisted of two hundred small dry-cells, giving a potential of about 190 volts. By means of the switch *V* the battery could be connected either with the lower platinum plate in the measuring tube or with the movable brass bar *D* by means of which the needle of the electrometer was charged. One

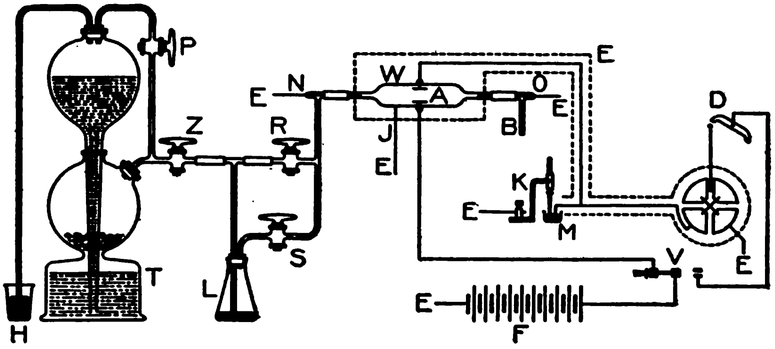

Fig. 1.

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[1] Heumann-Kühling, Anleitung zum Experimentieren, p. 697.

pair of quadrants of the Dolezalek electrometer was grounded; the other pair was connected with the upper platinum plate and also with the mercury cup *M* through which it could be grounded by means of the key *K*. The electrometer, the measuring tube, and all the connections between them were screened by brass wire gauze which is represented in the diagram by dotted lines. All parts marked *E* in the diagram are ground connections.

In making the measurements the radium-barium chloride solution was first placed in the flask *L*. The apparatus was then connected up throughout and was allowed to stand for three days. The passage of hydrogen through *L* was then begun and the emanation was led into the tube *W*. After the amount of the emanation had been measured, hydrogen was passed through the solution in *L* until the emanation ceased to be given off. *S* was then closed, *R* was opened and hydrogen was passed through *W* until the emanation was entirely removed from the tube. The exit tube *B* was connected with a flue to prevent the escape of the emanation into the room. The apparatus was then allowed to stand for three days, after which interval it was found that the excited activity on the walls of the tube *W* had entirely disappeared. The measurements were then repeated in the manner described above. After a series of such measurements, the hydrogen generator was disconnected at *Z* and atmospheric air was used instead of hydrogen. The measurements obtained with hydrogen and air are given in the following table, the readings in the table being scale divisions per second.

Hydrogen.	Air.	Hydrogen.	Air.
3.0	8	2.5	9
2.5	10	2.0	11
2.0	11	2.5	8
3.0	9	3.0	10
2.0	12	2.5	10
2.0	10	2.0	11
		Average 2.41	9.9

The above readings may be regarded as indicative of the amount of emanation if one assumes that the conditions, such as sensitiveness of electrometer, proportion of emanation to total gas, etc., were always the same. The constancy of the readings in both series appears to support the validity of this assumption.

The ratio of the average of the results in hydrogen to the average of the results in air, 0.243, closely approaches the relative conductivities of the two gases for the alpha-ray, 0.226. If the amount of emanation were appreciably greater in air than in hydrogen, the ratio obtained might naturally be expected to vary more widely from that of the relative conductivities of the two gases. Inasmuch as this result was not obtained the above experiments would seem to support the statements of Rutherford that the emanation is not due to the excited activity of a constituent of atmospheric air.

CORNELL UNIVERSITY,
March, 1907.

THE SCIENTIFIC WORK OF DEWITT BRISTOL BRACE.

By Edward L. Nichols.

PROFESSOR BRACE received the degree of doctor of philosophy twenty years ago; at which time he presented to the faculty at Berlin a thesis upon the *Magnetic Rotation of the Plane of Polarization of Light.*[1] His career as a scientific investigator, as indicated by published work, may, however, be said to date from 1897. For twelve years no scientific paper by his pen had appeared, and for this long period he had been isolated in a sense which only those can realize who know the conditions at that time, as regards physics, in the regions lying beyond the Mississippi river. His paper published that year in the Philosophical Magazine under the title: *Observations on Light Propagated in a Dielectric Normal to the Lines of Force*,[2] showed that although silent he had not been idle. The result of this search for an optical effect of the magnetic field upon light moving at right angles to the lines of force was negative. That the effect looked for, if it existed, must necessarily be very small seems to have been fully recognized, and the experiment was planned with this fact in mind. The magnet employed was of unusual size with cores 8″ in diameter and 36″ long, carrying 650 pounds of wire and having a total weight of about two tons. The Faraday effect in glass was eliminated by sending a beam of light through two cylinders each 22 cm. long and placed end to end in the field, and reflecting the beam backward upon its course by a mirror adjusted so as to be parallel with the lines of force. The sensibility of the apparatus was seriously vitiated by depolarization due to accidental double refraction in the glass, for which reason in the final experiments a tube 48 cm. long containing water was employed. This was placed between the poles of the electromagnet, the pole pieces being 50.7 cm. by 8.7

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[1] Wiedemann's Annalen, Vol. 26, page 576.

[2] Philosophical Magazine, Series 5, Vol. 44, page 342.

cm. A beam of sunlight was passed four times through the tube by reflection at the mirrors, making a total path of 198 cm. A Lippich half shade polarizer served to polarize the ray in any desired plane with respect to the magnetic field. The plane of polarization of the emerging ray could be determined by means of an analyzer reading to .003° and capable of detecting with certainty changes in the plane of polarization of .01°. No effect could be detected even with this exceedingly sensitive apparatus, indicating entire absence of double refraction within the limits of observation. The conclusion was that, according to Maxwell's view of the state of polarization and stress in such a medium, the pressure at right angles to and the tension along the lines of force affect the propagation of light by an amount less than $2 \times 10^{-14}\lambda$ for a C.G.S. unit of intensity per centimeter.

In 1899 Brace brought out his new spectrophotometer, in which the introduction of an arrangement such as the Lummer-Brodhun body is rendered unnecessary by the very simple and ingenious device of splitting the dispersion prism and silvering a portion of its face so that rays from the two collimators entering the prism in directions symmetrical to the section will issue in parallel paths and give similar spectra suitably located for comparison.

Brace's continued interest in the action of the magnetic field upon light is indicated by his next paper, which appeared in 1901 and which dealt with the resolution of light into its circular components in the Faraday effect.[1] The principle established by Fresnel that two opposite circular vibrations will be equivalent to a linear vibration and *vice versa*, which that investigator had verified for quartz, had been generally accepted as the explanation of the Faraday effect. Direct experimental confirmation of this theory was, however, lacking. It is true that Brace in his thesis work in 1885 had obtained a displacement of the bands when one of the two interfering rays in a Jamin refractometer passed through a cylinder of glass placed between the poles of a magnet, the two rays being circularly polarized. A like result had been obtained by Righi, Becquerel and Cornu. Experiments by the interference method were, how ever, not regarded as conclusive and Brace in this paper undertook to apply the more direct method of refraction.

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[1] Philosophical Magazine, Series 6, Vol. I., page 464.

If natural light is broken up into two opposite circular components when propagated along the lines of force, and if these travel with slightly different velocities, a linear ray moving over a sufficiently long path along the lines of force should be sufficiently separated as to direction to give a double image. By a series of total reflections within a pair of glass prisms placed in the field of a powerful electromagnet, the necessary conditions for resolution were obtained and the existence of the two circular vibrations opposed to each other in direction was established. The phenomnon predicted by Fresnel in his theory and demonstrated by him in the case of quartz was thus found to hold likewise of magnetic rotary polarization. The demonstration in the latter case is of vastly greater experimental difficulty, requiring a very long path through the glass and a powerful field. To observe the separation of the rays the optical conditions must be very carefully preserved and the surfaces at which the successive total reflections occur must be figured with great accuracy. The attainment of a definite result in this experiment is indeed to be regarded as a notable achievement.

At the Denver meeting of the American Association for the Advancement of Science Brace acted as chairman of Section B and in the following year, 1902, he delivered the vice-president's address before that section. He chose for his subject the *Group Velocity and the Wave Velocity of Light* and a more lucid and altogether admirable semi-popular presentation of a difficult subject would be hard to find.

The following year, 1903, marks the opening of the most important, productive and brilliant period of the scientific activity of Professor Brace. In that year he published the first of the series of three papers on his very beautiful and ingenious device, the sensitive strip spectro-polariscope,[1] an instrument which makes it possible to determine the plane of polarization of light with a degree of accuracy hitherto impossible and to detect traces of double refraction unobservable by other means. The second paper dealing with this method of studying polarized light appeared in the PHYSICAL REVIEW in 1904, under the title, *Half Shade Elliptical Polarizer and Compensator.*[1] The theory of the instrument was further developed in a paper on the *Errors of the Half Shade Elliptical Polarizer.*[2] The new instrument found immediate application in a series of bold attempts to detect double refraction in matter moving through the ether.[3]

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[1] Philosophical Magazine, Series 6, Vol. 2, Page 161.

The negative results obtained by Michelson and Morley in their careful experiments upon the interference of light in the direction of the earth's motion and at right angles to the same had led to the suggestion by Fitzgerald and by Lorentz of a mechanical action of the ether upon the refracting medium such as to counterbalance and annul the effect looked for. The medium used in Brace's experiments was water, which was contained in a trough 413 cm. long, 15 cm. wide and 27 cm. deep. A beam of polarized sunlight was sent several times through the entire length of this body of water by a series of reflections, the total path being 756 cm., after which it passed through the sensitive strip and the compensator of the polariscope and through the analyzing nicol. By means of this exceedingly delicate device the slightest trace of double refraction in the medium could have been detected and any such effect due to motion through the ether would show itself in the slight change of optical condition when the trough was placed in the direction of the earth's orbital motion and then at right angles to the same. No such change was observed and when the instrument was set to match in one position and then shifted to 90° no variation of the setting was necessary after the rotation. Tests of the sensitiveness of the method showed that 7.8×10^{-13} represented the greatest difference in velocity or in index between the two components of the ray which could exist without being detected. The effect which might be expected under the theory due to an increase in density in the direction of the motion was computed to be about 1,600 times greater than the smallest effect which could be observed.

Experiments were likewise tried with glass using two prisms cemented end to end and giving a single body 42 cm. long with polished ends. These end surfaces were silvered with the exception of a strip at the top and bottom, and a beam of light from the polarizer was reflected backwards and forwards from ten to eighteen times. The total path through the glass was from 420 to 756 cm. Difficulties with flexure of the trough necessitated, however, a modification of this arrangement in which two cylinders of flat glass each 22.3 cm. long were mounted between the polarizing and analyzing systems. The greatest difference in velocity or in index between the two components which could exist in this glass was found to be

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[1] PHYSICAL REVIEW, Vol. 18, page 70.

[2] PHYSICAL REVIEW, Vol. 20, page 218.

[3] Philosophical Magazine, Series 6, Vol. 7, page 317.

$$4.5 \times 10^{-11},$$

whereas the contraction in the medium from the change computed under Rayleigh's assumptions should have been fifty times greater than that which could have been detected. The conclusion is that the contraction hypothesis cannot explain the negative results of previous experiments and that if the ether does not move with the matter which it surrounds, the effect of the relative motion on the dimensions of a substance must be exceedingly small.

To the problem of the ether drift Professor Brace devoted two further papers, the last to appear before his death. In the first of these[1] he discussed the recent theoretical work of Larmor, Lorentz, Poincaré, Hasenoehrl and Abraham and considered the possibility of the experimental determination of a quiescent or mobile ether. Preliminary tests of one such method, in which he used a very ingenious combination of condensers giving the Kerr effect with the half shade elliptical polarizer, were found to give promise of sufficient delicacy for the direct comparison of the velocities of light with and against the drift.

The second of these[2] important papers is entitled *The Ether Drift and Rotary Polarization.* Mascart, who studied the effect of the motion of the earth on the rotary power of quartz, found that when the direction of the ray is reversed no effect upon the rotary power as large as one twenty thousandth could exist. Rayleigh, who repeated the work in 1902, found that reversal of the ray in the direction of the motion of the earth did not change the rotation by one part in a hundred thousand. By substituting oil of caraway seed, which although inferior in rotary power to quartz is free from the optical imperfections of the latter substance, Brace succeeded in showing that the effect of the motion of the earth on the rotation of the plane of polarization in optically active media is probably less than one part in 10,000,000. In some of the measurements the ray of white light was, by means of a suitable series of reflections, sent four times throughout the length of a trough containing the oil thus securing a total path of 1,640 cm. and a total rotation of 21,300 degrees. With the sensitive strip polariscope a direct change of one one hundred and twentieth of a degree could have been detected. The success of this investigation, which pushes the limit beyond which the effect of the earth, if it exist, must lie, probably fifty times as far as had been attained by any previous observer, depends upon the fact that by sending the beam over equal and opposite paths through the liquid complete neutralization of the rotatory effect is obtained whereas the effect due to the earth's motion would be additive.

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[1] Philosophical Magazine, Series 6, Vol. 10, page 71.

[2] Philosophical Magazine, Series 6, Vol. 10, page 83.

The experimental difficulties of researches of this character are almost undescribably great and the results obtained by Brace although uniformly negative are to be accounted among the greatest achievements in modern physics. Those of us who have had the pleasure of listening to the presentation of Brace's papers at the American Physical Society and elsewhere are aware that the published results are only a small part of what he had in mind in his attack upon the formidable problem of the ether drift, and there is reason to believe that but for his premature death other and still more striking and important results would have followed.

In addition to the papers already considered in this brief sketch of the scientific work of Professor Brace another important paper which appeared posthumously in the PHYSICAL REVIEW[1] should be mentioned. This paper, which is perhaps the latest and final product of the scientific activity of the author, is on the *Application of Spectral Bands in determining Anomalous Dispersion.* In it the method of determining sudden changes in the index of refraction of a medium by noting the crowding together or the separation of the dark bands observed in the spectrum of two systems of interfering rays is fully developed and the advantage of this method with substances such as fuchsine over the usual prismatic method is pointed out. The actual determination of the existence of anomalous dispersion in many cases in which the absorption spectrum indicates its presence is of great importance. This method when properly applied will doubtless add greatly to our quantitative knowledge of the subject.

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[1] PHYSICAL REVIEW, Vol. 21, page 289.

PUBLICATIONS BY DEWITT B. BRACE.

	Pages.	Figs.
1. "Ueber die magnetische Drehung der Polarisationsebene und einige besondere Fälle der Refraction." (Dissertation, Berlin, 1885).	33	8
2. "On the Transparency of the Ether." (University Studies, 1888).	16	1
3. "Observations on Light Propagated in a Dielectic Normal to the Lines of Force." (Phil. Mag., 1897.)	8	2
4. "Description of a New Spectrophotometer and an Optical Method of Calibration." (Phil. Mag., 1899.)	10	5
5. "On a New System for Spectral Photometric Work." (Astrophys. Journal, 1900.)	18	9
6. "The Observation of the Resolution of Light into its circular components in the Faraday Effect." (Phil. Mag., 1901.)	12	4
7. "The Group Velocity and the Wave Velocity of Light." (Vice-presidential address before Section B of the A.A.A.S., 1902.)	20	6
8. "A Sensitive Strip Polariscope." (Phil. Mag., 1903.)	9	0
9. "A Half Shade Elliptical Polarizer and Compensator." (PHYS. REVIEW, 1904.)	18	0
10. "On Double Refraction in Matter moving through the Æther." (Boftzmann Festschrift and Phil. Mag., 1904.)	12	1
11. "Notes on errors in the Half Shade Elliptical Polarizer." (PHYS. REVIEW, 1904.)	13	0
12. "The Negative Results of Second and Third Order Tests of the 'Æther Drift' and Possibly First Order Methods." (Phil. Mag., 1905.)	10	1
13. "The 'Æther Drift' and Rotary Polarization." (Phil. Mag., 1905.)	14	3
14. "On Application of Spectral Bands in determining Anomalous Dispersion and Achromatic Systems of Various Types." (PHYS. REVIEW, 1905.)		
15. "A Repetition of Fizeau's Experiment on the Rotation of a Refracted Ray Produced by the Earth's Motion." (Phil. Mag., 1905.)		

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE THIRTY-SEVENTH MEETING.

THE Spring meeting of the Physical Society was held in Washington, D. C., on Friday, April 19, and Saturday, April 20, 1907. In the absence of the president, Mr. A. G. Webster was made chairman of the meeting.

The Friday afternoon session was held at the Cosmos Club, and the Saturday sessions at the Bureau of Standards.

The Report of the treasurer for the year 1906 was read by the Secretary and on motion accepted.

Announcement was also made of the resignation of Mr. Hallock as treasurer of the Society, the resignation to take effect on July 1, 1907, and of the election of Mr. J. S. Ames as treasurer for the remainder of the unexpired term.

It was moved, and carried unanimously, that the Society express its thanks to the retiring treasurer, Mr. Hallock, for his long and faithful service to the Society since its founding.

The following papers were presented:

1. An Exact Formula for the Mutual Inductance of Coaxial Solenoids. LOUIS COHEN.

2. The Mutual Inductance of Coaxial Solenoids. E. B. ROSA and LOUIS COHEN.

3. A New Method for the Absolute Measurement of Inductance. E. B. ROSA.

4. Calibration of Calorimetric Platinum Resistance Thermometers. H. C. DICKINSON and E. F. MUELLER.

5. Some New Measurements with the Gas Thermometer. JOHN KAY CLEMENT.

6. An Accurate Calorimeter. W. P. WHITE.

7. Temperature Control of the Cheltenham Magnetic Observatory. JOHN E. BURBANK.

8. Finding Direction by Means of Submarine Sound Signals. (Read by title.) LUCIEN IRA BLAKE.

9. The Heat of Vaporization of Water. A. W. SMITH.

10. The Plan and Object of the Terrestrial Magnetic Work of the Carnegie Institution of Washington. L. A. BAUER.

11. The Influence of Atomic Weight upon the Maxima of Absorption and Reflection Bands. W. W. Coblentz.

12. The Fluorescence, Absorption and Magnetic Rotation Spectra of Potassium Vapor. T. S. Carter.

13. A Redetermination of the Wave-lengths of the Iron Lines. A. H. Pfund.

14. An Apparatus for the Production of a Rotating Electric Arc under Heavy Pressure. W. J. Humphreys.

15. Some Effects of Heavy Pressures on Arc Spectra. W. J. Humphreys.

16. Absorption and Emission Spectra of the Salts of Neodymium and Erbium. J. A. Anderson.

17. The Change of Phase due to the Passage of Electromagnetic Waves through Thin Plates and the Index of Refraction of Water for such Waves with Applications to the Optics of Thin Films and Prisms. Wm. R. Blair.

18. On the Measurement of the Intensity and Damping of Hertzian Waves in Free Air. W. P. White.

19. Lecture Experiment on Magnetically Coupled Systems. A. G. Webster.

20. Mercurous Sulphate, Cadmium Sulphate and the Cadmium Standard Cell. Geo. A. Hulett.

21. A New Method of using the Ultra-violet Microscope. L. J. Briggs and W. T. Swingle.

22. The Effect of the Time of Passage of a Quantity of Electricity on the Throw of a Ballistic Galvanometer. F. Wenner.

23. The Operation of a Direct Current Shunt Wound Motor in Synchronism with a Tuning Fork. R. H. Hough and F. Wenner.

24. (*a*) On the Final Value found for the Ratio of the Electrical Units, v. (*b*) On the Methods Available for the Accurate Determination of v. E. B. Rosa and N. E. Dorsey.

25. The Simultaneous Measurement of the Capacity and Power-factor of Condensers. Frederick W. Grover.

26. On the Variation of Resistances with Atmospheric Humidity. E. B. Rosa and H. D. Babcock.

Ernest Merritt, *Secretary*.

Treasurer's Report for the Year 1906.

Receipts,		
Balance brought forward	$1,742.53	
Interest on balance	69.70	
Annual dues, including exchange, etc.	1,742.30	
Arrears, of annual dues,	41.50	
Initiation fees	51.00	
Life-member fees	100.00	
Total	$3,747.03	$3,747.03

Expenditures,		
Paid the Physical Review	$1,123.50	
Paid Science abstracts	556.00	
Secretary's office	146.90	
Treasurer's office	31.50	
Exchange	4.20	
Printing	54.70	
Total	$1,916.80	$1,916.80
Balance carried forward		1,830.23
		$3,747.03
Normal Income, approximately as above:		
Interest on balance including life fees	$ 75.00	
Annual dues	1,750.00	
Initiation fees, very doubtful, say	30.00	
Total regular income	$1,855.00	$1,855.00
Annual deficit over income		61.00
Regular annual expenditures as above for 1906		$1,916.00

Wm. Hallock,
Treasurer.

Columbia University,
April 19, 1907.

The Physical Nature of Meteor Trains.[1]

By C. C. Trowbridge.

PERSISTENT meteor trains are luminous clouds occasionally formed by meteors, which sometimes remain visible for many minutes. There have been numerous cases where trains have been visible to the naked eye for three-quarters of an hour, and bright enough to be seen two hundred miles distant. The trains are usually ten miles long and gradually expand to about a mile in diameter and are evidently self-luminous.

Previous Investigations on the Subject.

Meteor trains have been observed by many astronomers but are still regarded as an unsolved mystery. But few papers have been published on the subject, other than those which are merely a record of observations.

J. Ennis has described seven different trains, and E. E. Barnard has published a paper relating to the drift of five trains observed at Nashville, U. S. No systematic attempts to collect the records together appear to have been made heretofore, yet it is evident that a large store of useful scientific information concerning the earth's atmosphere lies in these records.

The Source of Data.

The observations of meteor trains have been made chiefly by well-known astronomers at astronomical observatories, who have recorded the facts in scientific journals. Most of the records have been since 1866. Some of the astronomers are as follows:

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

In England: Herschel, Denning, Barkas, Macgregor, etc.

In the United States: Twining, Newton, Young, Barnard, Gilman, etc.

The records are very complete in many cases, in others only a few important items are given. The greater part are trustworthy and scientific observations.

The facts given in this abstract and determined by a comparative study are necessarily of the nature of a preliminary report but they are of sufficient interest to warrant their presentation at this time. About 175 observations have been studied.

SUMMARY OF THE FACTS DETERMINED.

Among the facts determined by this study of meteor trains are the following:

1. Comparisons of drawings and descriptions seem to show that meteor trains often fade first in the center, or that the luminosity is greatest near the outside of the train.

2. The meteor train resembles in a number of respects the phosphorescent afterglow formed in an electrodeless discharge tube, since: (1) The rate of the diffusion of each is of the same order; (2) the afterglow persists at liquid air tempertures; (3) the afterglow under some circumstances persists for nearly 20 minutes; (4) both appear to have line or narrow band spectra.

3. The trains assume curious shapes apparently owing chiefly to air currents at different altitudes in different directions, there being little if any downward movement apparent. The results of a study of the meteor train drifts at great altitudes are given in another paper.

4. *The Duration of Meteor Trains.*

The time that a meteor train remains visible depends largely on the distance between the meteor and the observer. Some idea, however, of the average duration can be had from the following facts. In a list ot meteor trains collected, 53 trains having a duration of more than one minute were as follows:

6 remained visible over 40 minutes (40–60 min.)
7 " " " 20 " (20–40 min.)
12 " " " 10 " (10–20 min.)
12 " " " 5 " (5–10 min.)

Thus there were 37 trains which remained visible to the naked eye from five minutes to one hour, the average of 53 trains being 14.8 minutes.

5. *Color of Meteor Trains.*

The observations on the color of meteor trains given are only those where very definite statements regarding the persistent train were made.

Thus the possibility was avoided of confusion of the color of the train proper with the color of the nucleus, or of the mass of sparks which sometimes persists for a few seconds in the track of a meteor.

The colors of the trains observed varied considerably, being red, orange yellow, yellow, green, bluish, silvery and white, but the greater part were green or greenish white. The colors of 25 trains seen at night were as follows:

Red	1
Yellow	1
Green (emerald green and bluish green)	12
Blue (bluish and greenish blue)	4
Silver or grey	4
White	3
	25

In several cases green trains changed gradually to white, and in one case to a dull reddish white. In another a red train changed to bluish.

Thus out of 25 trains, 12 are green and 11 either bluish, silver or white. If, as in several cases observed, the trains change from green to white, a number of white trains are to be expected as a second stage or lower degree of luminosity of an originally green train.

Eleven other trains evidently illuminated by sunlight were colored as follows: Red 2, pink 3, white 3, light blue 1, white changing to red 1, yellow to red 1. These colors are to be expected in the case of trains shining in sunlight.

6. *Altitude of Meteor Trains.*

The observations on the height of meteor trains have been very accurately made in a few cases and approximately made in a number of others. The following are the heights of ten trains that have been computed, the trains having been observed from two or more stations widely apart. The figures below are the number of miles above the surface of the earth: 50, 75, "over 60," 54, 51, 58 (65 to 52), 77, 70, 59, 54. The average of the heights of ten trains given above is 60.8. It is evident that trains seldom if ever occur below 50 miles or over 70, which agrees with a statement of W. F. Denning. In this zone 50 to 70 miles above the surface of the earth. We must expect, therefore, that there are conditions which are favorable to both the formation and the persistence of the mysterious luminosity of the meteor trains. The condition which is most important to electrical discharges and glows in gases is pressure. We may therefore look to the pressure of the atmosphere prevailing at 50 to 65 or 70 miles height as a probable condition which determines the fact that meteor trains are observed at that height only.

7. *Diffusion of Meteor Trains.*

It is evident that the lateral expansion of the train, about one mile in ten minutes, is due mainly to rapid gas diffusion at the low pressure at the altitude of 60 miles. Six trains, three of which the altitude was measured and three estimated at 60 miles, give a mean rate of diffusion of 2.3 meters per second. The diffusion of the afterglow of the electrodeless discharge in small tubes is of the order of several meters per second. Moreover, the calculated diffusion rate of air and a gas like CO_2 at 0.1 mm. gas pressure (which is also the pressure at which the afterglow occurs for long duration), and a temperature of — 150° C. is about 2 meters per second. Therefore, if the meteor train is an afterglow phenomenon it would appear as if the pressure of the atmosphere at 60 miles might be about 0.1 mm. and the temperature not far from — 150° C.

ON ATMOSPHERIC CURRENTS ABOVE FIFTY MILES FROM THE SURFACE OF THE EARTH.[1]

BY C. C. TROWBRIDGE.

THE systematic study of meteor trains is of much importance to meteorology because it is the only means by which atmospheric currents at very great altitudes above the surface of the earth can be studied. The drift of certain trains has been described and discussed to some extent by Newton and Barnard, the latter has described five trains seen in the United States having southeasterly and northeasterly drifts, and also called attention to the importance of this method of studying the movements of the atmosphere.

The writer has made a systematic study of meteor trains chiefly in order to find an explanation of their long-enduring luminosity in the light of recent advances in physics. The investigation has necessitated the making of a catalogue of meteor train observations which has not been done hitherto.

Thus far about sixty-five observations giving meteor train drifts have been found.

The observations when tabulated and studied add considerably to our knowledge of the movements of the atmosphere at great altitudes.

Recently much has been accomplished by cloud observations and by means of kites and balloons provided with recording apparatus, but at far lower heights than those where meteor train movements occur. The limit to which kites can be raised is about three miles, and while cloud observations have been made to a height of eight miles, the usual region for cloud study is below this level. In a few cases so-called ice clouds have been observed at a considerably higher altitude.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

Balloons with recording meteorological apparatus have been sent to an altitude of about fifteen miles, but the usual limit for successful operation is below twelve miles. The distribution of volcanic dust over the earth's surface by air currents has served as a means of information concerning atmospheric drifts at altitudes at the equator from twenty and perhaps thirty miles, but the only successful studies of importance in this direction are the well known facts of the drifts of the dust of Krakatoa following the eruption of that volcano in 1883 showing a westerly current of 70 to 80 miles per hour at the equator at an altitude probably between 20 and 25 miles above the earth's surface.

Far above these regions lies what may be called the meteor train zone. This extends from 50 to 65 (or perhaps 70) miles above the surface of the earth. Of the hundred or so meteor trains, chiefly observed in England and the United States, in the catalogue made by the writer, in the case of over sixty-five the drift has been observed. Of these sixty-one show the following directions:

Trains seen at night.		Trains seen in daylight or twilight.[1]	
North	9		
Northeast	4		
East	11	Northeast	3
Southeast	12	West	5
South	7		
Southwest	4		
West	0		
Northwest	3		
Drift in two strata in different directions	3		
Total	53		8

The greatest velocities observed were as follows: In one case 125 miles per hour northward drift at over 60 miles altitude above New England in the United States, and in another 61 miles in 30 minutes southeast drift or 122 miles per hour at 54 miles altitude above England.

It is seen from these facts that the drift at an altitude of 50 to 65 miles may be in any direction in northern United States and in England, but that the drift toward the eastward predominates.

In the case of over 25 of the above trains the rate of drift in degrees or miles was recorded.

It has generally been supposed that at great altitudes above the surface of the earth, in the northern hemisphere, there would be a steady eastward drift. The facts determined thus far from a comparative study of meteor train drifts clearly show that, at least in the temperate zone, the drift may be in any direction but predominating toward the eastward.

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[1] Some evidently below 50 miles.

The indications are that there are superimposed currents of different velocities and often in opposite directions in zones of five to ten miles depth; and also that in some cases in one zone there will be a calm, while directly above or below it there will be a current of considerable velocity.

On November 14, 1866, in England and Scotland, there was a current to the south or southeast shown by eight out of ten observations; and on November 14, 1901, the drift in the western part of the United States appeared to be north, while that in the middle and eastern portions was towards the northeast.

CALIBRATION OF CALORIMETRIC PLATINUM RESISTANCE THERMOMETERS.[1]

BY H. C. DICKINSON AND E. F. MUELLER.

THE problem was presented of measuring the difference of temperature between the inlet and outlet of a flow calorimeter under conditions requiring thermometers of high sensibility, small size and small time lag. Some form of resistance thermometer seemed to offer the greatest promise of success and after preliminary experiments a pair of such thermometers was built which proved to be so reliable and so adapted to general use that a careful study of them was made. These thermometers consist of four leads of thin sheet copper (.12 mm.), two of them connected to a coil of .02 mm. platinum wire wound on a flat mica strip, and the other two connected together for compensation. The leads and coil are enclosed in a thin copper sheath with mica insulation. This finished sheath is about 1 mm. thick, 7 mm. wide and 15 cm. long, outside dimensions.

The resistance of these thermometers is approximately 30 and 40 ohms in ice and steam respectively, temperature equilibrium is established to 0°.001 in 3 or 4 seconds and the water equivalent of the thermometers is less than one gram. Comparisons were made with the temperature scale of the Bureau of Standards by means of four primary standard mercurial thermometers at 30°, 40°, 50°, 60°, and 70°. These comparisons showed that the δ of the Calendar formula is constant within the limits of error of the best mercurial thermometer comparisons between 0° and 100°. The values for the two thermometers in question were respectively $\delta = 1.554 \pm .001_5$ and $\delta = 1.556 \pm .002$, representing a probable error (computed) of 0°.0005 at 50° C. It is therefore shown that these thermometers may be used to represent the mean scale of the Bureau of Standards to within 0°.001 between 0° and 100°.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

Since this factor δ may be taken as constant it might be equally well determined by observations at a single sufficiently well known point. For the establishment of such a point it was decided to make a redetermination of the transition temperature of sodium sulphate by means of these calibrated resistance thermometers. This temperature was found by Richards and Wells to be 32°.383 C. The use of resistance thermometers offers some decided advantages over a direct determination with mercurial thermometers as it gives a scale about 100 times more open. The results of this investigation are as follows. The transition temperature of a single sample of sodium sulphate is constant to within 0°.001 even for considerable changes of external temperature; some impurities such as sodium chloride may lower this temperature by 0°.001 for .001 per cent. The transition temperature of a sample of "Kahlbaum" sodium sulphate as obtained was found to be 32°.380, and of the purest samples obtained by recrystalization and preparation by different methods gave a temperature between 32°.384 and 32°.385.

ABSORPTION AND EMISSION SPECTRA OF THE SALTS OF NEODYMIUM AND ERBIUM.[1]

By J. A. Anderson.

As is well known, some of the rare earths give three kinds of spectra, besides those of the arc and spark.

These are:

1. Absorption by solutions.
2. Absorption by solid salts observed when white light is diffusely reflected from them.
3. Emission by solid oxides and phosphates when heated to incandescence.

The object of the present investigation is to try to find some relation between these.

The results show that the absorption spectrum of the solid salts changes with temperature, and that in case of the oxides at high temperature it becomes identical with the emission spectrum, hence showing not only that we are dealing with a pure temperature radiation, but also that the two spectra are due to the same vibrators.

The effect of raising the temperature is, in general, to widen the absorption bands, and a similar change in the absorption of any salt is produced by dissolving it in water. These changes may be explained by assuming that all absorption is caused by electrons inside the metallic atom, the periods of which depend upon the distances and orientation of the other atoms in the molecule. Anything which would increase the amplitude of the swings of these combined atoms, such as a rise in temperature, or the presence of a solvent, would cause a widening of the absorption bands.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

JOHNS HOPKINS UNIVERSITY,
April 4, 1907.

THE CHANGE OF PHASE DUE TO THE PASSAGE OF ELECTROMAGNETIC WAVES THROUGH THIN PLATES AND THE INDEX OF REFRACTION OF WATER FOR SUCH WAVES WITH APPLICATIONS TO THE OPTICS OF THIN FILMS AND PRISMS. PART 2.[1]

BY WM. R. BLAIR.

THIS paper deals with the last part of the above subject, the first or experimental part having been presented at the November meeting of the Society in Chicago.

The matter to be presented is as follows:

1. A summary of the experimental results given in Part 1.
2. Theoretical discussion of the change of phase due to the passage of electric waves through thin films and prisms and comparison of computed and observed results.
3. The possible bearing of this work upon the experiments of Johonnot and others on thin films and those of Kundt and Lampa on thin prisms.

RESEARCH OBS.,
MT. WEATHER, VA.

SOME NEW MEASUREMENTS WITH THE GAS THERMOMETER.[1]

BY J. K. CLEMENT.

A NEW gas thermometer has been designed and built in the Geophysical Laboratory, for the measurement of high temperatures, in which a conscientious effort has been made to reduce the magnitude of the errors common to the well-known European gas thermometers. The constant volume principle and the thermoelement as the medium of comparison are retained, but the sensibility of the instrument has been increased until the error of an individual reading is less than the error in the corresponding barometer reading. The bulb (10 per cent. iridium, 90 per cent. platinum, 200 c.c. capacity) is suspended in an air-tight bomb by means of which the pressure outside of the bulb is maintained nearly equal to the inside pressure. This makes it possible to use a high initial pressure — about 300 mm. Hg. — without deforming the bulb at high temperatures, and thus to obtain a sensibility of about 1 mm. of the manometer scale per degree C. without diminishing the range of the instrument.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

By taking advantage of the fact that the increased viscosity of the expanding medium (nitrogen) due to rise in temperature affects only the heated portion of the dead space, it has been found possible to reduce the diameter of the cold portion of the capillary tube which connects the bulb with the manometer tube, to 0.5 mm., so that the ratio of the volume of unheated space to the volume of the bulb, $v_s/V = 0.0015$. (Holborn and Day, $v_s/V = 0.0046$; Jaquerod and Perrot, $v_s/V = 0.018$.) The entire correction for the unheated space is therefore only 3.8° at 1,000° and 5.° at 1,200°.

One of the most formidable obstacles encountered was the contamination of the thermocouples with the iridium. To overcome this, it was found necessary to use a heating coil of very pure platinum wire, a nitrogen atmosphere outside the bulb, and to cover the platinum-iridium bulb with a very viscous glaze. Notwithstanding these precautions, the thermocouples deteriorate somewhat when heated for any considerable time at temperatures above 1,000°.

Perhaps the most important source of error which has been overcome is the variation of temperature along the bulb itself due to the cooling at the ends of the furnace tube. By the use of auxiliary heating coils at either end of the main coil, the gradient between the center of the bulb and either end of it has been made less than 0.5°.

We have made a number of comparisons between this instrument and *Pt–PtRh* thermocouples from 300° to 1,200° at intervals of 25°, in which the aggregate error from all sources appears to be well within 1°. Time has not yet permitted a complete series of melting-point determinations, but the following are offered tentatively, subject to minor changes when a sufficient number of determinations shall have been made:

Zinc,	418.4°	Greatest deviation from the mean,	± 0.4°	15 determinations.
Silver,	957.5°	" " " " "		1 determination.
Gold,	1059.1°	" " " " "		1 "
Copper,	1080.2°	" " " " "	± 0.5°	5 determinations.

SOME EFFECTS OF HEAVY PRESSURES ON ARC SPECTRA.[1]

BY W. J. HUMPHREYS.

ARC spectra of a number of metals were obtained at pressures of 42, 69, and 101 atmospheres. A Rowland concave grating of 21½ feet focal length was used, and the work done in the second and third orders of the spectra. The compression cylinder, obtained through a grant from the Rumford Fund, is described elsewhere.

The following are some of the effects noted:

1. Reversals, especially in the shorter wave-lengths, are much more pronounced and frequent under heavy than under light pressures.

2. Pressure seems to increase the widths of all lines, though quite unequally. Occasionally lines are found narrower when taken under heavy pressure, but apparently this is due to decrease in exposure, since in these cases the lines are rather weak with gradually fading edges.

3. The lines of the carbon and cyanogen bands are not certainly displaced even at the highest pressures.

4. The wave-lengths of all other lines examined increase approximately proportional to the increase in pressure up to the highest pressures used, though this increase or shift is very different for different lines even of the same element.

5. A number of these shifts amount to approximately half an Ångstrom unit, and a few are displaced to even a distinctly greater extent.

6. There is to me no convincing evidence that any considerable portion of the shift is due to anything other than the *total* pressure.

7. The extent of the shift of a line is practically independent of the amount of material used, and whether or not it is reversed.

8. So far as I can judge from the scanty data on the Zeeman effect, those lines which are strongly separated by a magnetic field have correspondingly large pressure shifts, and conversely.

9. The increase in wave-length is only a small fraction of what it would be if the luminous particles obeyed the same laws that control a Hertzian oscillator.

10. The intensities of titanium lines seem to be invariably increased by pressure; while iron, in this particular, behaves very irregularly. Some of its lines are increased in intensity, others are decreased, and a number even disappear.

The many spectrum negatives used in this work were obtained by the aid of the large spectroscope and the pump of the liquid air plant of the University of Virginia, and I wish to thank President Alderman and Professor Smith for their kindness in placing this excellent equipment at my disposal.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

AN APPARATUS FOR THE PRODUCTION OF A ROTATING ELECTRIC ARC UNDER HEAVY PRESSURE.[1]

BY W. J. HUMPHREYS.

IN the production of arc spectra under heavy pressure it is necessary for one of the poles to rotate, since it is not possible at pressures of fifty or more atmospheres, at least with ordinary voltages of two hundred and fifty or less, to maintain a steady arc between fixed poles.

The apparatus that proved efficient for this purpose consists of a cylindrical forged steel shell, with spherical ends terminating in short extensions through which the carbon carriers pass, and onto which attachments for tightening the packing around these carriers and for moving them endwise are screwed. The inside diameter of the shell, which is one and one half inches thick, is six inches, and its length sixteen inches. One side of the shell carries, at its middle, a thick tube onto the end of which is screwed a cap containing a quartz window cemented in place. A steel tube, carrying a pressure gauge and leading through a moisture separator provided with a stopcock to a compression pump, is screwed into the shell near one end. The carbon carriers, heavy steel tubes, pass through stuffing boxes at the ends of the shell and are easily adjusted lengthwise by means of wheel-shaped nuts fitting on screw threads cut some four or five inches along their outer ends. The tube that carries the fixed pole has a rod for the current running along it axially and screwed at the outer end through a piece of ebonite held in place by a suitable cap, and passing at the inner end through a block of mica. The space between the rod and the tube is tightly packed with ground mica, which produces both an airtight condition and electric insulation. The rotating pole, excentrically touched by the fixed pole, is a disk of carbon as large as will pass through the packing of the stuffing box — about an inch in diameter — suitably mounted on the end of a rod running through a steel tube similar to the one carrying the fixed pole. The inner end of this tube has an iron plug through which the rod passes in a loose bearing. The outer end is provided with a cap through a part of which the rod passes with a tolerably close fit. The end thrust of this rod is supported by a ball bearing, and a grooved pulley is provided near this bearing for rotation. Appreciable air leak is prevented by packing the space between the rotating rod and steel tube with Ceylon graphite, which is micaceous in structure, and readily blown into any leaky opening. It is also a lubricant and a conductor, both desirable qualities in this case.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

The apparatus worked nicely at all pressures tried, the highest being one hundred and one atmospheres.

I wish to thank the committee of the Rumford Fund, American Academy of Sciences, for a grant that met most of the expense incident to the construction of the above piece of pressure apparatus.

THE OPERATION OF A DIRECT CURRENT SHUNT WOUND MOTOR IN SYNCHRONISM WITH A TUNING FORK.[1]

BY R. H. HOUGH AND F. WENNER.

THE work here reported was undertaken as a first step in a problem in alternating currents where it is necessary to have a very constant frequency even when the power is changed by as much as 1 or 2 k. w. Several different machines were used in this work but the one used most was a 5 k. w. Westinghouse converter. A machine sometimes used in connection with the converter was a ½ h. p. Edison motor. The converter was generally used as a motor though at times it was used either as a dynamo or an inverted converter.

Let us consider the case where the regulation is effected through the dynamo. On the shaft of the motor is a half ring commutator. Two brushes bearing on this commutator are connected to the terminals of an auxiliary resistance in the field circuit of the dynamo. This arrangement shunts the auxiliary resistance of the dynamo during one half of each revolution of the motor armature. This increases the field current of the dynamo and therefore the voltage generated. In parallel with these brushes is a contact operated by the tuning fork and closed during one half of each of its vibrations.

When the phase of the commutator is 90° behind the phase of the tuning fork the auxiliary resistance is shunted during three fourths of each revolution of the motor armature and the dynamo generates a definite voltage corresponding to this phase difference. Now if due to any cause whatever the speed of the motor is decreased slightly, the phase difference continually increases (and therefore the field current of the dynamo and consequently the voltage generated) until the power supplied to the motor is sufficient to keep it from falling below sychronism. Or if due to any cause whatever the speed of the motor increases slightly the phase difference continually decreases and consequently the power supplied to it. This prevents it from going above synchronism.

In the above arrangement which is one of the several used the dynamo voltage is so regulated as to keep the motor at a constant speed. When the motor is not loaded the dynamo generates a constant voltage even when its speed or the load upon it is changed.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

THE FLUORESCENCE, ABSORPTION, AND MAGNETIC ROTATION SPECTRA OF POTASSIUM VAPOR.[1]

BY T. S. CARTER.

THE red fluorescence, which is exhibited by the vapor of metallic potassium when it is illuminated with a beam of white light, was first observed by Wiedmann and Schmidt.[2]

The metal was enclosed in a glass bulb which was evacuated and sealed. The bulb was heated until the metal was vaporized, and upon concentrating a beam of arc or sunlight upon the vapor, an intensely red cone of fluorescent light was observed. Examined with a spectroscope the spectrum showed a band in the red whose position was given by the observers as 695–615.

In view of the remarkable properties which have been shown by the vapor of metallic sodium, it was thought very desirable to take up the study of potassium vapor along the same lines.

The vapor has been studied from the standpoint of its absorption, its fluorescence when excited by white light and its magnetic rotation of the plane of polarization. Photographs of all these spectra have been obtained.

One of the most interesting facts brought out by these investigations is the almost exact similarity between these spectra and the corresponding spectra which are shown in the case of sodium vapor.

The vapor was investigated in glass bulbs during the earlier experiments, but it was found that it could be studied to much greater advantage in a highly exhausted steel tube. The arrangement of apparatus was the same as that made use of in the work on sodium vapor.[3]

All the photographs were taken with a grating spectrograph containing lenses of long focal length. The exposures in the case of the fluorescence spectra were from two to three hours. About fifty lines could be measured upon the dividing engine and their wave-lengths determined by comparison with the iron arc. These lines extend from wave-length 6,300 to 6,770.

The absorption spectrum observed with the vapor in a sufficiently dense condition consists of an immense number of very fine, sharp lines. The wave-lengths of the principal lines, which extend from 6,295 to 6,860, were determined.

When the negatives of the fluorescence and absorption spectra are superimposed the principal dark emission lines of the former coincide with the light absorption bands of the latter throughout the plates. Although there are a great many more lines in the absorption spectrum.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

[2] Wied. Ann., 57, 477, 1896.

[3] The Fluorescence of Sodium Vapor, Phil. Mag., Nov., 1905, R. W. Wood.

The magnetic rotation spectrum which is shown when polarized light is passed in succession through magnetized potassium vapor and a mica prism, set originally for extinction, has also been photographed with the same apparatus. The magnetic rotation lines are very sharp. The spectrum contains about eight very strong lines which form a series, the lines being nearly equidistant in the spectrum. These lines are represented in the fluorescence spectrum and are due to the very strong absorption lines. In all about twenty-five lines were measured up.

THE MUTUAL INDUCTANCE OF COAXIAL SOLENOIDS.

By E. B. Rosa and Louis Cohen.

VARIOUS formulæ for the calculation of the mutual inductance of coaxial solenoids have been given from time to time. Although few of these formulæ are exact, several of the approximate formulæ permit inductances to be calculated with very great accuracy by using a sufficient number of terms of the series by which they are expressed. We have collected a number of these formulæ, extended some of them to improve the accuracy, and have compared and tested them by numerical calculations. The following are the more important of the formulæ:

1. Maxwell's formula (extended) for coaxial solenoids of equal length, the inner having a radius considerably smaller than the outer, the length being several times the diameter for accurate results.

2. Roiti's formula (extended) for coaxial coils, the inner being considerably shorter than the outer. This is a very convenient and accurate formula. The proof is given in the complete paper for the first time.

3. Gray's formula for coaxial coils the length being $\sqrt{3}$ times the radius for both coils.

4. Searle and Airey's formula for coaxial coils, the length of the inner being considerably shorter than the outer.

5. Cohen's formula for concentric coaxial coils in general, which is used as a test for some of the others.

The numerical examples given show what degree of accuracy is attainable, and what form of coils can be computed by the various formulæ. The full paper will shortly be published in the Bulletin of the Bureau of Standards.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society April 19–20, 1907.

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ERRATA.

PAGE 310, seventh line from bottom omit the word "*find*."

PAGE 317, eighth line from the top read "*maximum*" for "*minimum*."

PAGE 318, fourth line from top read "*curve, c*," for "*curve, b*," and *transmission curve, b*," for "*transmission curve c*"; eighth line from top and fifth line from bottom read "*curves a, b, c*," for "*curves b, c, d*." Curve *d* in Fig. 7 is curve *c* in Fig. 6.

One of the Books of
Hark! the world so loud, and these, the movers of the world, so still.
George W. Pierce